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Double Stage Shrinkage Estimator of Two Parameters Generalized Rayleigh Distribution

مُقدر التقلص ذو المرحلتين لتوزيع رالى ذو المعلمتين

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Abstract:

This paper is concerned with double stage shrinkage estimator (DSSE) for lowering the mean squared error of classical estimator (MLE) for the shape parameter (α) of generalized Rayleigh (GR) distribution in a region (R) around available prior knowledge (α_0) about the actule value (α) as initial estimate in case when a scale parameter (λ) is known as well as to reduce the cost of experimentations.

In situation where the experimentations are time consuming or very costly, a double stage procedure can be used to reduce the expected sample size needed to obtain the estimator.

This estimator is shown to have smaller mean squared error for certain chóice of the shrinkage weight factor $\psi(\cdot)$ and for acceptance mentioned region R.

Expressions for Bias, Mean square error (MSE), Relative Efficiency [R.Eff($\cdot$)], Expected sample size [E(n/α,R)], Expected sample size proportion [E(n/α,R)/n], probability for avoiding the second sample [$p(\hat{\alpha}_1 \in R)$] and percentage of overall sample saved [$\frac{n_2}{n} p(\hat{\alpha}_1 \in R) * 100$] for the proposed estimator are derived. Numerical results and conclusions are established when the consider estimator (DSSE) are testimator of level of significance Δ. Comparisons with the classical estimator and with the last studies shown the usefulness of the proposed estimator.

المستخلص:

أقترح في هذا البحث مقدر التقلص ذي المرحلتين (DSSE) لتقليل متوسط مربعات الخطأ (MSE) لمقدر الامكان الاعظم (MLE) لمعلمة الشكل (α) لتوزيع رالي العام (GR) عند المنطقة (R) حول المعلومات المسبقة (α_0) المتوافرة حول المعلمة الحقيقية (α) بشكل تقدير ابتدائي وعندما تكون معلمة القياس (λ) معلومة بالاضافة الى تقليل كلفة المعاينة والتجارب.

عندما يكون استهلاك الوقت أو كلفة المعاينة او التجارب عالي جداً فان طريقة التقلص ذي المرحلتين تكون مناسبة للحصول على مقدر يقلل من حجم العينة المتوقع وبالتالي التقليل من هذه الكلف. ومن خواص هذا المقدر أيضاً انه ذو متوسط مربعات خطأ (MSE) صغير خصوصاً عند اختيار عامل تقلص موزون $\psi(\cdot)$ ومنطقة قبول R بشكل مناسب.

اشتقت معادلات التحيز، متوسط مربعات الخطأ (MSE)،الكفاءة النسبية [R.Eff($\cdot$)]، حجم العينة المتوقع [E(n/α,R)]، حجم العينة المتوقع النسبي [E(n/α,R)/n]، احتمالية تجنب العينة الثانية [$p(\hat{\alpha}_1 \in R)$] ونسبة الادخار الكلي المئوية للعينة [$\frac{n^2}{n} p(\hat{\alpha}_1 \in R) * 100$] للمقدر المقترح (DSSE). أعطيت النتائج العددية والاستنتاجات لمقدر الاختبار الأولي (DSSE) المقترح بمستوى معنوية Δ. أجريت المقارنات مع المقدر الكلاسيكي وبعض المقدرات المقترحة في الدراسات الاخيرة لبيان فائدة المقدر المقترح.

1. Introduction

Several authors considered different aspects of the Burr Type X and Burr Type XII distribution, see for example [6], [8] and [14]. Also, Burr Type X has been studied by [3] and [7].

Two parameters Burr Type X distribution and correctly named as two parameter generalized Rayleigh distribution are introduced in [10] and [11]. They showed that the two parameters generalized Rayleigh distribution can be used quite effectively in modeling strength data and also in modeling general life time da.a. Different estimators are considered in [5] and they studied how the estimator of the different unknown parameter behave for different sample size.

The two parameters generalized Rayleigh (GR) distribution has the following distribution function:-

$$F(x;\alpha,\lambda)=[1-e^{-(\lambda x)^2}]^{\alpha} \quad \text{for } x>0,\ \alpha>0,\ \lambda>0 \qquad \ldots(1)$$

Thus, the probability density function (p.d.f.) of (GR) distribution is

$$f(x;\alpha,\lambda)=\begin{cases}2\alpha\lambda^2 x e^{-(\lambda x)^2}(1-e^{-(\lambda x)^2})^{\alpha-1} & \text{for } x>0,\alpha,\lambda>0\\ 0 & \text{o.w.}\end{cases} \qquad \ldots(2)$$

where, α and λ are the shape and scale parameters respectively.

In this paper we introduce the problem of estimating of the shape parameter (α) of GR distribution with known scale parameter (λ) when some prior information (α_0) regarding the actual value (α) available due past experiences such a prior estimate may arise for any one of anumber of resons [12], e.g., we are estimating α and ;

i. We belive α_0 is close to true value of α, or

ii. We fear that α_0 may be near the true value of α, i.e.; Something bad happens if $\alpha = \alpha_0$ and we do not know about it.

In such a situation it is natural to start with an estimator $\hat{\alpha}$ (e.g. MLE) of α and modify it by moving it closer to α_0, so that the resulting estimator, though perhaps biased, has smaller mean square error than that of $\hat{\alpha}$ in some interval around α_0. This method of constructing an estimator of α that incorporates the prior value α_0 leads to what is known as a shrinkage estimator.

It is an important aspect of estimation that one should be able to get an estimator quickly using minimum experimentation. This also economizes cost of experimentation. To achieve this, double stage shrinkage estimator were introduced.

A double stage shrinkage estimator procedure is defined as follows:
Let x_{1i}; i = 1,2, …, n_1 be a random sample of n_1 from GR distribution and $\hat{\alpha}_1$ be a "good" estimator of α based on these n_1 observation. Construct a preliminary test region R in the parameter space based on α_0 and an appropriate criterion.

If $\hat{\alpha}_1 \in R$ shrink $\hat{\alpha}_1$ towards α_0 by shrinkage weight factor $0 \le \psi(\hat{\alpha}) \le 1$ and use the shrinkage estimator $\psi(\hat{\alpha}_1)\hat{\alpha}_1 + (1-\psi(\hat{\alpha}_1))\alpha_0$, for estimate α.

If $\hat{\alpha}_1 \notin R$, obtain x_{2i}; i = 1,2, …, n_2, an additional sample of size n_2 and use a pooled estimator $\hat{\alpha}_p$ of α based on combined sample of size $n = n_1 + n_2$, i.e.; $\hat{\alpha}_p = \dfrac{n_1\hat{\alpha}_1 + n_2\hat{\alpha}_2}{n}$.

Thus, the double stage shrinkage estimator (DSSE) of α will be:

$$\tilde{\alpha} = \begin{cases} \psi_1(\hat{\alpha}_1)\hat{\alpha}_1 + (1-\psi_1(\hat{\alpha}_1))\alpha_0 & ,\text{if } \hat{\alpha}_1 \in R \\ \hat{\alpha}_p & ,\text{if } \hat{\alpha}_1 \notin R \end{cases} \qquad \text{...(3)}$$

The motivation of this study was provided by the work of [4], [13], [2] and [1].

The aim of this paper is to employ the double stage shrinkage estimator (DSSE) $\tilde{\alpha}$ defined by (3) for estimate the shape parameter (α) of two parameters generalized Rayleigh (GR) distribution when the scale parameter (λ) is known.

The expression of Bias, Mean squared error (MSE), Relative Efficiency [R.Eff(·)], Expected sample size proporation, probability for avoiding the second sample and sample size, Expected sample size proporation, probability for avoiding the second sample and percentage of overall sample saved are derived and obtained for the estimator $\tilde{\alpha}$.

Numerical results and conclusions due mentiond expressions including some constants are performed and displayed in annexed tables.

Comparisons between the proposed estimator with the classical estimator ($\hat{\alpha}$) and with some of the last studies are demonstrated.

2. Unbiased - Maximum Likelihood Estimator of α

In this section, we consider the maximum likelihood estimator (MLE) of GR distribution with shape and scale parameter α and λ respectively i.e. GR(α,λ).

Assume $x_{11}, x_{12}, \ldots, x_{1n_1}$ be a random sample of size n_1 from GR(α,λ) then the log-likelihood function L(α,λ) can be written as:

$$L(\alpha,\lambda) = c + n_1 \ln\alpha + 2n_1 \ln\lambda + \sum_{i=1}^{n_1} \ln x_{1i} - \lambda^2 \sum_{i=1}^{n_1} x_{1i}^2 + (\alpha-1)\sum_{i=1}^{n_1} \ln(1-e^{-(\lambda x_{1i})^2})$$
...(4)

where c is constant.
In this paper we take $\lambda = 1$ (λ is known).

So, $\frac{\partial L}{\partial \alpha}=\frac{n_1}{\alpha}+\sum_{i=1}^{n_1}\ln(1-e^{-x_{1i}^2})=0$ ...(5)

then, the MLE of α, say $\hat{\alpha}_{1_{MLE}}$ is

$$\hat{\alpha}_{1_{MLE}}=-\frac{n_1}{\sum_{i=1}^{n_1}\ln(1-e^{-x_{1i}^2})} \quad ...(6)$$

Note that, if $x_{1i} \overset{iid}{\sim} GR(\alpha,1)$, then $-\alpha\sum_{i=1}^{n_1}\ln(1-e^{-x_{1i}^2}) \sim G(n_1,1)$, see [8].

i.e.: $E(\hat{\alpha}_{1_{MLE}})=\frac{n_1}{n_1-1}\alpha$ and $var(\hat{\alpha}_{1_{MLE}})=\frac{n_1^2\alpha^2}{(n_1-1)^2(n_1-2)}$.

Using (6), an unbiased estimator $\hat{\alpha}_1$ of α can be easily obtained as:

$$\hat{\alpha}_1=\frac{n_1-1}{n_1}\hat{\alpha}_{1_{MLE}}=-\frac{n_1-1}{\sum_{i=1}^{n_1}\ln(1-e^{-x_{1i}^2})} \quad ...(7)$$

i.e. $E(\hat{\alpha}_1)=\alpha$ and $var(\hat{\alpha}_1)=MSE(\hat{\alpha}_1)=\frac{\alpha^2}{n_1-2}$ ...(8)

3. Double Stage Shrinkage Estimator (DSSE) $\tilde{\alpha}$

In this section, we consider the (DSSE) $\tilde{\alpha}$ which is defined in (3) using $\hat{\alpha}_1$ defined by (7), when $\psi(\hat{\alpha}_1)=k$ is a constant weight factor $(0<k<1)$ for estimate the shape parameter α of GR distribution when $\lambda=1$.

$$\tilde{\alpha}=\begin{cases} k\hat{\alpha}_1+(1-k)\alpha_0 & ,\text{if } \hat{\alpha}_1\in R \\ \hat{\alpha}_p=\frac{n_1\hat{\alpha}_1+n_2\hat{\alpha}_2}{n} & ,\text{if } \hat{\alpha}_1\notin R \end{cases} \quad ...(9)$$

where R is a pretest region for testing the hypothesis $H_0: \alpha=\alpha_0$ vs $H_A: \alpha\neq\alpha_0$ with level of significance (Δ) using test statistic funcion $T(\hat{\alpha}_1/\alpha_0)=\frac{2(n_1-1)\alpha_0}{\hat{\alpha}_1}$

i.e.; $R=\left[\frac{2(n_1-1)\alpha_0}{b},\frac{2(n_1-1)\alpha_0}{a}\right]$ ...(10)

where $a(=X^2_{1-\Delta/2,2n_1})$ and $b(=X^2_{\Delta/2,2n_1})$, ...(11)

are respectively the lower and upper 100(Δ/2) percentile point of chi-square distribution with degree of freedom ($2n_1$), [5], [9].

The expression for Bias of DSSE ($\tilde{\alpha}$) is defined as below

$$Bias(\tilde{\alpha}/\alpha,R)=E(\tilde{\alpha}-\alpha)$$

$$=\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\in R}\left[k(\hat{\alpha}_1-\alpha_0)+(\alpha_0-\alpha)\right]f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2+$$

$$\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\notin R}\left[\hat{\alpha}_p-\alpha\right]f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2$$

where $\bar{R}$ is the complement region of R in real space and we derived the p.d.f. of $\hat{\alpha}_i$ (i=1,2) as below.

$$f(\hat{\alpha}_1)=\begin{cases}\dfrac{\left[\dfrac{(n_1-1)\alpha}{\hat{\alpha}_1}\right]^{n_1+1} e^{-\frac{(n_1-1)\alpha}{\hat{\alpha}_1}}}{\Gamma(n_1)\cdot(n_1-1)\alpha} & \text{for } \hat{\alpha}_1>0, \alpha>0 \\ 0 & \text{o.w.}\end{cases} \quad \ldots(12)$$

We conclude,

$$Bias(\tilde{\alpha}/\alpha,R)=\alpha\left\{(\zeta-1)J_0(a^*,b^*)-\frac{1}{1+u}[(n_1-1)J_1(a^*,b^*)-J_0(a^*,b^*)]\right\}$$

...(13)

$$\text{where } J_\ell(a^*,b^*)=\int_{a^*}^{b^*} y^{-\ell}\frac{y^{n_1-1}e^{-y}}{\Gamma(n_1)}dy; \ell=0,1,2, \quad \ldots(14)$$

also $y=\dfrac{(n_1-1)\alpha}{\hat{\alpha}_1}$, $a^*=\zeta^{-1}\cdot a$, $b^*=\zeta^{-1}\cdot b, u=\dfrac{n_2}{n_1}$ (the ratio of second sample size to first sample size), $n=n_1+n_2$ and $\zeta=\dfrac{\alpha_0}{\alpha}$ (the ratio of prior estimate α_0 to actual value α)

...(15)

The Bias ratio $[B(\cdot)]$ of DSSE ($\tilde{\alpha}$) is defined as :

$$B(\tilde{\alpha})=\frac{Bias(\tilde{\alpha}/\alpha,R)}{\alpha} \quad \ldots(16)$$

The expression of Mean squared error $[MSE(\cdot)]$ of $\tilde{\alpha}$ derived as below:-

$$MSE(\tilde{\alpha}/\alpha,R)=E(\tilde{\alpha}-\alpha)^2$$

$$=\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\in R}[k(\hat{\alpha}_1-\alpha)+(\alpha_0-\alpha)]^2 f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2 +$$

$$\int_{\hat{\alpha}_2=0}^{\infty}\int_{\hat{\alpha}_1\notin R}\left[\hat{\alpha}_p-\alpha\right]^2 f(\hat{\alpha}_1)f(\hat{\alpha}_2)d\hat{\alpha}_1 d\hat{\alpha}_2$$

and by simple computations, one can get:

$$MSE(\tilde{\alpha}/\alpha,R)=\alpha^2\left\{(\zeta-1)^2J_0(a^*,b^*)+\left(\frac{1}{1+u}\right)^2\left(\frac{1}{n_1-2}\right)\left(\frac{u}{1+u}\right)^2\left(\frac{1}{n_1u-2}\right)-\left(\frac{1}{1+u}\right)^2[(n_1-1)^2J_2(a^*,b^*)-\right.$$
$$\left.2(n_1-1)J_1(a^*,b^*)+J_0(a^*,b^*)]-\left(\frac{u}{1+u}\right)^2\cdot\left(\frac{1}{n_1u-2}\right)J_0(a^*,b^*)\right\}$$

...(17)

Now, the Efficiency of $\tilde{\alpha}$ relative to $\hat{\alpha}$ denoted by R.Eff($\tilde{\alpha}$/α,R) is defined by:

$$R.Eff(\tilde{\alpha}/\alpha,R)=\frac{MSE(\hat{\alpha})}{MSE(\tilde{\alpha}/\alpha,R)\cdot[E(n/\alpha,R/n]} \quad \ldots(18)$$

where $E(n/\alpha,R)$ is the Expected sample size, which is defined as:

$$E(n/\alpha,R)=n\left[1-\frac{u}{1+u}J_0(a^*,b^*)\right], \quad \ldots(19)$$

see for example [10], [11], [12] and [13].

As well as the Expected sample size proportion $E(n/\alpha,R)/n$ equal to

$$1-\frac{u}{1+u}J_0(a^*,b^*) \quad \ldots(20)$$

Also, we have to define the percentage of the overall sample saved (p.o.s.s.) of $\tilde{\alpha}$ as:

$$\text{p.o.s.s.} = \frac{n_2}{n} J_0(a^*, b^*) * 100 \qquad \ldots(21)$$

And, finally, $P(\hat{\alpha} \in R)$ represent the probability of a voiding the second sample (stage).

4. Conclusions and Numerical Results

The computations of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)], Expected sample size [E(n/α,R)], Expected sample size proportion [E(n/α,R)/n], Percentage of the overall sample saved (p.o.s.s.) and propability of a voiding the second sample [$P(\hat{\alpha} \in R)$] were used for the estimator $\tilde{\alpha}$. These computations were performed using MathCAD software program for n_1 = 4,6,8,10,12, 16,18, u(= n_2/n_1) = 0.5, 1, 2, 4, 8, 12, ζ (= α_0/α) = 0.25(0.25)2, Δ = 0.01,0.05,0.1 and k = 0.01, 0.1(0.1)0.9.

Some of these computations are given in the tables (1)-(8).

The observation mentioned in the tables lead to the following results:

i. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}$ are adversely proportional with small value of Δ especially when ζ = 1, i.e. Δ = 0.01 yield highest efficiency

ii. The Relative Efficiency [R.Eff(·)] of $\tilde{\alpha}$ has maximum value when $\alpha=\alpha_0(\zeta=1)$, for each n_1, Δ, and decreasing otherwise (ζ≠1). This feature shown the important usefulness of prior knowledge which given higher effects of proposed estimator as well as the important role of shrinkage technique and its philosophy.

iii. Bias ratio [B(·)] of $\tilde{\alpha}$ are reasonably small when $\alpha=\alpha_0$ for each n_1, Δ, and increases otherwise. This property shown that the proposed estimator $\tilde{\alpha}$ is very closely to unbiasedness property especially when $\alpha=\alpha_0$.

iv. The Effictive interval of $\tilde{\alpha}$ [the value of $\tilde{\alpha}$ which makes R.Eff(·) of $\tilde{\alpha}$ greater than one] is [0.5,1.5].

v. Bias ratio [B(·)] of $\tilde{\alpha}$ are reasonably large with small value of u.

vi. R.Eff($\tilde{\alpha}$) is decreasing function with increasing of the first sample size n_1, for each Δ and ζ.

vii. The Expected value of sample size of $\tilde{\alpha}$ is close to n_1, specially when ζ ≥ 1 and start faraway otherwise.

viii. Percentage of the overall sample saved $\left[\frac{n_2}{n} J_0(a^*, b^*) * 100\right]$ is increasing value with increasing value of u (u = n_2/n_1) and ζ.

ix. R.Eff($\tilde{\alpha}$) is an increasing function with respect to u. This property shown the effective of proposed estimator using small n_1 relative to n_2 (or large n_2) which given higher efficiency and reduce the observation cost.

x. The considered estimator $\tilde{\alpha}$ is better than the classical estimator especially when $\alpha\approx\alpha_0$, this will given the effective of $\tilde{\alpha}$ relative to $\hat{\alpha}$ and also given an important weight of prior knowledge, and the augmentation of efficiency may be reach to tens times.

xi. The considered estimator $\tilde{\alpha}$ is more efficient than the estimators introduced by [8], in the sense of higher efficiency.

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Table (1): Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}$ w.r.t. Δ, n_1 and ζ when u = 2

			ζ					
Δ	n_1	R.Eff. Bias	0.25	0.75	1	1.25	1.75	2
0.01	4	R.Eff(·) B(·)	0.513 (− 0.122)	2.342 (− 0.17)	6.923 (0.035)	3.076 (0.268)	0.508 (0.754)	0.291 (0.997)
	8	R.Eff(·) B(·)	0.9 (-28×10^{-4})	1.215 (− 0.105)	6.288 (0.037)	1.743 (0.257)	0.238 (0.748)	0.135 (0.993)
	16	R.Eff(·) B(·)	0.956 (− 0.138)	0.864 (− 0.02)	2.958 (0.034)	0.829 (0.234)	0.114 (0.743)	0.065 (0.991)
	20	R.Eff(·) B(·)	0.965 (0)	0.902 (-7×10^{-3})	2.196 (0.028)	0.65 (0.218)	0.09 (0.74)	0.051 (0.991)
0.05	4	R.Eff(·) B(·)	0.722 (− 0.013)	1.627 (− 0.092)	3.90 (0.065)	2.38 (0.276)	0.477 (0.742)	0.277 (0.968)
	8	R.Eff(·) B(·)	0.904 (-16×10^{-6})	1.041 (− 0.038)	3.042 (0.05)	1.423 (0.243)	0.226 (0.726)	0.129 (0.958)
	16	R.Eff(·) B(·)	0.956 (0)	0.942 (-3×10^{-3})	1.617 (0.025)	0.748 (0.185)	0.108 (0.71)	0.062 (0.953)
	20	R.Eff(·) B(·)	0.965 (0)	0.959 (-72×10^{-4})	1.341 (0.016)	0.63 (0.158)	0.086 (0.704)	0.049 (0.953)

Table (2)
Shown Bias ratio [B(·)] and R.E.ff of $\tilde{\alpha}$ w.r.t. Δ, n_1 and ζ when u = 6

			ζ					
Δ	n_1	R.Eff. Bias	0.25	0.75	1	1.25	1.75	2
0.01	4	R.Eff(·) B(·)	0.301 (− 0.145)	2.442 (− 0.2)	25.507 (0.015)	3.494 (0.254)	0.454 (0.748)	0.256 (0.993)
	8	R.Eff(·) B(·)	0.942 (-35×10^{-4})	0.837 (− 0.134)	14.97 (0.016)	1.562 (0.245)	0.218 (0.744)	0.123 (0.991)
	16	R.Eff(·) B(·)	0.98 (− 0.178)	0.612 (− 0.03)	3.927 (0.015)	0.591 (0.22)	0.105 (0.74)	0.061 (0.991)
	20	R.Eff(·) B(·)	0.985 (0)	0.737 (− 0.011)	2.539 (0.012)	0.425 (0.204)	0.082 (0.738)	0.048 (0.99)

0.05	4	R.Eff(·)	0.69	1.475	9.375	2.502	0.385	0.216
		B(·)	(-0.017)	(-0.135)	(0.028)	(0.247)	(0.726)	(0.958)
	8	R.Eff(·)	0.956	0.769	4.549	1.074	0.182	0.105
		B(·)	(-21×10^{-6})	(-0.059)	(0.021)	(0.219)	(0.716)	(0.951)
	16	R.Eff(·)	0.98	0.867	1.742	0.452	0.085	0.051
		B(·)	(0)	(-55×10^{-3})	(0.011)	(0.166)	(0.703)	(0.951)
	20	R.Eff(·)	0.985	0.942	1.381	0.361	0.066	0.041
		B(·)	(0)	(-13×10^{-3})	(68×10^{-3})	(0.141)	(0.696)	(0.951)

Table (3)
Shown Bias ratio |B(·)| and R.E.ff of $\tilde{\alpha}$ w.r.t. Δ, n_1 and ζ when u = 10

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·)	0.208	2.062	47.744	3.313	0.428	0.24
		B(·)	(-0.151)	(-0.209)	(96×10^{-3})	(0.25)	(0.746)	(0.99)
	8	R.Eff(·)	0.949	0.609	20.853	1.344	0.206	0.117
		B(·)	(-36×10^{-4})	(-0.142)	(0.01)	(0.241)	(0.743)	(0.991)
	16	R.Eff(·)	0.987	0.47	4.28	0.449	0.098	0.058
		B(·)	(-0.189)	(-0.032)	(93×10^{-3})	(0.216)	(0.739)	(0.99)
	20	R.Eff(·)	0.99	0.62	2.649	0.31	0.077	0.046
		B(·)	(0)	(-0.012)	(77×10^{-3})	(0.2)	(0.737)	(0.99)
0.05	4	R.Eff(·)	0.613	1.16	13.333	2.126	0.328	0.183
		B(·)	(-0.018)	(-0.147)	(0.018)	(0.239)	(0.722)	(0.955)
	8	R.Eff(·)	0.971	0.588	5.174	0.816	0.154	0.09
		B(·)	(-22×10^{-6})	(-0.065)	(0.014)	(0.213)	(0.713)	(0.952)
	16	R.Eff(·)	0.987	0.793	1.779	0.317	0.071	0.044
		B(·)	(0)	(-61×10^{-3})	(68×10^{-3})	(0.161)	(0.7)	(0.951)
	20	R.Eff(·)	0.99	0.916	1.393	0.25	0.054	0.035
		B(·)	(0)	(-15×10^{-3})	(43×10^{-3})	(0.136)	(0.694)	(0.951)

Table (4)
Shown Bias ratio |B(·)| and R.E.ff of $\tilde{\alpha}$ w.r.t. Δ, n_1 and ζ when u = 12

			ζ					
Δ	**n_1**	**R.Eff. Bias**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
0.01	4	R.Eff(·)	0.18	1.893	59.067	3.202	0.418	0.234
		B(·)	(-0.153)	(-0.211)	(81×10^{-3})	(0.249)	(0.746)	(0.992)
	8	R.Eff(·)	0.948	0.535	23.054	1.253	0.201	0.115
		B(·)	(-37×10^{-4})	(-0.144)	(85×10^{-3})	(0.24)	(0.743)	(0.991)
	16	R.Eff(·)	0.989	0.421	4.384	0.4	0.095	0.057
		B(·)	(-0.192)	(-0.033)	(79×10^{-3})	(0.215)	(0.739)	(0.99)
	20	R.Eff(·)	0.992	0.574	2.68	0.273	0.074	0.045
		B(·)	(0)	(-0.012)	(65×10^{-3})	(0.199)	(0.737)	(0.99)
0.05	4	R.Eff(·)	0.578	1.038	14.866	1.957	0.306	0.171
		B(·)	(-0.018)	(-0.15)	(0.015)	(0.237)	(0.721)	(0.954)
	8	R.Eff(·)	0.975	0.525	5.366	0.726	0.144	0.084
		B(·)	(-22×10^{-6})	(-0.066)	(0.011)	(0.211)	(0.712)	(0.952)
	16	R.Eff(·)	0.989	0.76	1.79	0.276	0.065	0.042
		B(·)	(0)	(-63×10^{-3})	(57×10^{-3})	(0.159)	(0.7)	(0.951)
	20	R.Eff(·)	0.992	0.902	1.396	0.216	0.05	0.033
		B(·)	(0)	(-16×10^{-3})	(36×10^{-3})	(0.135)	(0.694)	(0.951)

Table (5)
Shown Probability of a Voiding Second Sample w.r.t. Δ, u, n_1 and ζ

			ζ					
u	**n_1**	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**

2	4	0.01	0.216	0.893	0.952	0.976	0.991	0.99
		0.05	0.026	0.668	0.823	0.9	0.952	0.95
6	8	0.01	53×10^{-4}	0.62	0.851	0.942	0.988	0.99
		0.05	322×10^{-6}	0.3	0.612	0.806	0.944	0.95
10	16	0.01	278×10^{-12}	0.147	0.544	0.837	0.983	0.99
		0.05	0	0.029	0.264	0.606	0.929	0.95
12	20	0.01	0	0.055	0.409	0.771	0.981	0.99
		0.05	0	722×10^{-3}	0.159	0.511	0.921	0.95

Table (6) Shown Expected Sample Size of $\tilde{\alpha}$ w.r.t. Δ, u, and ζ when $n_1 = 4$

		ζ					
u	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	0.01	10.271	4.859	4.381	4.192	4.075	4.080
	0.05	11.793	6.655	5.414	4.803	4.384	4.40
6	0.01	22.812	6.578	5.143	4.575	4.226	4.240
	0.05	27.38	11.965	8.242	6.408	5.153	5.20

Table (7) Shown Expected Sample Size Proportion w.r.t. Δ, u, n_1 and ζ

			ζ					
u	**n_1**	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	4	0.01	0.856	0.405	0.365	0.349	0.340	0.340
		0.05	0.983	0.555	0.451	0.40	0.365	0.367
6	8	0.01	1	0.468	0.27	0.193	0.153	0.151
		0.05	1	0.743	0.475	0.309	0.190	0.186
10	16	0.01	1	0.866	0.496	0.239	0.106	0.1
		0.05	1	0.974	0.760	0.449	0.155	0.136
12	20	0.01	1	0.949	0.623	0.289	0.095	0.086
		0.05	1	0.993	0.853	0.528	0.150	0.123

Table (8) Shown Percentage of Overall Sample Saved w.r.t. Δ, u, n_1 and ζ

			ζ					
u	**n_1**	**Δ**	**0.25**	**0.75**	**1**	**1.25**	**1.75**	**2**
2	4	0.01	14.410	59.505	63.492	65.068	66.039	66.000
		0.05	1.723	44.542	54.883	59.977	63.463	63.333
6	8	0.01	0.045	53.157	72.976	80.726	84.715	84.857
		0.05	276×10^{-4}	25.698	52.493	69.096	80.954	81.428
10	16	0.01	253×10^{-10}	13.408	50.397	76.505	89.404	89.999
		0.05	135×10^{-14}	2.644	24.032	55.058	84.456	86.363
12	20	0.01	292×10^{-15}	5.3097	37.734	71.142	90.537	91.385
		0.05	218×10^{-20}	0.667	14.655	47.164	85.028	87.692