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On Simply* Compact Spaces

En Simplemente * Espacios Compactos

S. Noori and **Y. Y. Yousif**
Debarment of Mathematics, College of Education for Pure Sciences (Ibn- Al-Haitham), University of Baghdad
sarah.asas90@gmail.com and yoyayuosif@yahoo.com

ABSTRACT/ In the present paper, a simply* compact spaces was introduced it defined over simply*- open set previous knowledge and we study the relation between the simply* separation axioms and the compactness, in addition to introduce a new types of functions known as αS^{M*}_irresolte , αS^{M*}_$continuous$ and $R\,S^{M*}$_ continuous, which are defined between two topological spaces. **Keywords:** simply* compact , S^{M*}_regular , S^{M*}_$normal$, S^{M*} _ $Lindelöf$, S^{M*}_$homemorphism$. **Key words :** Breast cancer , Support Vector machine , Wisconsin Breast Cancer , Confusion Matrix, Information Gain. . RESUMEN/ En el presente trabajo, se introdujo un simple * espacios compactos definidos sobre simple * - conjunto de conocimientos previos y estudiamos la relación entre los axiomas de separación simple * y la compacidad, además de introducir un nuevo tipo de funciones conocidas como αS ^ (M *) _irresolte, αS ^ (M *) __ continuo y RS ^ (M *) _ continuo, que se definen entre dos espacios topológicos.
Palabras clave: simplemente * compacto, S^{M*} _regular, S^{M*} _normal, S^{M*} _ Lindelo ¨f, S^{M*} _homemorphism.

Introduction:
In 1969 M. K. $Singal$ and $Asha$ Mathur presented the concept of nearly compact if (every regular open cover of X has a finite sub cover)[13], which depends on the regular open set if $(S = int(cl(S)))$ was used for the first time in 1937 by M. H. Stone[14], it symbolizes by $RO(X)$. In 1985 S. N. $Masheshwari$ and S. S. Thakur presented the concept of α- compact if (for all α-open cover of X has a finite sub cover)[5], which depends on the α-open sets if $(K \subset int(cl(int(K))))$ was used for the first time in 1965 by $Njasted$ [9], it symbolizes by $\alpha O(X)$).
In 2007, the term of " simply* _compact" was used for the first time by M. El- $Sayed$[11], he was adopted in his definition of anew set is said a simply*open set, it symbolizes by $S^{M*}O(X)$), it is considered an amendment to the set simply open set the researcher A. $Neubrunnove$ presented it in 1975[8] if ($H = k \cup N$ such that K is open set and N is $nowhere\ dense$ (N is $nowhere\ dense$ if $cl(int\,N) = \emptyset$[15])), it symbolizes by $S^{M}O(X)$. He also studied the basic concepts on this set and some of the separation axioms(for example S^{M*}_regular and S^{M*}_$normal$), and simply* - connect if (a subset M of apace(X,τ) is said simply* connect relative to(X,τ) if there are no subsets E and F of X such that E and F are $S^{M*} - separated\ i.e$ (two nonempty subsets E and F in a topological space (X,τ) are said to be $S^{M*} - separated$ if $E \cap S^{M*}$cl(F)= $\emptyset = F \cap S^{M*}$cl(F)) relative to$(X,\tau)$ and $M = E \cup F$) . In 2013 M. El- $Sayed$ and I A. $Noaman$ presented a transformed definition of simply open set if (A subset O of a topological

space (X,τ) is simply open set if $int(cl(O)) \subseteq cl(int(O))$)[12]. The aim of this paper is to introduce some results on Simply* compact and present a new types of functions known as αS^{M*}_irresolte , $\alpha S^{M*}\_continuous$ and $R\,S^{M*}$_ continuous, which are defined between two topological spaces.

1.Basic concepts

Definition 1.1 :[11] A subset F of a topological space (X,τ) is said to be Simply* open set (for short, S^{M*}_open) set if $F \in \{X, \emptyset, G \cup N;\ G$ is a proper open set and N is a $nowhere\ dense$ set$\}$.

It is symbolizes by S^{M*}O(X). The complement of an simply* open set is said to be simply* closed (for short, S^{M*}_closed) set and it symbolizes is by $S^{M*}C(X)$.

Remark 1.1 :[11] The following diagram show the relationship between this species and other species:

Regular open $(RO(X)) \longrightarrow$ α-open $(\alpha O(X)) \longrightarrow$ simply open$(S^M O(X))$

$\downarrow$ $\nearrow$

Simply* open $(S^{M*}$O(X))

Diagram 1.1

Example 1.1 : Let $X = \{1,2,3,4\}$, $\tau = \{X, \emptyset, \{1\}, \{2,3\}, \{1,2,3\}\}$ then the set **{4}** $\in S^M O(X)$ but **{4}** $\notin S^{M*}$O(X) **.**

Definition 1.2 :[11] A space (X,τ) is said to be simply* compact (for short, S^{M*}_compact) if every S^{M*}_open cover of X has a finite sub cover.

Definition 1.3 :[11] A space (X,τ) is said to be S^{M*}_regular if for every $A \in S^{M*}C(X)$; $x \notin A$ then there exist $U,V \in S^{M*}O(X)$; $U \cap V = \emptyset$ such that $x \in U$ and $A \subset V$.

Definition 1.4 :[11] A space (X,τ) is said to be S^{M*}_normal if for every $U,V \in S^{M*}C(X)$; $U \cap V = \emptyset$ then there exist $H,F \in S^{M*}O(X)$ such that $U \subset H$ and $V \subset F$.

Definition1.5 :[6] A space (X,τ) is said to be a $Lindel\ddot{o}f$ space if every open cover of X has a countable sub cover.

Definition 1.6 :[11] If a function $f: (X,\tau) \longrightarrow (Y,\sigma)$ is said to be $S^{M*}\_irresulte$ if every $U \in S^{M*}O(Y)$, $f^{-1}(U) \in S^{M*}O(X)$.

Definition 1.7:[11] For any topological space (X,τ), a subset $N_x$of X is called S^{M*}_neighborhood (for short, S^{M*}_nbd) of a point $x \in X$ if there exist S^{M*}_open set W such that $x \in W \subset N_x$. The class of all S^{M*}_nbd of x is called S^{M*}_neighborhood system of x and denoted by $S^{M*}\_N_x$.

Definition 1.8 :[11] For any topological space (X,τ) and $A \subseteq X$. We define simply* interior (resp. simply* closure) of A as the following:

-$S^{M*}\_int(A) = \cup\{G \subset X : G \in S^{M*}O(X), G \subset A\}$

-$S^{M*}\_cl(A) = \cap\{F \subset X : F \in S^{M*}C(X)\ F \supset A\}$

Theorem 1.1 :[11] A S^{M*}_closed subset of $S^{M*}\_compact$ space (X,τ) is $S^{M*}\_compact$.

Theorem 1.2 :[11] If a function $f: (X,\tau) \longrightarrow (Y,\sigma)$ is $S^{M*}\_irresulte$ $surjectiv$ and X is $S^{M*}\_compact$ then Y is $S^{M*}\_compact$.

2.Main results

Definition 2.1 : A space (X,τ) is said to be $S^{M*}T_1$ space if for any two distinct points x and y of X, there exist $U,V \in S^{M*}O(X)$;such that $(x \in U \wedge y \notin U)$ and $(x \notin V \wedge y \in V)$.

Definition 2.2 : A space (X,τ) is said to be $S^{M*}T_2\_\ space$ if for all $x,y \in X;\ x \neq y$ then there exist $U,V \in S^{M*}O(X)$ such that $(x \in U \wedge y \in V)$ or $(x \in V \wedge y \in U)$and $U \cap V = \emptyset$.

Theorem 2.1 : Any $S^{M*}\_compact$ space of a $S^{M*}T_2$_ space is S^{M*}_closed.

Proof: Let A be a S^{M*}_compact subset of a $S^{M*}\_T_2$ space(X,τ),and suppose $x \in X - A$. We must find a S^{M*}_neighborhood of x which does not meet A (thus showing that $X - A$ is S^{M*}_open), since X is $S^{M*}\_T_2$ space, given any $y \in A$, there are S^{M*}_neighborhoods U_y and V_y of x and y, respectively such that $U_y \cap V_y = \emptyset$. Then $\{V_y\}, y \in A$ is an S^{M*}_open cover of A. Hence there are finitely many y say $y_1, y_2, \dots, y_n$ such that $\{V_{y_1}, \dots, V_{y_n}\}$ is an S^{M*}_open cover of A. Set $U = U_{y_1} \cap \dots \cap U_{y_n}$ and $V = V_{y_1} \cup \dots \cup V_{y_n}$ then $x \in U, A \subset V$ and $U \cap V = \emptyset$ and hence U is S^{M*}_neighborhood of x such that $U \cap A = \emptyset$. Then A is S^{M*}_closed .

Definition 2.3 : A space (X,τ) is said to be $S^{M*}T_3$_space if X is both S^{M*}_regular and $S^{M*}T_1$ space.

Definition 2.4 : A space (X,τ) is said to be $S^{M*}T_4$_space if X is both $S^{M*}\_\ normal$ and $S^{M*}T_1$ space.

Remark 2.1 : The following diagram show the relation between the types of Separation Axioms

$S^{M*}T_4$_space $\longrightarrow S^{M*}T_3$_space $\longrightarrow S^{M*}T_2$_ space $\longrightarrow S^{M*}T_1$ space

Diagram 2.1

Combining Theorem(1.1) and (2.1) we obtain the following.

Corollary 2.1 : A subset of a $S^{M*}\ \_compact$ and $S^{M*}T_2$_ space is $S^{M*}\ \_compact$ if and only if it is $S^{M*}\ \_closed$.

Corollary 2.2 : A $S^{M*}\ \_\ compact$ and $S^{M*}T_2$_ space is $S^{M*}T_3$ _space.

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Proof: Let $A \subset X$ by Theorem(2.1) A is S^{M*} _closed, since there exist $U,V \in S^{M*}O(X)$ such that $x \in U, A \subset V$ and $U \cap V = \emptyset$ (by proof Theorem(2.1)), hence X is S^{M*}_regular. Since X is $S^{M*}T_2$_ space, by Diagram (2.1) X is $S^{M*}T_1$ space. Hence X is $S^{M*}T_3$_space .

Definition 2.5 : A space (X,τ) is said to be a S^{M*} _ *Lindelöf* space if every S^{M*} _ *open* cover of X has a countable sub cover.

Remark 2.2 : Any S^{M*} _ *compact* is S^{M*} _ *Lindelöf*.

Proposition 2.1 : A S^{M*}_regular S^{M*} _ *Lindelöf* is S^{M*}_ *normal*.

Proof: Let (X,τ) be S^{M*}_regular *Lindelöf* space and let A,B be disjoint S^{M*}_closed subsets of X. If $x \in A$,then $X - B$ is a S^{M*}_neighborhood of x. Since X is S^{M*}_regular, there is a neighborhood U_x of x such that $S^{M*}Cl\,U_x \subset X - B$(a space (X,τ) is S^{M*}_regular if and only if given any $x \in X$ and any S^{M*}_neighborhood U of x, there is a S^{M*}_neighborhod V of x such that $S^{M*}ClV \subset U$).

Similarly, if $x \in B$, there is a S^{M*}_neighborhood U_x of x such that $S^{M*}Cl\,U_x \subset X - A$.

If x is not an element of either A or B, then $X - (A \cup B)$ is a S^{M*}_neighborhood of x; hence we may find a S^{M*}_neighborhood U_x of x such that $S^{M*}Cl\,U_x \subset X - (A \cup B)$(and thus $S^{M*}Cl\,U_x \cap (A \cup B) = \phi$).

The family of U_x for each $x \in X$ is an open cover for X. Since X is S^{M*}_*Lindelöf* , this S^{M*}_cover has a countable sub cover$\{U_{xn};\ n = 1,2,3,\ldots\}$. Let U_1, U_2, … be the U_{xn}(relabeled for convenience) which meet A, and let V_1, V_2, … be the V_{xn} which meet B.

Then for each positive integer n , $S^{M*}Cl\,U_n \cap B = \phi$ and $S^{M*}Cl\,V_n \cap A = \phi$; moreover $A \subset \cup_N U_n$ and $B \subset \cup_N V_n$. Define $W_1 = U_1$ and set $Y_1 = V_1 - S^{M*}Cl\,W_1$.

Let $W_2 = U_2 - S^{M*}Cl\,Y_1$ and $Y_2 = V_2 - (S^{M*}Cl\,W_1 \cup S^{M*}Cl\,W_2)$. Suppose W_n and Y_n have been defined, then set $W_{n+1} = U_{n+1} - S^{M*}(ClY_1 \cup ClY_2 \cup \ldots \cup ClY_n)$ and

$Y_{n+1} = V_{n+1} - S^{M*}(Cl\,W_1 \cup Cl\,W_2 \cup \ldots \cup Cl\,W_{n+1}), W_n$ is always an open set since

$$W_n = U_n \cap (X - S^{M*}(ClY_1 \cup ClY_2 \cup \ldots \cup ClY_{n-1}))$$
$$= U_n \cap (X - S^{M*}Cl(Y_1 \cup Y_2 \cup \ldots \cup Y_{n-1})) \quad ;$$

hence W_n is the intersection of two S^{M*}_open sets, and is therefore S^{M*}_open .

Similar reasoning shows that Y_n is S^{M*}_open for each n. Set $H = \cup_N W_n$ and $K = \cup_N Y_n$, since H and K are the union of S^{M*}_open sets , then are S^{M*}_open. Suppose $a \in A$, then $a \in U_n$ for some n, and

$W_n = U_n - S^{M*}(ClY_1 \cup ClY_2 \cup \ldots \cup ClY_{n-1})$. But for any k, $S^{M*}Cl\,Y_k \subset S^{M*}Cl\,V_k$ and $S^{M*}Cl\,V_k \cap A = \phi$. Therefore $a \notin S^{M*}ClY_k$ for any k. We have then that $a \in W_n$. Therefore $A \subset \cup_N W_n = H$.

Similarly B ⊂ K. In order to show that X is S^{M*}_normal, we now have merely to prove $H \cap K = \phi$. Suppose that $x \in H \cap K$, then $x \in W_n \cap Y_m$ for some m and n. Suppose $m \geq n$.Then $x \in Y_m = V_m - S^{M*}(Cl\,W_1 \cup \ldots \cup ClW_n \cup ClW_m)$; hence x could not be in ClW_n, a contradiction.

On the other hand $m < n$, then $x \in W_n = U_n - S^{M*}(ClY_1 \cup \ldots \cup ClY_m \cup \ldots \cup ClY_{n-1})$. Thus $x \notin S^{M*}ClY_m$, again a contradiction .

There for H and K are disjoint S^{M*}_open subsets of X, which contain A and B, respectively, and hence X is S^{M*}_normal.

On the other hand, by Remark 2.2 and Proposition 2.1 we have the following stronger result.

Corollary 2.3: A S^{M*} _ compact $S^{M*}T_2$_ space is $S^{M*}T_4$_space.

Proof: Since X is S^{M*} _ compact then X is S^{M*} _ *Lindelöf* (by Remark 2.2), and any S^{M*}_ compact, $S^{M*}T_2$_ space is $S^{M*}T_3$_space (by Corollary 2.2),then X is $S^{M*}T_3$_space , hence X is S^{M*}_regular. Then X is S^{M*}_ *normal* (by Proposition 2.1**)**. Hence X is $S^{M*}T_4$_space.

Definition 2.6 : Let (X,τ) and (Y,σ) be two topological spaces, then X is said to be S^{M*}_*homemorphic* to Y iff there exist a S^{M*}_*homemorphism* function (f is *bij* , f is S^{M*}_*irresulte* , f^{-1} is S^{M*}_*irresulte*) from (X,τ) onto (Y,σ) and we denoted for that $X \cong^{M*} Y$.

Since any S^{M*}_*homemorphism* is S^{M*}_*irresulte* (Theorem 1.2:[11]) we have the following

Corollary 2.4 : If (X,τ) is S^{M*}_*compact* then any space S^{M*}_*homemorphic* to X is S^{M*}_*compact*.

Proposition 2.2 :Let f be a S^{M*}_*irresulte* one-one function from a S^{M*}_compact space (X,τ) on to a $S^{M*}T_2$_ space (Y,σ). Then f is a S^{M*}_*homemorphism*.

Proof: We must show that f^{-1} is S^{M*}_*irresulte*. We use (Proposition8 Chapter4[6]**)** Let (X,τ) and (Y,σ) be two topological spaces, then a function from X to Y is continuous if and only if given any closed subset F of Y, f^{-1}(F) is a closed subset of X.).Suppose F is any S^{M*}_cloced subset of X. Since F is S^{M*}_ closed, F is S^{M*}_compact (Theorem 1.1[11]); hence f(F) is a S^{M*}_compact**(** Theorem 1.2[11]). Then f(F) is a S^{M*}_ compact subset of a $S^{M*}T_2$_ space and is therefor S^{M*}_closed(Theorem 2.1). But $f(F) = (f^{-1)})^{-1}(F)$. We have therefore shown that if F is any S^{M*}_ closed subset of X, $(f^{-1})^{-1}$(F) is a S^{M*}_$closed$ subset of Y. therefore f^{-1} is S^{M*}_$irresulte$; hence f is S^{M*}_$homemorphism$.

Definition 2.7 : A function $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be αS^{M*}_irresolte if for every $U \in S^{M*}O$(Y) , $f^{-1}(U)$ is α_open set in X.

Corollary 2.5 : A α S^{M*}_ irresolute surjection image of α_compact space is S^{M*}_compact space .

Proof: Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be an αS^{M*}_irresolute and surjection function, let $\{Gi, i \in I\}$ be an S^{M*}_ _open subset cover of Y. Then $f^{-1}(Gi), i \in I\}$ be an α_open sub set cover of X, since X is α_compact , then there exist finite sub set I_0 of I such that $X = \cup f^{-1}(Gi), i \in I_0\}$ and hence $Y = f(X) = \cup \{f(f^{-1})(Gi)), i \in I_0\} \subset \cup \{Gi, i \in I_0\}$ which is an S^{M*}__open cover of Y. Thus Y is S^{M*}_compact.

Theorem 2.2 : The image of an α_compact subset under on αS^{M*}_$irresulte$ function is S^{M*}_compact.

Proof : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be an αS^{M*}_irresolute and surjection function, let A be α_compact subset of X, then let $\{Ui, i \in I\}$ be S^{M*}_open set cover of f(A) such that f(A) ⊂ $\cup\{U_i;\ i \in I\}$. Thus A ⊂ $f^{-1}(f(A)) \subset f^{-1}(\cup\{U_i;\ i \in I\}) \subset \cup\{f^{-1}(U_i);\ i \in I\}$, A ⊂ $\cup\{f^{-1}(U_i);\ i \in I\}$. Since f is αS^{M*}_irresolute, then $\{f^{-1}(U_i);\ i \in I\} \in \alpha O(X)$. Since A is α_$compact$ and $\{f^{-1}(U_i);\ i \in I\}$ is an α_open cover of A. Then there exists a finite sub set I_0 of I such that A⊂$\cup\{f^{-1}(U_i);\ i \in I_0\}$, this implies that f(A) ⊂ $\cup\{ff^{-1}(U_i);\ i \in I_0\} \subset \cup\{U_i;\ i \in I_0\}$. Then f(A) is S^{M*}_$compact$.

Definition 2.8 : A function $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be αS^{M*}_continuous if for every $U \in \alpha O$(Y) , $f^{-1}(U)$ is S^{M*}_open set in X.

Theorem 2.3 : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be αS^{M*}_continuous and $surjective$ function, if X is S^{M*}_compact then Y is α_compact.

Proof : Let$\{V_\alpha;\ \alpha \in I\}$ is a α_open set cover of Y, since f is αS^{M*}_continuous then $\{f^{-1}(V_\alpha);\ \alpha \in I\}$ be S^{M*}_open set cover of X. Since X is S^{M*}_compact then there exists a finite sub set I_0 of I such that $X = \cup\{f^{-1}(V_\alpha);\ \alpha \in I_0\}$, since f is $surjective$, then $f(x) = Y = \cup\{V_\alpha;\ \alpha \in I_0\}$. Then Y is α_compact.

Definition 2.9 : A function $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be $R\,S^{M*}$_ continuous if for every $V \in RO(Y)$, $f^{-1}(V) \in S^{M*}O(X)$.

Theorem 2.4 : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be $R\,S^{M*}$_ continuous and $surjective$ function, if X is S^{M*}_compact then Y is nearly compact.

Proof : Let$\{V_\alpha;\ \alpha \in I\}$ is regular _open cover of Y, since f is NS , then $\{f^{-1}(V_\alpha);\ \alpha \in I\}$ be S^{M*}_open set cover of X , since X is S^{M*}_compact. Then there exists finite sub set I_0 of I such that $X = \cup\{f^{-1}(V_\alpha);\ \alpha \in I_0\}$, since f is $surjective$, then $f(x) = Y = \cup\{V_\alpha;\ \alpha \in I_0\}$, then Y is nearly compact.

Definition 2.10 : A space (X, τ) is called S^{M*}_locally indiscrete space if every S^{M*}_open subset of X is S^{M*}_closed.

Corollary 2.6 : For a topological space (X, τ) these phrases will be equal:

(1) X is S^{M*}_$compact$.

(2) X is a S^{M*} _C_3 space, i.e. ∀ hS^{M*}_closed set is finite.

Remark : A subset B of a topological space (X, τ) is called S^{M*}_closed, if every subset of B is also S^{M*}_closed in (X, τ), then B is called hereditarily S^{M*}_closed (hS^{M*}_closed).

Corollary 2.7 : Let O a subset of a topological space (X, τ) these phrases will be equal :

(1) O is hS^{M*}_closed.

(2) $N(X) \cap Int(cl(A)) = \emptyset$, where $N(X)$ point out the set of nowhere dense singletons in X.

Corollary 2.8 : Let (X, τ) by any space and let (Y, σ) by S^{M*}_indiscrete. Let $A \subseteq X \times Y$ and let P: $X \times Y \longrightarrow X$ denote the projection. Then $int(cl(A)) = int\left(cl(p(A))\right) \times Y$.

Proposition 2.3: Let $(X_\alpha, \tau_\alpha)_{\alpha \in \Omega}$ be a family of pairwise disjoint topological spaces. For the topological sum $X = \sum_{\alpha \in \Omega} X_\alpha$ the following conditions are equivalent:

(1) X is a S^{M*}_compact space.

(2) Each X_α is a S^{M*}_compact space and $|\Omega| < \aleph_0$.

Theorem 2.6 : If (X, τ) is S^{M*}_compact and (Y, σ) is finite and S^{M*}_locally indiscrete then $X \times Y$ is S^{M*}_compact.

Proof : Since Y is a finite topological sum of indiscrete spaces**,** by(Proposition 2.3) it suffices to assume that Y is S^{M*}_indiscrete. Suppose that $A \subseteq X \times Y$ is infinite and hS^{M*}_closed. Then $p(A)$ is infinite and hence, by Corollary 2.6 and Corollary 2.7 we have $N(X) \cap Int(cl(p(A))) \neq \emptyset$. Pick $x \in N(X) \cap Int(cl(p(A)))$ and $y \in Y$. Then $\{(x, y)\}$ is nowhere dense in $X \times Y$ and by Corollary 2.8, we have $((x, y) \in int(cl(A))$, a contradiction to the hS^{M*}_$closednees$ of A. Thus $X \times Y$ is S^{M*}_compact.

Conclusion: The simply* compact spaces was introduced which are defined over simply*-open set**.** In addition relation between the simply* separation axioms and the compactness were studied. It can be concluded that some of the theories to study stacking more broadly is presented and it is necessary to provide a lot of other theoretical research to study the topic fully.

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