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Noor A. Hussein and Luma N. M. Tawfiq

Efficient Approach for Solving High Order (2+1)D-Differential Equation

Noor A. Hussein[1, a)] and Luma N. M. Tawfiq[2,b)]

[1]*Department of Mathematics, College of Education, University of AL-Qadisiyah, AL-Diwaniyah, Iraq.*
[2]*Department of Mathematics, College of Education for Pure Science Ibn Al-Haitham, University of Baghdad-Iraq*

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[a)] noor.alli@qu.edu.iq
[b)]Corresponding author : dr.lumanaji@yahoo.com

Abstract. This article presents an exact analysis solution for high order (2+1) dimensional differential equations by using efficient approach based on coupled method via LA-transform with decomposition method to overcome the computational difficulties. Convergence of series solution has been discussed with two illustrated examples, and the method has shown a high-precision, fast approach to solve non-linear (2+1) dimensional PDEs with initial conditions, there is no need any discretization of domain or assumption for a small parameter to be present in the problem. The steps of suggested method are easy implemented, high accuracy and a rapid convergence to the exact solution compared with other methods can be used to solve this type of PDEs.

Keywords: (2+1) D-PDEs, ADM, LA-transform, Coupled Method, Convergence Analysis.

INTRODUCTION

Partial differential equations (PDEs) have been use to described many important models in real life, such as contamination, heat, waves, contamination and reaction model [1-4]. So it is important to find their solution. As exact analytical solutions are only available in few cases, the construction of efficient approximate or numerical methods is essential [5]. In recent years, many authors have focused on solving non-linear PDEs using various methods such as HAM [6], VIM [7, 8], DTM [9], HPM [10-11], ADM [12-13], coupled method [14], semi analytic technique [15] and parallel processing technique [16]. Recently, modifications of efficient methods have been used in wider scope to solve different types of PDEs [17-19]. From these types is (2+1) dimensional differential equations ((2+1)D-PDEs). In 2001 Wazwaz and Gorguis [20] solved linear (2+1)D-PDEs by using ADM. Soufyane and Boulmaf [21] solved non-linear parabolic equation with variable physical coefficient in space and time and got analytical solution by using ADM. Achouri and Omrani [22], get numerical solution of PDE of type damped generalized regularized long-wave equation (DGRLW) by using ADM. Also, several numerical methods are used to find numerical solution of nonlinear (2+1)D-PDEs such as [23-24]. Many researchers find solitary wave soliton solutions for (2+1)D-PDEs for more details see [25-26]. Other researchers used new approach to get exact solutions of the generalized (2 + 1) D- PDEs for more details see [27-31]. In this article new combine of LA-transform with decomposition method is presented to solve (2+1)D-PDEs to get exact analytic solution.

LA- Transformation [32]

In 2018 Jabber and Tawfiq defined LA- transformation for a function $f(t)$ as follow:

$$\bar{f}(s) = \mathbb{T}\{f(t)\} = \int_0^\infty e^{-t} f\left(\frac{t}{s}\right) dt \ ,$$

Where (s) is a real number, for those values of (s) the improper integral converge.

This transformation has domain wider than in Laplace transform (LT) and has some interesting properties which make it rival to the Laplace Transform. Some of these properties are:

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1. The domain of the LAT is wider than the domain of LT. This feature makes the LAT more widely used in problems.
2. Depending on [32] the LAT has the duality with LT, therefore, the LAT can be solve all the problems which be solved by LT
3. In the t-domain, the unit step function was transformed to unity in the u-domain.
4. In the t-domain, the differentiation and integration are equivalent to multiplication and division of the transformed function F(u) by u in the u-domain.
5. By Linear property we have that for any constant $a \in$R, $\mathbb{T}\{a\} = a\mathbb{T}\{1\} = a$, and hence, $\mathbb{T}^{-1}\{a\} = a$, that is, we
don't have any problem when we dealing with the constant term (the constant with respect to the parameter u). For more details see [32].

Suggested Approach for Solving Nonlinear (2+1) D- Differential Equations

Here we suggest new approach based on combine LA-transform with ADM and denoted by **LATDM**. Firstly, we write the form of nonlinear (2+1)D-PDEs as:

$$u_t(x, y, t) = g(x, y, t) + R + N \qquad (1)$$

Subject to initial condition: $u(x, y, 0) = f(x, y)$

where R represent the linear operator part, N represent the nonlinear operator part and g is inho-mogeneous source.

The implemented of suggested approach is started by taking LA-transform (denoted by $\mathbb{T}$) on both sides of the equation (1), to get

$$\mathbb{T}\{u_t(x, y, t)\} = \mathbb{T}\{g(x, y, t)\} + \mathbb{T}\{R + N\}$$

Using the property of derivative for LA-transform, i.e.

$$s^n\mathbb{T}\{u(x, y, t)\} - s^n f(x, y, 0) = z(x, y, s) + \mathbb{T}\{R + N\}$$

So, $\mathbb{T}\{u(x, y, t)\} = w(x, y, s) + \mathbb{T}\{R + N\}$

Then using the decomposition series for the linear part and for the nonlinear part the infinite series of Adomian polynomials will be used. Where Adomian polynomials$A_m$defined as:

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}[N(\textstyle\sum_{i=0}^{\infty} \lambda^i y_i)]_{\lambda=0}, \quad m = 0,1,2, \dots \ . \qquad (2)$$

So, using the linearity property of the LA- transform. Hence, the solution $u(x, y, t)$ are obtained easily by applying the inverse of LA- transform. Putting these components into the expansion given by:

$$u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, , t) = u_0 + u_1 + u_2 + \cdots \qquad (3)$$

We get required solution.

Illustrative Problems

This section two examples has been presented to illustrate the efficiency and accuracy for suggested method.

Example 1 (Kadomtsev-Petviashvili equation) [10]

Consider the following 4th order nonlinear (2+1)D-Differential equation:

$$u_{xt} - 6uu_{xx} - 6(u_x)^2 + u_{xxxx} + 3u_{yy} = 0 \ ,$$

With IC: $u_x(x, y, 0) = -\frac{1}{2}csc^2\left(\frac{1}{2}(\mathrm{x} + \mathrm{y})\right)\coth\left(\frac{1}{2}(\mathrm{x} + \mathrm{y})\right)$

Take new transform to above equation

$s\mathbb{T}\{u_x\} - su_x(\mathrm{x}, \mathrm{y}, 0) = \mathbb{T}\{6uu_{xx} + 6(u_x)^2 - u_{xxxx} - 3u_{yy}\}$,

using the decomposition procedures and taking (T^{-1})

$u_{nx} = T^{-1}\left\{u_x(x, y, 0) + \frac{1}{s}T\{6A_n + 6B_n - u_{nxxxx} - 3u_{nyy}\}\right\}$

$$u_{0x} = \frac{-1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$u_0 = \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$u_{1x} = T^{-1}\left\{\frac{1}{s}T\{6A_0 + 6B_0 - u_{0xxxx} - 3u_{0yy}\}\right\}$$
$$u_{0xx} = u_{0yy} = \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$u_{0xxx} = -csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^3\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$u_{0xxxx} = \frac{1}{2} csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{11}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$

Calculate Adomian polynomials for nonlinear parts as follow:

$$A_0 = F(u_0) = u_0 u_{0xx}$$
$$A_0 = \frac{1}{8} csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$B_0 = (u_{0x})^2 = \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$

$$u_{1x} = t\left[\frac{3}{4} csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{3}{2} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{3}{2} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{11}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right]$$

$$u_{1x} = t\left[\frac{1}{4} csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right]$$

$$u_{1x} = t\left[\frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + \frac{1}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{1}{2} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{4} csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - \frac{3}{2} csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right]$$

$$u_{1x} = \left[-csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - 2csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right] t$$
$$u_{1x} = td\left(2csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right)$$, by integrate w.r.to x
$$\Longrightarrow\ u_1 = 2tcsch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)$$
$$u_{1x} = t\left[-csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - 2csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right]$$
$$u_{1xx} = \frac{t}{2}\left[4csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + 4csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) + 4csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^3\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right]$$
$$u_{1xxx} = 2t\left(-csch^6\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - 4csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right) + t\left(-3csch^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) - 2csch^2\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right) coth^4\left(\frac{1}{2}(\mathrm{x}+\mathrm{y})\right)\right)$$

$$u_{1xxxx}=t\left(6csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-8csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+16csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right)+\frac{t}{2}\left(6csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+12csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)+8csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)+4csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^5\left(\frac{1}{2}(\mathrm{x+y})\right)\right)$$

$$u_{1xxxx}=17tcsch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+26tcsch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)+2tcsch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^5\left(\frac{1}{2}(\mathrm{x+y})\right)$$

$$u_{2x}=\mathbb{T}^{-1}\left\{\frac{1}{s}\mathbb{T}\{6A_1+6B_1-u_{1xxxx}-3u_{1yy}\}\right\}$$
$$A_1=\left[\frac{d}{d\lambda}\mathrm{F}(u_0+\lambda u_1)\right]_{\lambda=0}$$
$$A_1=u_0u_{1xx}+u_1u_{0xx}$$
$$A_1=\left(\frac{1}{2}csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)\right)\left(4tcsch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+2tcsch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right)+\left(2tcsch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)\right)\left(\frac{1}{4}csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)+\frac{1}{2}csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^2\left(\frac{1}{2}(\mathrm{x+y})\right)\right)$$
$$A_1=\frac{5}{2}tcsch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+2tcsch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)$$

$$B_1=\left[\frac{d}{d\lambda}\mathrm{F}(u_0+\lambda u_1)\right]_{\lambda=0}$$
$$B_1=\frac{d}{d\lambda}((u_0+\lambda u_1)_x)^2$$
$$B_1=2u_{0x}u_{1x}$$
$$B_1=tcsch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+2tcsch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)$$

$$u_{2x}=\mathbb{T}^{-1}\left\{\frac{1}{s^2}\left\{15csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+12csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)+6csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+12csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)-17csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-26csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)-2csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^5\left(\frac{1}{2}(\mathrm{x+y})\right)-12csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-6csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right\}\right\}$$
$$u_{2x}=\frac{t^2}{2!}\left[4csch^6\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-2csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)-2csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)-2csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)-12csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-6csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right]$$
$$u_{2x}=\frac{t^2}{2!}\left[-16csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-8csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right]$$

$$u_{2x}=\frac{t^2}{2!}\left[-8csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)-8\left(csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)coth\left(\frac{1}{2}(\mathrm{x+y})\right)+csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^3\left(\frac{1}{2}(\mathrm{x+y})\right)\right)\right]$$

$$u_2=\frac{t^2}{2!}\left[4csch^4\left(\frac{1}{2}(\mathrm{x+y})\right)+8csch^2\left(\frac{1}{2}(\mathrm{x+y})\right)coth^2\left(\frac{1}{2}(\mathrm{x+y})\right)\right]$$

and so on

$$u = \sum_{n=0}^{\infty} u_n = u_0 + u_0 + u_0 + \cdots$$

$$u = \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x+y})\right) + 2t csch^2\left(\frac{1}{2}(\mathrm{x+y})\right) coth\left(\frac{1}{2}(\mathrm{x+y})\right) + \frac{t^2}{2!}\left(4 csch^4\left(\frac{1}{2}(\mathrm{x+y})\right) + 8 csch^2\left(\frac{1}{2}(\mathrm{x+y})\right) coth^2\left(\frac{1}{2}(\mathrm{x+y})\right)\right) + \cdots$$

$$u = \left[\frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x+y-4t})\right)\right]_{t=0} + t\left[\frac{\partial}{\partial t}\left(\frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x+y-4t})\right)\right)\right]_{t=0} + \frac{t^2}{2!}\left[\frac{\partial^2}{\partial t^2}\left(\frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x+y-4t})\right)\right)\right]_{t=0} + \cdots$$

That is closed to the exact solution:

$$u = \frac{1}{2} csch^2\left(\frac{1}{2}(\mathrm{x+y-4t})\right)$$

Example 2 (Boussinesq equations) [7]

Consider the following 4[th] order nonlinear (2+1)D-Differential equation:

$$u_{tt} - u_{xx} - \frac{1}{2}(u^2)_{xx} - u_{yy} - u_{xxxx} = 0\ ,$$

with ICs: $u(x,y,0) = 6sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)$

$$u_t(x,y,0) = 24\left(\frac{1}{\sqrt{2}}\right) sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

Take new transform of equation

$$\mathbb{T}\{u_{tt}\} = \mathbb{T}\left\{u_{xx} + \frac{1}{2}(u^2)_{xx} + u_{yy} + u_{xxxx}\right\}$$

$$s^2\mathbb{T}\{u\} - s^2u(x,y,0) - su_t(x,y,0) = \mathbb{T}\left\{u_{xx} + \frac{1}{2}(u^2)_{xx} + u_{yy} + u_{xxxx}\right\}$$

$$\mathbb{T}\{u\} = u(x,y,0) + \frac{1}{s}u_t(x,y,0) + \frac{1}{s^2}\mathbb{T}\left\{u_{xx} + \frac{1}{2}(u^2)_{xx} + u_{yy} + u_{xxxx}\right\}$$

$$\mathbb{T}\{u\} = 6sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}24\left(\frac{1}{\sqrt{2}}\right) sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s^2}\mathbb{T}\left\{u_{xx} + \frac{1}{2}(u^2)_{xx} + u_{yy} + u_{xxxx}\right\}$$

By using decomposition procedure

$$\mathbb{T}\{u_0\} = 6sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}24\left(\frac{1}{\sqrt{2}}\right) sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$u_0 = \mathbb{T}^{-1}\left\{6sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}24\left(\frac{1}{\sqrt{2}}\right) sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)\right\}$$

$$u_0 = 6sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24t\left(\frac{1}{\sqrt{2}}\right) sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right) tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$\mathbb{T}\{u_n\} = \frac{1}{s^2}\mathbb{T}\left\{u_{nxx} + \frac{1}{2}A_n + u_{nyy} + u_{nxxxx}\right\}$$

$u_n = \mathbb{T}^{-1}\left\{\frac{1}{s^2}\mathbb{T}\left\{u_{nxx} + \frac{1}{2}A_n + u_{nyy} + u_{nxxxx}\right\}\right\}$, where A_n is Adomian polynomial defined as equation (2)

$$\mathrm{A_0 = F(u_0) = (u_0{}^2)_{xx} = (2u_0u_{0x})_x = 2u_0u_{0xx} + 2(u_{0x})^2}$$

$$\mathrm{u_{0x} = u_{0y}} = \frac{-12}{\sqrt{2}}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(\mathrm{x+y})\right)\mathrm{tanh}\left(\frac{1}{\sqrt{2}}(\mathrm{x+y})\right) + 12\mathrm{tsech}^4\left(\frac{1}{\sqrt{2}}(\mathrm{x+y})\right) - 24\mathrm{tsech}^2\left(\frac{1}{\sqrt{2}}(\mathrm{x+y})\right)\mathrm{tanh}^2\left(\frac{1}{\sqrt{2}}(\mathrm{x+y})\right)$$

$$\mathbf{u_{0xx}} = \frac{-12}{\sqrt{2}}\left(\frac{-1}{\sqrt{2}}\right)\left[\mathbf{sech^4}\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2\mathbf{sech^2}\left(\frac{1}{\sqrt{2}}(x+y)\right)\mathbf{tanh^2}\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] + \left[12(-4)\left(\frac{1}{\sqrt{2}}\right)\mathbf{tsech^4}\left(\frac{1}{\sqrt{2}}(x+y)\right)\mathbf{tanh}\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] - 24t\left(\frac{1}{\sqrt{2}}\right)\left[2\mathbf{sech^4}\left(\frac{1}{\sqrt{2}}(x+y)\right)\mathbf{tanh}\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2\mathbf{sech^2}\left(\frac{1}{\sqrt{2}}(x+y)\right)\mathbf{tanh^3}\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]$$

$$u_{0xx} = -6sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{96}{\sqrt{2}}tsech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{48}{\sqrt{2}}tsech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$u_{0xxx} = \frac{24}{\sqrt{2}}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{12}{\sqrt{2}}\left[2sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] - \frac{96}{2}t\left[sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - 4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] + \frac{48}{2}t\left[3sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]$$

$$u_{0xxxx} = 12\left[sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - 4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] + 12\left[sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - 4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] - 12\left[3sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] - \frac{48}{\sqrt{2}}t\left[-6sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] + \frac{48(4)}{\sqrt{2}}t\left[2sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - 4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] + \frac{24(3)}{\sqrt{2}}t\left[2sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - 4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\right] - \frac{48}{\sqrt{2}}t\left[4sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - 2sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^5\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]$$

$$\mathrm{u_{0xxxx}} = 24\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - 132\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{816}{\sqrt{2}}\mathrm{tsech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1248}{\sqrt{2}}\mathrm{tsech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{96}{\sqrt{2}}\mathrm{tsech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^5\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$\mathrm{u_0u_{0xx}} = \left[6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24t\left(\frac{1}{\sqrt{2}}\right)\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]\left[-6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{96}{\sqrt{2}}\mathrm{tsech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{48}{\sqrt{2}}\mathrm{tsech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]$$

$$u_0u_{0xx} = -36\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right) + 72\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{720}{\sqrt{2}}\mathrm{tsech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{576}{\sqrt{2}}\mathrm{tsech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - 1152t^2\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576t^2\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$(u_{0x})^2 = \left[\frac{-12}{\sqrt{2}}sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12tsech^4\left(\frac{1}{\sqrt{2}}(x+y)\right) - 24tsech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]\left[\frac{-12}{\sqrt{2}}sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12tsech^4\left(\frac{1}{\sqrt{2}}(x+y)\right) - 24tsech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]$$

$$(u_{0x})^2 = 72\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{288}{\sqrt{2}}t\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{576}{\sqrt{2}}t\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + 144t^2\mathrm{sech}^8\left(\frac{1}{\sqrt{2}}(x+y)\right) - 576t^2\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576t^2\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

$$u_1 = \mathbb{T}^{-1}\left\{\frac{1}{s^2}\mathbb{T}\left\{u_{0xx} + \frac{1}{2}A_0 + u_{0yy} + u_{0xxxx}\right\}\right\}$$

$$u_1 = \mathbb{T}^{-1}\left\{\frac{1}{s^2}\left[-12sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s}\frac{192}{\sqrt{2}}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}\frac{96}{\sqrt{2}}sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - 36sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right) + 72sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s}\frac{720}{\sqrt{2}}sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}\frac{576}{\sqrt{2}}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - 1152\frac{2!}{s^2}sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576\frac{2!}{s^2}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 72sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s}\frac{288}{\sqrt{2}}sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}\frac{576}{\sqrt{2}}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + 144\frac{2!}{s^2}sech^8\left(\frac{1}{\sqrt{2}}(x+y)\right) - 576\frac{2!}{s^2}sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576\frac{2!}{s^2}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - 132sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 24sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}\frac{816}{\sqrt{2}}sech^6\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s}\frac{1248}{\sqrt{2}}sech^4\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{96}{\sqrt{2}}\frac{1}{s}sech^2\left(\frac{1}{\sqrt{2}}(x+y)\right)tanh^5\left(\frac{1}{\sqrt{2}}(x+y)\right)\right]\right\}$$

$$u_1 = \mathbb{T}^{-1}\left\{\frac{-12}{s^2}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{24}{s^2}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s^3}\frac{192}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s^3}\frac{96}{\sqrt{2}}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{36}{s^2}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{72}{s^2}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s^3}\frac{720}{\sqrt{2}}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s^3}\frac{576}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - 1152\frac{2!}{s^4}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576\frac{2!}{s^2}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 72\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s}\frac{288}{\sqrt{2}}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s}\frac{576}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + 144\frac{2!}{s^2}\mathrm{sech}^8\left(\frac{1}{\sqrt{2}}(x+y)\right) - 576\frac{2!}{s^2}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 576\frac{2!}{s^2}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{24}{s^2}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{132}{s^2}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{24}{s^2}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{1}{s^2}\frac{816}{\sqrt{2}}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{1}{s^3}\frac{1248}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{96}{\sqrt{2}}\frac{1}{s^3}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^5\left(\frac{1}{\sqrt{2}}(x+y)\right)\right\}$$

$$u_1 = t^2\left(-6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) - 6\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right) + 6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right) + 12\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\right) + t^3\left(-\frac{32}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) + \frac{16}{\sqrt{2}}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right) - \frac{32}{\sqrt{2}}\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right) - \right.$$

$\frac{16}{\sqrt{2}}\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)+\frac{16}{\sqrt{2}}\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^5\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+t^4\Bigg(-144\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+12\mathrm{sech}^8\left(\frac{1}{\sqrt{2}}(x+y)\right)+96\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)$

$$u_1=t^2\Bigg(-6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)+12\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)-6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)+6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+6\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+12\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)-12\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+\frac{t^3}{\sqrt{2}}\Bigg(-32\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)+16\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)-32\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)+32\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)-16\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)+16\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)-16\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+t^4\Bigg(-144\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+12\mathrm{sech}^8\left(\frac{1}{\sqrt{2}}(x+y)\right)+96\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)$$

$$u_1=t^2\Bigg(-12\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)+24\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+\frac{t^3}{\sqrt{2}}\Bigg(-64\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)+32\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+t^4\Bigg(-144\mathrm{sech}^6\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+12\mathrm{sech}^8\left(\frac{1}{\sqrt{2}}(x+y)\right)+96\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)$$

and so on.

$$u=\sum_{n=0}^{\infty}u_n=u_0+u_1+u_2+\cdots$$

$$u=6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)+\frac{24}{\sqrt{2}}t\,\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)+t^2\Bigg(-12\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)+24\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+\frac{t^3}{\sqrt{2}}\Bigg(-64\mathrm{sech}^4\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh\left(\frac{1}{\sqrt{2}}(x+y)\right)+32\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y)\right)\tanh^3\left(\frac{1}{\sqrt{2}}(x+y)\right)\Bigg)+\cdots$$

$$u=\left[6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y-2t)\right)\right]_{t=0}+t\left[\frac{\partial}{\partial t}\left(6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y-2t)\right)\right)\right]_{t=0}+\frac{t^2}{2!}\left[\frac{\partial^2}{\partial t^2}\left(6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y-2t)\right)\right)\right]_{t=0}+\frac{t^3}{3!}\left[\frac{\partial^3}{\partial t^3}\left(6\mathrm{sech}^2\left(\frac{1}{\sqrt{2}}(x+y-2t)\right)\right)\right]_{t=0}+\cdots$$

That is closed to the exact solution: $u=6sech^2\left(\frac{1}{\sqrt{2}}(x+y-2t)\right)$

CONCLUSION

In this article, new coupled method biased on combine LA-Transform with decomposition method is suggested to solve high order (2+1)-D differential equation and we get exact analytical solution, where ADM or other modifications are used to solve the same examples but cannot be getting exact analytical solution. Moreover, in LATDM the nonlinear terms is easier computation than in ADM or its modifications. So, this approach is very efficient, easy implementation and rapid convergence to the exact solutions.

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