International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 10, 2020
ISSN: 1475-7192

Hybridization Methodology of ARMA-FIGARCH Model to Examine Gasoline Data in Iraq

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Abstract
The reason beyond this research is to investigate the long memory with volatility impact embedded in the daily fuel prices (Gasoline) via time-series behavior. Gasoline returns are assumed to follow ARMA-FIGARCH models. Here, a hybrid methodology is introduced in two main steps. Firstly, the results of the estimation of mean value have been achieved by using ARMA models, while the second step is to estimate the conditional variance value by using FIGARCH models. Among these formulating, the final step will be done by combining the previous two steps that yield ARMA-FIGARCH. Particularly, AR(2)-FIGARCH (1,d,2) model will be yielded under the normal distributional assumption of residuals, which indicated a better fit for price volatility of gasoline. Non-Gaussian residuals are also assumed by using student-t distribution. Moreover, AR(2)-FIGARCH (1,d,2) had been selected significantly for daily returns and was preferred due to its success in passing the goodness-of-test fit.

Keywords: hybrid model, ARMA-FIGARCH, fluctuations, Gaussian and No-Gaussian residuals, Quasi-maximum likelihood, Gasoline data.

1. Introduction

Crude oil is regarded as the most actively traded commodity followed by its various derivatives such as white oil (heating oil), Gasoline and benzene…etc. This is not surprising since crude oil and its derivatives are important factors of production in the world's economies, and oil prices fluctuations can significantly affect their performance. The dynamics of energy markets are similar to those of financial markets. The markets for oil products and many other commodities are characterized by high levels of volatility. Analyzing the volatility behavior of fuel prices has implications for both traders and market participants.

The classical methods that calculate the volatility are referred to be unconditional and cannot hold the features shown through data analysis's results. Many types of models can be listed according to this point of view. Engle (1982) initiated Autoregressive Conditional Heteroscedasticity (ARCH) model is a good example for this overview. ARCH model aims mainly to estimate the conditional variance of the financial data. Other related model named Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model upgraded by Bollerslev (1986), which considered as the most frequently used models holding the previous properties from financial data. The purpose of this generalization is to describe the conditional variance by its own lagged values and the square of the lagging values of the innovations or shocks values.

Tayefi, M., & Ramanathan (2012) reviews the theoretical and applicable point of views concerning (FIGARCH) models, to indicate the description of the volatility behavior of time series data. The long memory feature of FIGARCH models permits to act better competentnt than other models dealt with conditional heteroscedastic for modeling volatility.

Some hybridization methods have been attempted to capture changeable characteristics that indicate the modern finance phenomena. Wiphatthananthakul, C., & Sriboonchitta, S. (2010) have estimated ARFIMA-FIGARCH and ARFIMA-FIAPARCH which can capture long memory besides asymmetry associated with the conditional variance. Their application was about Thailand Volatility Index.

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Adejumo & Suleiman (2017) implemented a mixed model combining (ARMA) and (GARCH) models on the solar radiation series data for a certain meteorological station in Nigeria.

This research investigates the hybridization methodology ARMA-FIGARCH model in data analyzing of Gasoline prices. Moreover, after selecting the appropriate model, we take into account the forecasting achievement measured by (RMSE, MAE, and MAPE) to select the best FIGARCH. Both normal and student-t innovations were used via goodness-of test fit to select the best model.

2. Materials and Methods

2.1 Mean Model (Foundation of ARMA)

The general statistical (ARMA) model describes a time series that develops through time. The values at a specific time point have linearity effect associated with past values, noise too. In linear models, the time series return combining AR and MA, which produces (ARMA) model. We firstly use AR, MA, and ARMA model to identify the best fitted model in modeling conditional mean to control for autocorrelation in the data (Adejumo & Suleiman 2017) (Dong 2012)

2.1.1 Autoregressive (AR) Model

Autoregressive processes are as their name suggests regressions on themselves, i.e., a time series is explained by its lagged values. Specifically, a p-th order autoregressive processY_t denoted by $AR(p)$satisfies the equation;

$$Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + u_t \quad \text{...(1)}$$

$$= \delta + \sum_{i=1}^{p} \phi_i Y_{t-i} + u_t$$

In equation (1), Y_t denotes a time series, u_t denotes error term. The series Y_t which represents the present value can be expressed as a linear combination of the p most new past values of itself adding an "innovation" component u_t that includes all new in the series at time t which is not explained by the previous values. So, we impose that u_t is independent of $Y_{t-1} + Y_{t-2} + \dots + Y_{t-p}$ for every t (Cryer and Chan 2008).

2.1.2 Moving Average (MA) Model

The moving-average model is a linear function of the lag values of error term and un-predictable error term. Specifically, a q-th order moving-average process Y_t denoted by $MA(q)$satisfies the equation;

$$Y_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \dots - \theta_q u_{t-q} \quad \text{...(2)}$$

$$= u_t - \sum_{j=1}^{q} \theta_j u_{t-j}$$

So, a moving average process has constant mean, constant variance, and auto-covariance which may be non-zero to lag q and will always be zero thereafter. (Tsay 2002)(Cryer and Chan 2008)(Brooks 2008).

2.1.3 Autoregressive Moving Average (ARMA) Model

By assembling the AR(p) with MA(q) models, we will get ARMA(p, q) model. This model assumes that the present value of a certain series y has linear dependency on its latest values in addition the combination of present and latest values of a white noise error component (Brooks 2008). A general ARMA(p, q) model could be written:

$$Y_t = \phi_o + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + u_t - \theta_1 u_{t-1} - \dots - \theta_q u_{t-q}$$

$$= \phi_o + \sum_{i=1}^{p} \phi_i y_{t-i} + u_t - \sum_{j=1}^{q} \theta_j y_{t-j} \quad \text{...(3)}$$

Where; ϕ_o is constant, ϕ_i are the parameters of the autoregressive component of order p, and θ_j are the parameters of the moving average component of order q. the $u_t \sim$ i.i.d. $N(0, \sigma^2)$ is called a white noise process or error at time t. Where Y_t is a time series be modeled. Y_t is said to be a mixture (ARMA) of orders p and q, respectively; we can introduce a reduced formula to ARMA (p, q), with p and q are non-negative integers (Cryer and Chan 2008) (Tsay 2020).

2.2 Volatility Model (FIGARCH) Foundation

For an ARCH model has the process $\{ \varepsilon_t \}$ with $\varepsilon_t = \sigma_t z_t$ Where $E_{t-1}(Z_t) = 0$ and $var_{t-1}(Z_t) = 1$, and σ_t is finite measurable regarding to the time t-1 dataset. We can represent the mean equation and conditional variance of the ARCH(p) model linearly interms of the past squared values: mean equation $r_t = \mu + \varepsilon_t$, $\varepsilon_t \backsim N(0, \sigma_t^2)$

$$\varepsilon_t = \sigma_t z_t \quad , \quad z_t \backsim iid(0, 1)$$

variance equation $$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$$

$$\sigma_t^2 = \alpha_\circ + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 \qquad \dots(4)$$

Where $\alpha_\circ > 0$ $and\, \alpha_1, \dots, \alpha_q > 0$and r_t is return at time t (stationary time series), then $r_t$can be expressed as its mean plus a white noise if there is no significant autocorrelation in $r_t$itself.μis the mean of r_t. $\varepsilon_t$is the innovation (residual returns). $\sigma_t^2$is the conditional variance of the innovations (errors) at time t. Since ε_t has a zero mean, $\mathrm{var}(\varepsilon_t) = E(\varepsilon_t^2) = \sigma_t^2$. $z_t$is an i.i.d variable such that $z_t \backsim D$ where D is some distribution with mean zero and unit variance. In our case D will be the normal or Student's-t distribution. GARCH model has an (ARMA) form for the conditional variance σ_t^2 which is expressed as a function of past squared innovations and its own lags.

The GARCH(p,q) formulation, where the present conditional variance is parameterized to depend upon p lags of the squared error and q lags of the conditional variance. Then the GARCH (p,q) model can be written as follows (Brooks 2008) (Bollerslev 1986);

$$\text{mean equation} \quad r_t = \mu + \varepsilon_t \quad , \quad \varepsilon_t \backsim N(0, \sigma_t^2)$$

$$\varepsilon_t = \sigma_t z_t \quad , \quad z_t \backsim \mathrm{iid}(0, 1)$$

$$\text{variance equation} \quad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \alpha_\circ + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_q \sigma_{t-q}^2 \qquad \dots(5)$$

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

The conditional variance formulation can be determined as a function of three components:
ω: A constant term.
ε_{t-i}^2: News about volatility from the previous period, calculated as the lag of the squared residuals from the mean equation (the ARCH term).
σ_{t-j}^2: Last period forecast variance: (the GARCH term).
where; $\omega > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ to ensure the positive conditional variance. r_t = return of the series at time t. μ is the average return. $\varepsilon_t$is the residual returns.
The common assumption for the noise is that $z_t$is standardized residual returns. The volatility process $\{\sigma_t\}$ is a non-negative stochastic process such that $z_t$and σ_t are not dependent when fixing for a fixed t, where, p is the number of lagged of ε^2terms and q is the number of lagged σ^2 terms. The GARCH (p, q) model is strictly stationary with finite variance when the conditions ω> 0, and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ are required (Ding , Granger and Engle 1993) (Dong 2012).
$\alpha(L)$ and $\beta(L)$ are the lag polynomials with orders of q and p respectively. That is, by joining the parameter p to the ARCH model, longer lags can be expected in the GARCH model with low orders. The GARCH(p,q) can be expressed in terms of ARMA in ε_t^2. (Bentes 2014)(Nakatsuma and Tsuruma) (Engle 1982).

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]\upsilon_t \qquad \dots(6)$$

Where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$. The $\{v_t\}$ process can be considered as the innovations for the conditional variance. The essential assumption here is that all the roots of the polynomial $[1 - \beta(L)]$ lie outside the unit circle to ensure conditional variance to be nonnegative. Taking under consideration a unit root in the autoregressive polynomial $[1 - \alpha(L) - \beta(L)]$ introduced the IGARCH model, in other words, $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j \approx 1$. The IGARCH(p,q) is given by (Tayefi Ramanathan 2016)(Billie and Morana 2009)

$$\emptyset(L)[1 - L]\varepsilon_t^2 = \omega + [1 - \beta(L)]\upsilon_t$$

Where

$$\emptyset(L) = [1 - \alpha(L) - \beta(L)][(1 - L)^{-1}] \qquad \dots(7)$$

In the IGARCH process, current information is still significant for the forecast of the conditional variance for all horizons.
To capture long-range dependence in volatility, extended the IGARCH model to the FIGARCH model. A FIGARCH process of order (p,d,q) is defined by

$$\emptyset(L)(1-L)^d\varepsilon_t^2 = \omega + [1-\beta(L)]\upsilon_t$$

where is the parameter d is permitted to take the real interval (0 , 1) and

$$\emptyset(L) = 1 - \emptyset L - \Lambda - \emptyset_q L^q$$

and,

$$\beta(L) = \beta_1 L + \Lambda + \beta_q L^q$$

All the roots of $\emptyset(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The existence of shocks (or sudden effects) to the conditional variance, or the degree of long-term dependencies is computed by the fractional differencing parameter d. Concerning GARCH(p,q) model for d=0 and IGARCH(p,q) model for d=1, we can notice that the FIGARCH process has GARCH and IGARCH processes as special cases. (Baillie , Bollerslev and Mikkelsn 1996)
Changing υ_t in the above equation, and subsequently, the equation becomes:

$$[1-\beta(L)]\,\sigma_t^2 = [1-\beta(L) - \emptyset(L)(1-L)^d]\,\varepsilon_t^2$$

The variance equation then specified as:

$$\sigma_t^2 = \omega[1-\beta(1)]^{-1} + \eta(L)\varepsilon_t^2$$

$$\eta(L) = [1 - [1-\beta(1)]^{-1}\ \emptyset(L)(1-L)^d]$$

$$= \eta_1 L + \eta_2 L + \dots and\ \ \eta_k \geq 0\quad k = 1,2,\dots,n$$

The parameter (d) produces important information about the style and amount with which shocks take place to the volatility process. For values of $d > 1$, the conditional variance σ^2 becomes explosive, and impulsive response becomes undefined.
So, FIGARCH is regarded as more flexible version of long memory GARCH Models. ARMA representation is extended to the FIGARCH model of squared residuals, which results from the GARCH model, to a fractionally integrated model. However, to ensure that FIGARCH model is stationary and the conditional variance σ_t^2 is permanently positive, frequently complicated and ungainly limitations have to be supposed on the model coefficients (Shimizu 2010) (Tayefi and Ramanathan 2016) (Efimova 2013).

2.3 Maximum Likelihood Estimation

The parameters of ARMA-FIGARCH model can be estimated by the quasi- maximum likelihood (QMLE) method which is used with many assumptions according to the distribution of the errors (Bollerslev and Wooldridge 1992) (Baillie, Bollerslev and Mikkelsen 1996).

3. Results and Discussion

3.1 Financial Data Analysis and Results

A real financial data concern to Gasoline daily prices were can be accessed through this link. The data contain (2920) observations that cover the period from 01/ 01/ 2012 to 31/ 12/ 2019. Computer programs were used to obtain numerical results through **E-views version9**package and **R** programming language.
Simply, we calculate the returns of the daily time series from day to day to get a fair adjustment of the non-random effects. Therefore, the currency Gasoline prices are transformed into daily returns through the first difference of natural logarithm. This can be noticed in Figure (3.3) with squared ln-returns series for the Gasoline prices. Then the daily datasets are transformed into ln-returns (r_t), with P_t denoting the daily Gasoline prices series recorded at time t, by applying the following mathematical formula:

$$r_t = (\ln P_t - \ln P_{t-1}) \text{ for first difference}$$

Returns Series of White Oil = Diff(ln(White Oil))

Summary Descriptive Statistics and Normality Test are shown in Table (1) below.

Table 1: Descriptive Statistics and Normality Tests for Gasoline Prices Return Series

Series / Statistic	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	p-value of JB
	4.49e-05	0.0000	0.1335	-0.1708	0.0146	-0.6381	38.6315	154613.5	0.0000

From above Table, we notice that datasets are extremely volatile. The data exhibits both positive and negative spikes / jumps. The mean and median of daily returns does not differ from zero significantly. It suggests that returns fuel series in general decrease somewhat through time. The measures of skewness for the fuel returns series is 0.6381, not zero which means all the returns series is asymmetric and skewed to the left (negatively skewed). On the other hand all returns series exhibit positive excess kurtosis, 38.6315 is more than three, indicating the leptokurtic characteristic of the fuel daily returns distributions, which mean all return have the fat-tail characteristic, greater peak at the mean than normal distribution, indicating the necessity of using the fat-tailed distributions to describe these variables. According to the Jarque-Beratests, p-values are less than 0.05, and then we reject the null hypothesis of normality at 5% for returns series, so the distributions of the fuel returns are not normal distribution. In another speech, returns series distributions have significantly fatter tails than the normal distribution. This is a natural feature of financial time series.

We plot the data in terms of original observations to show their visualization. With Graphical representation, one may investigate the stationarity of time series. We begin with plotting the daily white oil price series.

Figure (1) illustrates the original daily Gasoline prices series. The observed data show that there are periods with higher fluctuations, followed by periods with lower movements. The data shows non-stationary, with sudden spikes and jumps, i.e., their means and variances are changing with time, the volatility seems to change over time as well, indicating heteroscedasticity. But just looking at the time series graph is not enough to know how non- stationary the series is, so we have to use Ljung-Box tests, correlogram, and the unit root tests for data series.

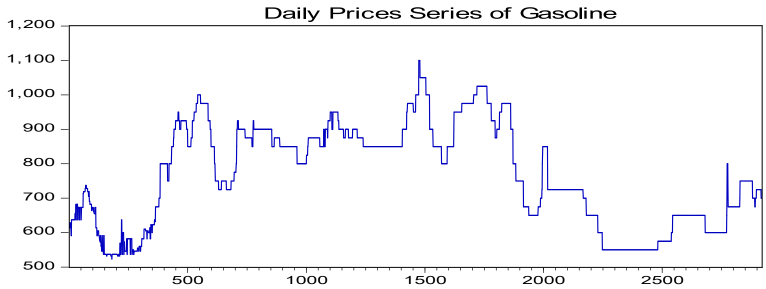

Figure 1: The Scatter Plots of the Daily Gasoline Price Series

The Figure (1) displays the mean returns to be constant while the variances fluctuate across the time about some normal level. It seemed that the volatility form clusters. It can be indicated that the high peak volatility periods can be noticed from the low bottom volatility periods. The presence of spikes and volatility clustering can be seen obviously.

3.2 Unit Root Tests for Returns Series (Stationary)

The unit root tests results for fuel returns series are shown in Table (2). This Table displays the results of unit root tests using the ADF and PP tests at level with p-values and critical values for returns series of fuel prices. The null hypothesis of unit roots can be rejected to all returns series at 5% level of significance.

Table 2: The Results of Unit Root Tests for Daily Gasoline Returns Series

Unit Root Test of White Oil Returns Series				
Null Hypothesis H_o: Gasoline Returns Series has a unit root (Not Stationary)				
Test Statistic	Type of Model	5% Critical Value	Value of Test of Statistics	p-value
Augmented Dickey Fuller (ADF)	Intercept	-2.8623	-19.5220	0.0000
	Trend and Intercept	-3.4114	-19.5337	0.0000
	None	-1.9409	-19.5239	0.0000
Phillips-Perron (PP)	Intercept	-2.8623	-52.2072	0.0001
	Trend and Intercept	-3.4114	-52.2080	0.0000
	None	-1.9409	-52.2164	0.0001

According to the results in Table (2), we investigate the stationary of the returns series, the p-values are less than 5%. Therefore, we reject the null hypothesis of "series have unit root" and conclude that all returns series are stationary. For this reason, we use the returns series in the subsequent analysis.

3.3 Estimation mean model

We can construct suitable linear ARMA(p, q) models using the daily returns series of fuel prices because they are stationary at level 5%. Many ARMA versions are fitted to the returns series and the standardized residuals analyzed. By observing the autocorrelation (ACF) and partial autocorrelation functions (PACF), the rough p and q can be obtained, and taking into account the significant of parameters, the more appropriate p and q will be picked up, to select the best fitted linear ARMA (p, q) models, by using different orders for fuel daily return series, chosen the optimal model among the competentnt models after several attempts, taking into account ARCH effect and serial correlation. It was found that the model ARMA(2, 0) without a constant are best models among several combinations of parameters p and q of ARMA(p, q) for Gasoline return series. Figure (2) observed the results of appropriate estimated linear ARMA(p, q) Models and graph comparison among residuals actual and fitted series of ARMA(p, q) Models for daily Gasoline returns series.

ARMA(2, 0) Model of Gasoline Returns

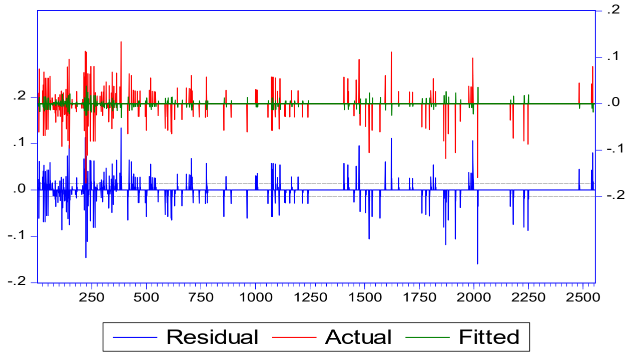

Figure (2)

3.4 Residuals Diagnostics of ARMA(p, q)

The diagnostics stage includes residuals analysis of estimated model. Now we want to test whether the heteroskedaticity (ARCH effect) and serial correlation problems are exist or not, These contain the results of the ARCH-LM tests, Ljung-Box tests, on Residuals of ARMA(p, q) models for the daily returns of fuel series.

Table 3: Results of the ARCH-LM tests, Ljung-Box tests Tests on Residuals of ARMA(p, q) Models for the Daily Returns of Gasoline Series

ARMA(2,0)					
Ljung-Box test			Arch-LM test		
Q-statistic	Lag	p-value	Q-statistic	Lag	p-value
122.9087	5	0.0003	291.3228	5	0.0000
148.9232	10	0.0000	501.3321	10	0.0000
149.4382	15	0.0000	522.1873	15	0.0000
151.4830	20	0.0002	676.5124	20	0.0001
190.3289	25	0.0001	733.0839	25	0.0000
Reject null hypothesis because : p-value < 0.05					

The results of p-value obtained at 5% significance level for the above table (*),are smaller than 0.05, which justify the null hypothesis rejection at 25thlag for the return series, i.e., which means residuals of ARMA(p, q) models of Gasoline returns have serial correlation and ARCH effect.

3.5 Estimating the Volatility Model

Concerning volatility models, it is used the analysis of the residual of the average model on the return of Gasoline data. It is necessary to investigate the impact of long memory before the estimation of the volatility models. The main method to estimate the parameter d is named GPH technique where can be seen in Table 1 are the results.

The following hypothesis confirms the long memory presence:

$$H_\circ: d = 0$$
$$H_1: d \neq 0$$

Calculating z-statistic is obtained with $\alpha = 0.05$ significance level

Gasoline	$\widehat{d}$	$SE(\widehat{d})$	Confidence of Interval	z	Long memory effect
	0.0272	0.0119	-0.3131 < d < 0.1327	3.1810	Significant

From above table we obtained the values of z are greater than $\boldsymbol{z_{1-0.05/2} = 1.96}$, can be noticed as significant, which means that there is a long memory effect indeed among Gasoline return dataset. We can indicate the confident interval to be within -0.5 < d < 0.5 .

Once the presence of ARCH effects is confirmed, then the optimal Lag of FIGARCH model has be determined before the construction of the final model. Various FIGARCH models for the Gasoline returns series, four models were considered; FIGARCH (1,d,1) model , FIGARCH (2,d,1), FIGARCH(1,d,2) model , FIGARCH (2,d,2) model under different error terms distributions assumption for residuals were chosen as competentnt (appropriate) model to represent the price volatility of Gasoline dataset. Furthermore, we consider the forecasting achievement via RMSE, MAE, and MAPE to select FIGARCH models as follows:

Table (4) comparison of competentnt models of Gasoline

Competentnt model	Error distribution	Forecasting accuracy measure		
		RMSE	MAE	MAPE
FIGARCH (1,$\hat{d}$,1)	Normal	0.2164	0.1031	113.3220
FIGARCH (2, $\hat{d}$,1)	Normal	0.2355	0.1203	112.4391
FIGARCH (1, $\hat{d}$,2)	Normal	0.2109	0.1009	112.0229
FIGARCH (2, $\hat{d}$,2)	Normal	0.2118	0.1091	130.7302
FIGARCH (1, $\hat{d}$,1)	Students-t	0.2411	0.1330	137.8330
FIGARCH (2, $\hat{d}$,1)	Students-t	0.2153	0.1127	119.0021
FIGARCH (1, $\hat{d}$,2)	Students-t	0.2118	0.1322	129.4800
FIGARCH (2, $\hat{d}$,2)	Students-t	0.2133	0.1421	112.3326

Result of table(4) shown that among the several FIGARCH models considered with price of Gasoline on Iraq by using the Three criteria of optimality were measured for normal residuals distribution assumption by RMSE, MAE, and MAPE for FIGARCH with (1, 0.0272, 2). It was noticed that this last model is better fit to price volatility of Gasoline dataset to have the smallest forecast error.

3.6 Estimation ARMA-FIGARCH Models for Daily Returns of Gasoline Prices

After volatility clustering are confirmed with returns series and stationarity using ADF and PP tests, heteroscedasticity effects using ARCH-LM ,Ljung-Boxtests, and showed that the fuel market has a memory longer , we specify the conditional mean equation using ARMA models , AR(2) was selected as the mean equation of Gasoline return series. The ARCH-LM ,Ljung-Boxtests tests also support the presence of ARCH effects in residuals, and from the conditional variance equation the FIGARCH (1,0.0272,2) is selected among several models , since the hybrid AR(2)-FIGARCH(1,0.0272,2) model with normal distribution of residuals was selected and forecast accuracy measures, using quasi maximum likelihood estimation method to estimate the parameters conditional mean and variance equations. The following Table shows the results of in-sample estimation of the important models

Table (5): Results of AR (2)-FIGARCH(1, $\hat{d}$,2) model estimation by Q-MLE method for Gasoline price returns In case residuals distributed is Normal distribution

	Parameter	Estimate (SE[parameter])
Mean equation	AR(1)	3.1e-02* (4.1e-01)
	AR(2)	3.5e-01* (2.5e-06)
Variance equation	Mu	2.7e-06* (2.3e-02)
	Omega α_0	3.7e-03* (3.0e-03)
	Alpha 1	7.3e-03* (4.3e-06)
	Beta 1	4.7e-04* (1.3e-02)
	Beta 2	4.1e-06* (2.2e-08)
	d-FIGARCH	0.0272* (2.3e-04

*: significant result.

According to the table (5) in the model of mean equation under investigation all the coefficients(except the constant) are significant at .95 level of confidence and indicate that ARCH and GARCH parameters are positive, Fractional difference parameter d, is found to be positive and statistically significant in all cases for higher parameters .

3.7 Diagnostic checking model

To check the fitted model are a good fit of the data or not, this is done through applied the two tests (Ljung-Box and ARCH-LM) to check the fitted models, which means tests of statistically significant from residuals and squared residuals, as followed:

Table 6: The results of Ljung-Box and ARCH-LM tests on the Residuals ofVolatility Models for Returns and Squared Returns of Gasoline Price Series

Standardized Residuals for AR(2)-FIGARCH (1,0.0272,2) models (Normal Distribution) of Gasoline Returns Series			
Ljung-Box test			
Series	Q-statistic	Lag	p-value
Res	21.2679	10	0.0554
Res	23.8703	15	0.0673
Res	29.2908	20	0.0829
Res^2	5.6532	10	0.8435
Res^2	6.3135	15	0.9580
Res^2	40.3234	20	0.0619
Arch-LM test			
Series	Q-statistic	Lag	p-value
Res	5.4310	10	0.8606
Res	6.2283	15	0.9604
Res	36.7754	20	0.0606
Squared Residuals:Res^2Residuals: Res			

The residual diagnostics checking for the best fitted model, according to Table (6), ARCH-LM test is employed to check ARCH effect in residuals and from the results, it is inferred that the p-values >0.05, which lead to conclude that the null hypothesis of 'no ARCH effect' is not rejected, which means there is no ARCH effect in the residuals of the model. Based on the results of Ljung-Box test at 5% significance level for squared standardized residuals Lags (20) of the best-fitting model, all the p-values in the Table are more than 0.05 (not significant), then we can't reject the null hypothesis at 20^{th}lags for residuals series, which means there is no serial correlation in the residuals of model. Therefore the selection of the AR(2)-FIGARCH(1,0.0272,2) model under normal distribution assumption to investigate the determinants of the price volatility of Gasoline was well justified. In sample forecasting using the AR(2)-FIGARCH(1,0.0272,2) model volatility models was done.

4. Discussion and conclusion

Most of the estimated ARMA-GARCH models are supported by normal, student t distribution while most of the ARMA-GARCH models exhibited high persistence values in the presence of gasoline. In particular case, the estimated AR (2)-FIGARCH(1,0.0272,2) model for daily return have been significantly impacted, while the preferred estimated models also passed the goodness-of-test fit. In the model we have chosen, it is noticed that the volatility for gasoline to be non-negative and significant, which indicates that the price increment has a larger influence the volatility than the price controlled decrement in gasoline prices. Of course, this research approve its benefit concerning investment decisions through selecting Gasoline prices. Recommendation was remarked by the conclusion of this research, which can be summarized by taking careful control of the price of gasoline as it shows volatility via the research period as it may affect the country's economy to a certain range.

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