Analysis of the Capacity, Spectral Efficiency and Probability of Outage of Adaptive Mobile Channel for WiMAX System

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ABSTRACT

This paper presents an analysis of the channel capacity, spectral efficiency and probability of outage for the 4th generation adaptive modulation ofWiMAX system. The adaptive modulation techniquesare Variable rate-Variable power, Variable rate-Constant power and Constant rate by using truncated channel inversion in the presence of Additive White Gaussian Noise (AWGN), Rayleigh fading and Nakagami fading.

The growing demand for wireless communication requires determining the capacity, spectral efficiency and probability of outage of these channels to evaluate the performance time varying channels.

The analysis results show the advantage of using WiMAX system with adaptive MQAM modulation techniques. Analytical results have proved that the channel capacity is improved by (38) M bit/sec when increasing the bandwidth fromMHz to 20MHz in the presence of AWGN, the capacity channel is improved by (35) Mbit/Secwhen increasing the bandwidth from 1MHz to 20MHz in the presence of Rayleigh fading with Variable rate-Variable power, the capacity channel is improved in the presence of Nakagami fading (m=2)which is (37) Mb/sec and improved (38) when (m=3) due to severe effect of Nakagami when increasing the 1MHz to 20MHz for a given SNR. However, the analysis of spectral efficiency shows that maximum spectral efficiency is accrues in Shannon capacity.The spectral efficiency decreases when Bit Error Rate (BER) increases. The analysis results show the spectral efficiency decreases (0.5) when BER=$10^{-3}$is used than when BER=10^{-6} is used. The probability of outage increases when the BER increase.

Keywords: Adaptive modulation, fading channels, spectral efficiency, WiMAX.

تحليل سعة ، كفاءة الطيف واحتمالية القطع قناة الموبايل المتكيفة لنظام الموجات الميكرويف

الخلاصـــة

في هذا البحث تم تحليل سعة القناة، كفاءة الطيف واحتمالية القطعللجيل الرابع لنظام الموجات المايكرويف. والتقنيات المستخدمة في التضمين التكيفي هي تغير الطاقة - معدل الارسال ، تغير الطاقة - تثبيت معدلا لارسال و تثبيت الطاقة- تغير معدل الارسال عند وجود الضوضاء والاضمحلال الدوري. التطور الكبير في الارسال اللاسلكي تطلب حساب سعة وكفاءة الطيفية للقناة لتقييم اداء القناةفي بيئات مختلفة.

تحليل النتائج التي تم الحصول عليها بينت فوائد استخدام التعامل المتبادل بموجات المايكرويف (WiMAX) مع التضمين التكيفي. النتائج التحليلية اثبتت ان سعة القناة تحسنت بمقدار(٣٨Mbit/sec) عندما زاد(bandwidth) من (1MHz t 20MHz) عند وجود الضوضاء، تحسين بمقدار Mbit/sec (٣٥) عند وجود الاضمحلال الدوري من نوع (Rayleigh fading) عند تغير كل من الطاقة ومعدل الارسال وتحسين بمقدار Mbit/sec (٣٧)عند وجود الاضمحلال الدوري من نوع (Nakagamifading (m=2))وتحسين بمقدار Mbit/sec (٣٨)عند وجود الاضمحلال الدوري من نوع (Nakagami fading (m=3)). تحليل كفاءة الطيف تبين ان اعظم طيف يتم الحصول علية في حالة (Shannon capacity) وان كفاءة الطيف تقل عند زيادة نسبة الخطأ (BER) وتحليل النتائج تبين ان كفاءة الطيف يقل بمقدار(0.5dB)عندما يتغير نسبة الخطأ من($BER=10^{-3}$) الى($BER=10^{-6}$) واحتمالية القطع (outageprobability)يقل بزيادة الخطأ.

INTRODUCTION

Worldwide Interoperability for Microwave Access (WiMAX) isessentially a powerful and advanced system of the 4G .WiMAX is similar to Wi-Fi but bandwidth, stronger encryption is more and also improved performance over long distance by connecting between receiving stations that are not in the line of sight [1]. WIMAX is also known as “Last Mile” broadband wireless access technology WIMAX gives an alternate and better solution compared to cable and Wi-Fi technologies [2].

The capacity limits dictate the maximum data rates that can be achieved any constraints on delay or the complexity of the encoder and decoder [3]. The spectral efficiency determines the number of bits per hertz of the channel also the data rate of the transmission. However, the probability of outage determines the probability that the received SNR falls below a specified threshold.The capacity, spectral efficiency and probability of outage depend on the characteristic of the channel, which depending on the receiver of the signal to noise ratio is a function of the fading.

Adaptive Modulation Techniques

Adaptive modulation techniques are based on the variation in the transmitted power, symbol transmission rate, constellation size, coding rate/schemes, or any combination of these parameters [4].Figure (1) shows, the main units of adaptive modulation (AM) system. The AM is based on SNR measurement and depending on the value of SNR selected the type of modulation

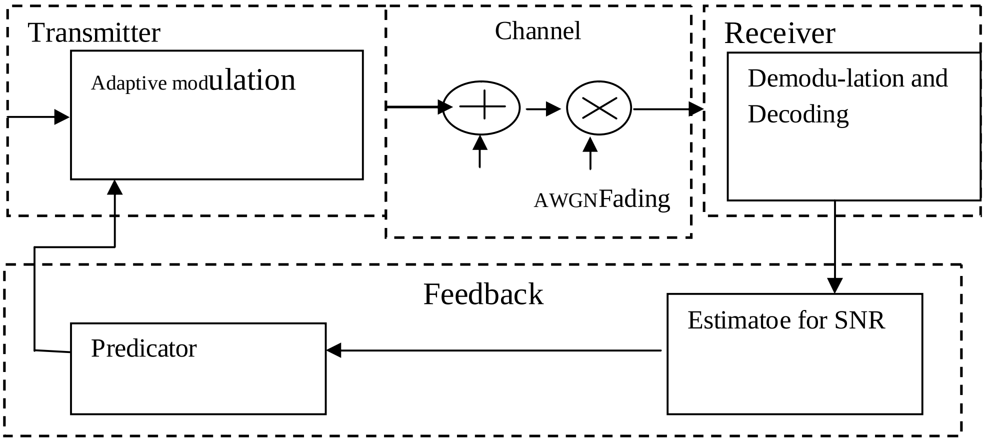

Figure (1) Typical adaptive modulation system

a. Variable-Power Techniques

Adaptive transmitted power is used to compensate for SNR variation due to fading. The goal is to maintain a constant received SNR [5].

b. Variable-Rate Techniques

In variable-rate modulation the data are varied relative to the channel gain. This can be done by fixing the symbol rate of the modulation and using multiple modulation schemes or constellation size [6].

c. Variable Error Probability

The instantaneous error probability in fading channel varies as the received SNR varies, resulting in an average. This is not considered an adaptive technique since the transmitter does not adapt to SNR. Thus, in adaptive modulation error probability is typically adapted along with some other form of adaptation such as constellation size or modulation type [5].

d. Variable Coding Techniques

In adaptive coding different codes are used to provide different amounts of coding gain to the transmitted bits. When a strong error correction code is used when SNR is small and vice to versa [3].

e. Hybrid Techniques

Hybrid techniques can adapt to multiple parameter of the transmission scheme, including rates, power, coding, and instantaneous error probability [3].

Channel Capacity in the Presence of AWGNfor WiMAX system

The channel capacity is given byShanon-Hartley equation [7],

$$C = W \log_2 \left(1 + \frac{\bar{P}}{N_0 W}\right) \qquad \ldots (1)$$

where $\frac{\bar{P}}{N_0 W}$ is the received signal-to-noise ratio (SNR). When the SNR is large (SNR >> 0 dB), the capacity $C \approx W \log_2 \frac{\bar{P}}{N_0 W}$ is logarithmic in power and approximately linear in bandwidth. This is called the *bandwidth-limited regime*. When the SNR is small (SNR << 0 dB), the capacity $C \approx \frac{\bar{P}}{N_0} \log_2 e$ is linear in power but insensitive to bandwidth. This is called the *power-limited regime* [7, 8].

Channel Capacity in the Presence of Nakagami Fading Channels

Closed-form expressions for the capacity channel for WiMAX system in the presence of Nakagami multipath fading (NMF) channels for various adaptive modulation techniques are adapted to the WiMAX system and pr
esented in this section.

Variable rate – Variable power

This method is called optimal adaptive because both rate and power are varied to obtain optimal adaptive. The channel capacity of a fading channel in this case with received SNR distribution $p_\gamma(\gamma)$ is given in [3, 4, 7 and 8] as

$$C = W \int_{\gamma_0}^{+\infty} \log_2 \left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma) \, d\gamma \qquad ...(2)$$

where

C = capacity of channel in (bit/sec)

W= the channel bandwidth in [Hz]

γ_0 = the optimal cutoff SNR, which is below this level no data is transmitted.

$p_\gamma(\gamma)$ = the probability distribution function (PDF) of the channel.

For NMF channel the probability distribution function (PDF) of the channel gain α is given by [8]

$$\mathcal{P}(\alpha) = 2(\frac{m}{\Omega})^m \frac{\alpha^{2m-1}}{\Gamma(m)} \exp\left(-m\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0 \qquad ... (3)$$

where

$\Omega = E(\alpha^2)$, is the average fading power.

m=the Nakagami fading parameter ($m \geq 1/2$).

$\Gamma(.)$ = is the gamma function.

The received SNR,γ, is then gamma distributed according to the PDF,$p_\gamma(\gamma)$is given by[8]

$$p_\gamma(\gamma) = (\frac{m}{\bar{\gamma}})^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\bar{\gamma}}\right) \quad ,\gamma \geq 0 \qquad ...(4)$$

Where

γ = the received SNR.

$\bar{\gamma}$ = the average received SNR.

The Nakagami fading represents a wide range of multipath channels via the m fading parameter. The Rayleigh distribution (m=1) isa special case [8, 9].

The complete derivation is given in Appendixes (A,B). The NMF channel capacity per bandwidth C/W[bits/sec/Hz] under the variable power and the rate adaptation policy is given by

$$\frac{C}{W} = \log_2(e) \sum_{k=0}^{m-1} \frac{\Gamma(k, m\frac{\gamma_0}{\bar{\gamma}})}{k!} \quad \ldots(5)$$

1.1 *Variable rate – constant power*

Shannon capacity of a fading with receiver Channel Side Information (CSI) for an average power transmitter $\bar{S}$ can be obtained from results [3, 4, 9and 10] as

$$C = \int_0^{\infty} B \log_2(1+\gamma) p_\gamma(\gamma) d\gamma \quad \ldots(6)$$

The complete derivation is given in Appendix (C).C/W [bits/sec/Hz] for Variable rate – constant powercan bewritten as

$$\frac{C}{W} = \log_2(e) e^{m/\bar{\gamma}} \sum_{k=0}^{m-1} (\frac{m}{\bar{\gamma}})^k \,\Gamma\left(-k, \frac{m}{\bar{\gamma}}\right) \quad \ldots(7)$$

1.2 *Fixed rate-Variable power*

In this case the received power is maintainedconstant and the transmitted power of the transmitter is varied. The channel capacity when the transmitter adapts its power to maintain a constant SNR at the receiver (by using the channel inversion or equalizer) is also investigated in [7, 8, 10, 11 and 13]. The channel after using ideal channel inversion or equalizer appears as a time-invariant AWGN channel. The channel capacity with this technique (C [bits/sec]) derives from the capacity of an AWGN channel and is given by

$$C = W \log_2(1 + \frac{1}{\int_0^{+\infty} \frac{p_\gamma(\gamma)}{\gamma} d\gamma}) \quad \ldots (8)$$

Another approach is to use a modified inversion policy which inverts the channel fading only above a fixed cutoff fade depth γ_0. The capacity with this truncated channel inversion and fixed rate policy (C [bits/sec]) is givenby [7,12]

$$C = W \log_2(1 + \frac{1}{\int_{\gamma_0}^{+\infty} \frac{p_\gamma(\gamma)}{\gamma} d\gamma}) \,(1 - P_{out}) \quad \ldots (9)$$

where P_{out} is given by equation (4) in Appendix B. The cutoff level $\gamma_0$can be selected to achieve a specified outage probability.

By substituting the SNR distribution equation (4) from equation (8) it is found that the capacity per bandwidth of an NMF channel with total channel inversion,C/W, is given for all $m \geq 1$by

$$\frac{<C>_{cifr}}{W} = \log_2(1 + \frac{m-1}{m}\bar{\gamma}) \quad \ldots (10)$$

Thus the capacity of a Rayleigh fading channel ($m = 1$) is zero in this case. It should be noted that the capacity of this policy for an NMF channel is the same as the capacity of an AWGN channel with equivalent $SNR = \frac{m-1}{m}\bar{\gamma}$.

With truncated channel inversion the capacity per bandwidth C/W[bits/sec/Hz] can be expressed in terms of $\bar{\gamma}$and γ_0. Substituting equation (4) intothe equation (9), yields

$$\frac{C}{W} = \log_2(1 + \frac{\bar{\gamma}\Gamma(m)}{m\Gamma(m-1, m\gamma_0/\bar{\gamma})})\frac{\Gamma(m, m\gamma_0/\bar{\gamma})}{\Gamma(m)}),\ m \geq 1 \qquad \dots\ (11)$$

For the special case of the Rayleigh fading channel (m = 1), the capacity per unit bandwidth with truncated channel inversion reduces to

$$\frac{C}{W} = \log_2(1 + \frac{\bar{\gamma}}{E_1(\gamma_0/\bar{\gamma})})e^{-\gamma_0/\bar{\gamma}} \qquad \dots\ (12)$$

Analysis of Spectral Efficiency

Let S(γ) denote the transmit power adaptation policy relative to an instantaneous value of γ, subject to the average power constraint

$$\int S(\gamma)\mathcal{P}(\gamma)d\gamma \ \leq\ \bar{S} \qquad \dots(13)$$

The capacity of a fading channel with bandwidth B and average power $\bar{S}$ is

$$C = \max_{S(\gamma)\int S(\gamma)\mathcal{P}(\gamma)d\gamma=\bar{S}} \int_0^\infty B\log_2\left(1 + \frac{S(\gamma)\gamma}{\bar{S}}\right)\mathcal{P}(\gamma)d\gamma \qquad \dots\ (14)$$

The power adaptation which maximizes is given by [3, 4, 9,10]

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma}, & \gamma \geq \gamma_0 \\ 0, & \gamma < \gamma_0 \end{cases} \qquad \dots(15)$$

Substituting equation (15) into equation (13), it is seen that γ_0 is determined by numerically solving

$$\int_{\gamma_0}^\infty \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right)\mathcal{P}(\gamma)d\gamma\ =\ 1 \qquad \dots\ (16)$$

For some (γ) distributions, a closed-form expression for γ_0 may be found. Once γ_0 is known as in equation (2)

The maximum spectral efficiency can be obtained by dividing both sides of the equation (2) by the channel bandwidth B.

It is noted that for a constant transmit power $S(\gamma) = \bar{\gamma}$, the capacity of the equation (14) reduces to equation (6)

Spectral efficiency is maximizeby using optimal power control equation (15) which achieves capacity [4, 9]:

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma K}, & \gamma \geq \gamma_0/K \\ 0, & \gamma < \gamma_0/K \end{cases} \qquad \dots\ (17)$$

whereγ_0/K is optimized cutoff fade depth and

$$K\ = \frac{-1.5}{\ln{(5BER)}} \qquad \dots\ (18)$$

If one defines$\gamma_K = \gamma_0/K$ and substitutes equation (17) into equation (2), the maximum spectral efficiency is obtained

$$\frac{R}{B} = \int_{\gamma_K}^{\infty} \log_2\left(\frac{\gamma}{\gamma_K}\right)\mathcal{P}(\gamma)d\gamma \quad \ldots(19)$$

Analysis of Probability of Outage

Outage Probability (P_{out}) is defined as the probability that the instantaneous error probability exceeds a specified value. Outage probability is given by, [7]

$$Pout = \int_0^{\gamma_0} \mathcal{P}_\gamma(\gamma)d\gamma \quad \ldots(20)$$

where

γ_0 = the “cutoff” value.

If the received signal power is below this level, then no power is allocated to data transmission, so the outage probability for this policy is p ($\gamma < \gamma_0$) [7] and $\mathcal{P}_\gamma(\gamma)$ is the probability density function (PDF) [7, 9]. In Rayleigh fading the outage probability becomes [1]:

$$P_{out} = \int_0^{\gamma_0} \frac{1}{\overline{\gamma_s}} e^{-\gamma_s/\overline{\gamma_s}}\, d\gamma_s = 1 - e^{-\gamma_s/\overline{\gamma_s}} \quad \ldots(21)$$

Inverting this formula shows that for a given outage probability, the required average SNR($\overline{\gamma_s}$) is

$$\overline{\gamma_s} = \frac{\gamma_0}{-\ln(1-P_{out})} \quad \ldots(22)$$

Since no data are sent when the receiver SNR falls belowγ_1, theMQAM scheme suffers an outage probability,

$$P_{out} = \int_0^{\gamma_1} \mathcal{P}_\gamma(\gamma)d\gamma = 1 - \frac{\Gamma(m, \frac{m\gamma_1}{\overline{\gamma}})}{\Gamma(m)} \quad \ldots(23)$$

where $\gamma_1 = [erfc^{-1}(2BER_0)]^2$, BER_0 is equal to target BER.

Results and Discussion

Fig (2) shows the channel capacity versus the SNR for differentWiMAX system bandwidths (BW=1, 1.25, 5, 10, 15, 20 MHz) in the presence of AWGN, by using the equation (1). Results show that when the bandwidthincrease, the capacity of channel will be increased.

Figs (3, 4 and 5) show the channel capacity versus the SNR for different bandwidths (BW=1, 1.25, 5, 10, 15, 20 MHz) by using equation (5) for NMF channel m=1(Rayleigh channel), m=2 and m=3. If m is increased, then the effect of the fading channel increase.

Figs (6, 7 and 8) show the channel capacity versus the SNR for different bandwidths (BW=1, 1.25, 5, 10, 15, 20 MHz) by using equation (7) in Variable rate-Constant power technique with NMF channel m=1, m=2 and m=3. Results show higher capacity is obtainedin the presence of Nakagami fading (m=3).

Figs (9, 10 and 11) show the channel capacity versus the SNR for different bandwidths (BW=1, 1.25, 5, 10, 15, 20 MHz) by using equation (11) in Truncated channel inversion technique with NMF channel m=1, m=2 and m=3. Results show minimum capacity is obtained in the presence of Rayleigh fading .

Figs (12 and 13) show the channel capacity versus the SNR for different bandwidths (BW=1, 1.25, 5, 10, 15, 20 MHz) by usingequation (10) in Total channel inversion technique with NMF channel m=2 and m=3.For m=1, the channel capacity is equal to zero. The minimum capacity is obtained in this case.

Fig (14) shows the spectral efficiency of adaptive MQAM for Variable rate-Variable power) versus the SNR in the presence of Rayleigh fading for $P_b = 10^{-3}, 10^{-4} \& 10^{-6}$ by using equation (19).

Figs (15 & 16) show the spectral efficiency of adaptive MQAM for Variable rate-Variable power versus the SNR in the presence of Nakagami fading m=2 & m=3 for $P_b = 10^{-3}, 10^{-4} \& 10^{-6}$ by using equation (19).Figs (13, 14 and 15) show the highest spectral efficiency is obtained in the precened of Nakagami fading.

Fig (17) shows the spectral efficiency of adaptive MQAM for Variable rate-Constant power versus the SNR in the presence of different types of fading by using equation (6).

Figs (18, 19 and 20) show the outage probability versus the SNR for Rayleigh fading and Nakagami fading by using equation (21). Resultsshow the probability of outage increases when Nakagami fading parameter(m) increases.

CONCLUSION

The following points represent the main conclusions drawn from this paper:-

1. The use of adaptive modulation allows a wireless system to choose higher order modulation depending on the SNR. Different order modulations allow sending more bits per symbol and achieving higher data rate and better spectral efficiency.
2. Adaptive modulation is suggested forWiMAX to enhance the capacity ofWiMAX.
3. The higher capacity is obtained in the presence of Variable rate- Variable power technique.
4. The capacity of NMF channels is always smaller than the capacity of an AWGN channel.
5. The different techniques of adaptive modulation have not changed the large value of capacity when the type of techniqueis change.
6. The highest spectral efficiency is obtained in the presence of Nakagami fading. The optimal spectral efficiency is obtained by using Shannon capacity.
7. The minimum outage is achieved in the presence of Nakagamifadingand whenit increases,the Nakagami fading parameter (m) decreases. The probability of outage as well as the higher BER (Bit Error Rate) give a higher probability of outage.

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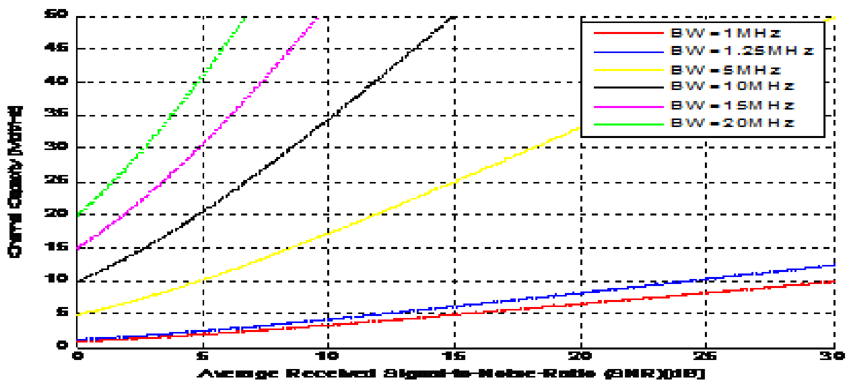

Figure (2) Channel capacity versus average received (S/N) of WiMax system in the presence AWGN.

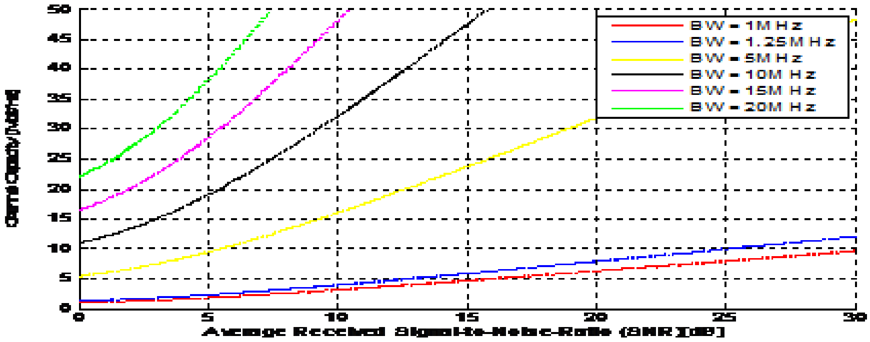

Figure (3) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Variable power in the presence of Rayleigh fading.

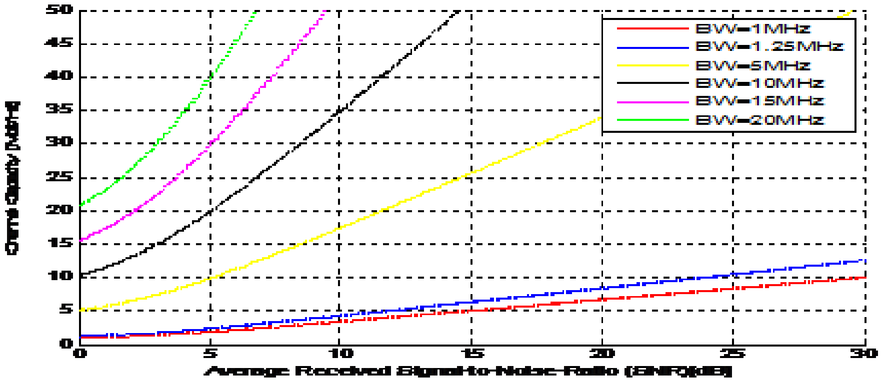

Figure (4) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Variable power in the presence of Nakagami fading (m=2).

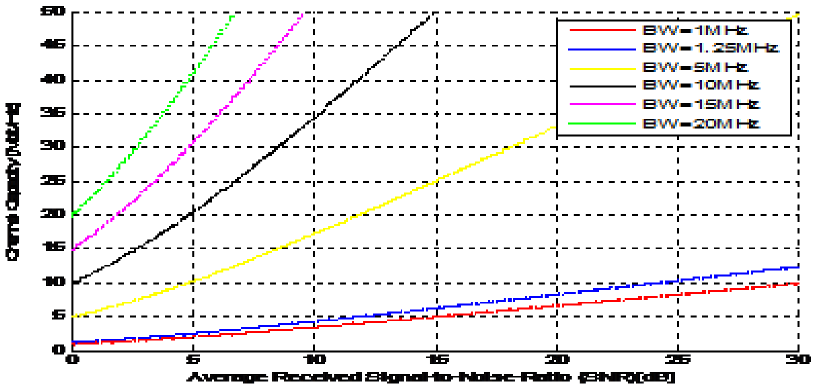

Figure (5) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Variable power in the presence of Nakagami fading (m=3).

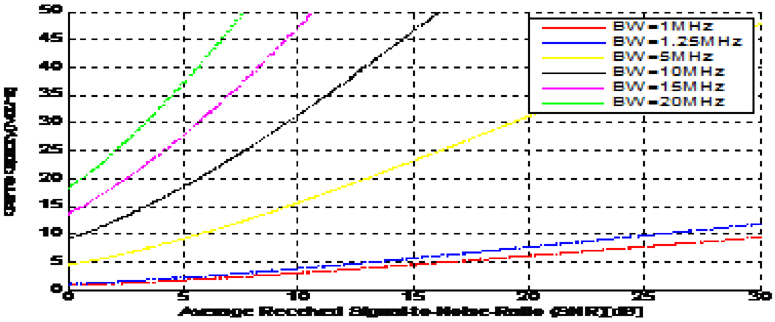

Figure (6) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Constant power in the presence of Rayleigh fading.

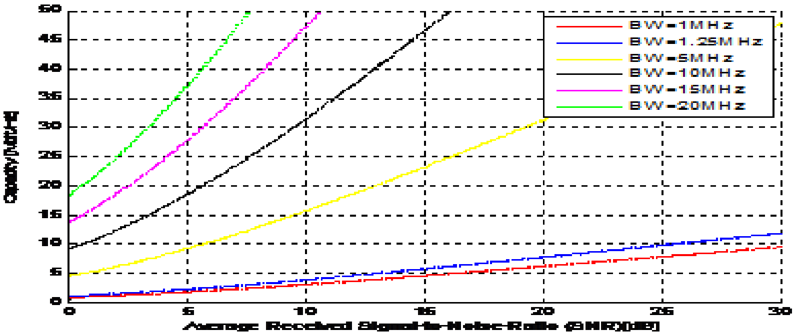

Figure (7) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Constant power in the presence of Nakagami fading (m=2).

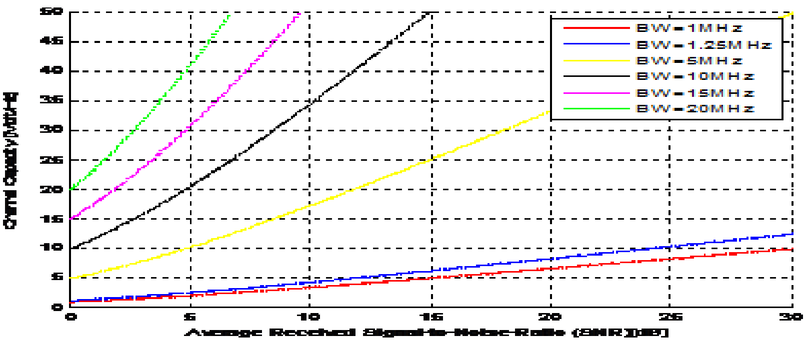

Figure (8) Channel capacity versus average received (S/N) for WiMax system with Variable- rate & Constant power in the presence of Nakagami fading (m=3).

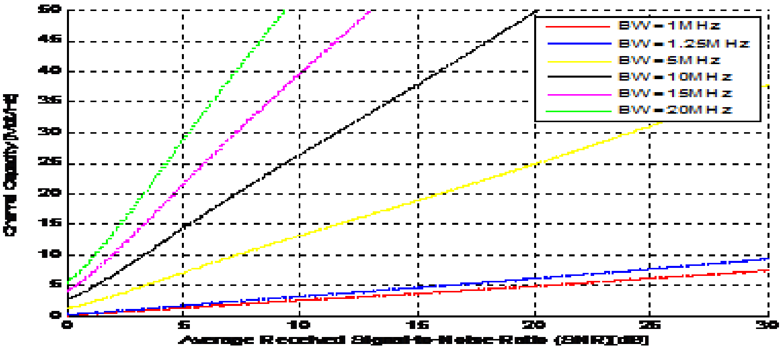

Figure (9) Channel capacity versus average received (S/N) for WiMax system with Truncated channel Inversion in the presence of Rayleigh fading (m=1).

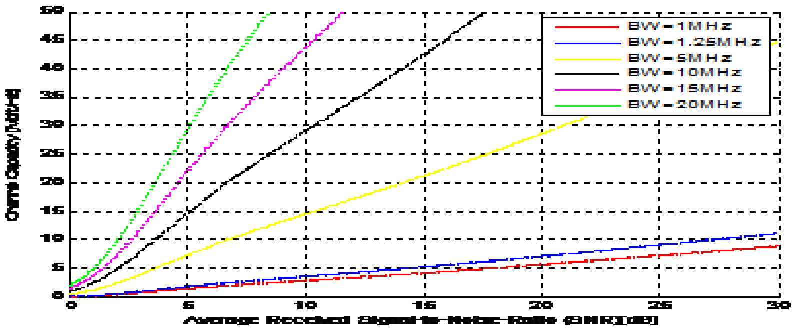

Figure (10) Channel capacity versus average received (S/N) for WiMax system with Truncated channel Inversion in the presence of Nakagami fading channel (m=2).

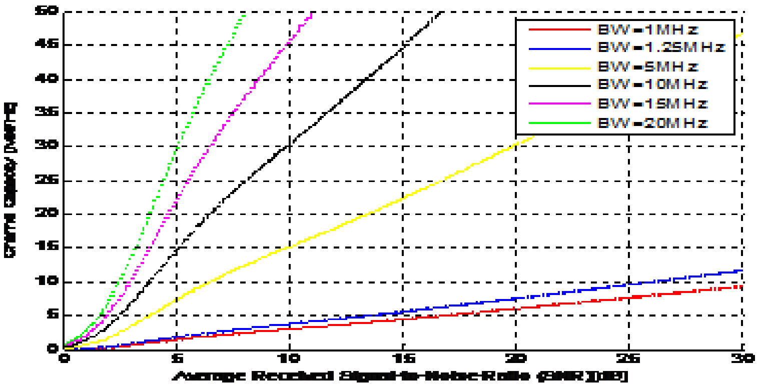

Figure (11) Channel capacity versus average received (S/N) for WiMax system with Truncated channel Inversion in the presence of Nakagami fading channel (m=3).

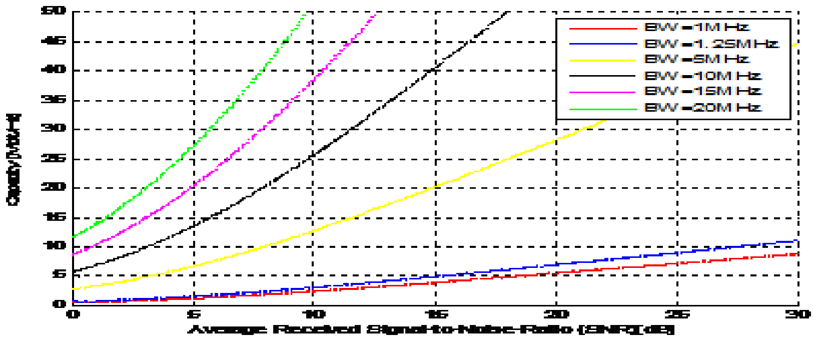

Figure (12) Channel capacity versus average received (S/N) for WiMax system with Total channel Inversion in the presence of Nakagami fading (m=2).

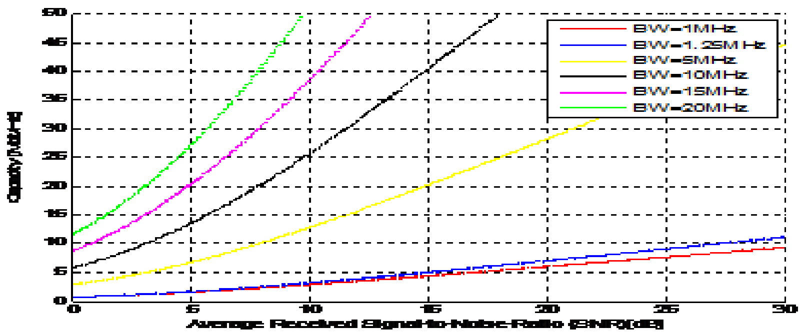

Figure (13) Channel capacity versus average received (S/N) for WiMax system with Total channel Inversion in the presence of Nakagami fading (m=3).

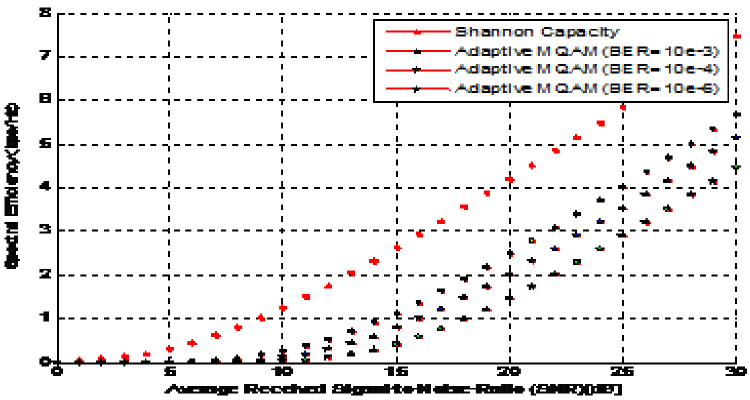

Figure (14) Spectral efficiency of Adaptive MQAM (Variable-rate Variable-power) in the presence of Rayleigh fading for $P_b = 10^{-3}, 10^{-4} \& 10^{-6}$

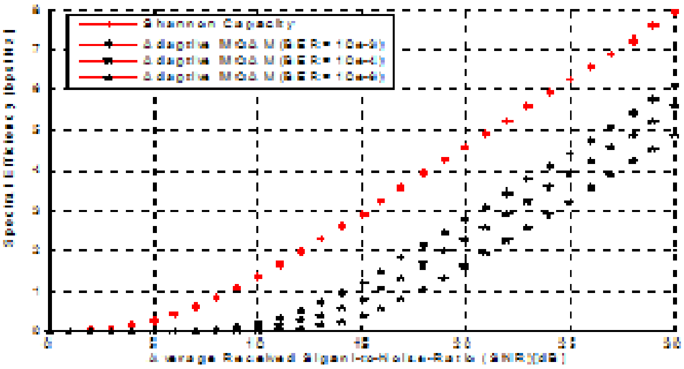

Figure (15) Spectral efficiency of Adaptive MQAM (Variable-rate Variable-power) in the presence of Nakagami fading (m=2) for ??$_{??}$ = ???? ??, ??? ??&???? ??.

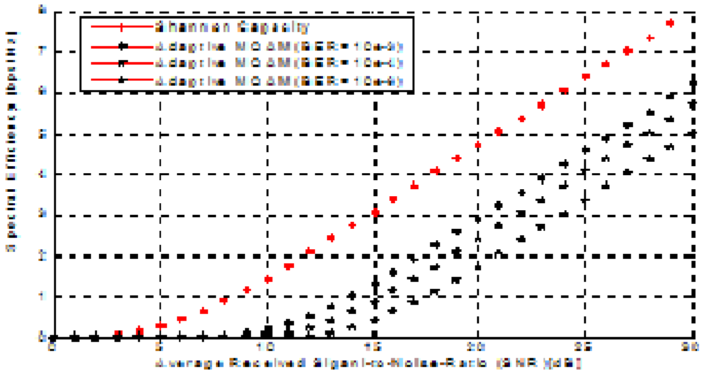

Figure (16) Spectral efficiency of Adaptive MQAM (Variable-rate Variable-power) in the presence of Nakagami fading (m=3) for ??$_{??}$ = ???? ??, ??? ??&???? ??.

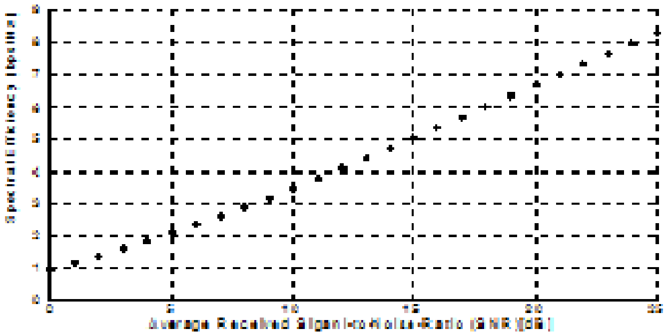

Figure (17) Spectral efficiency for constant transmit power-Variable rate

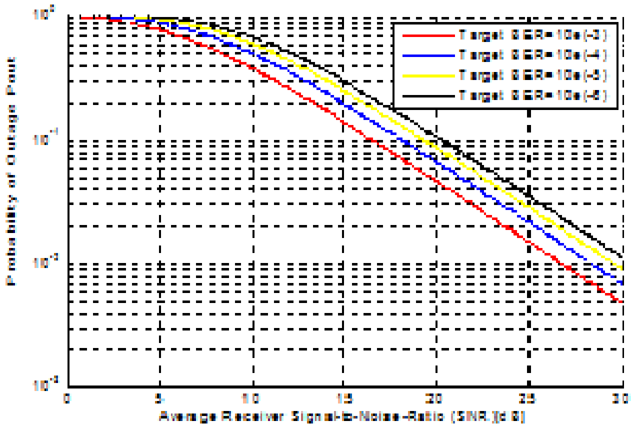

Figure (18) Outage probability for Rayleigh fading

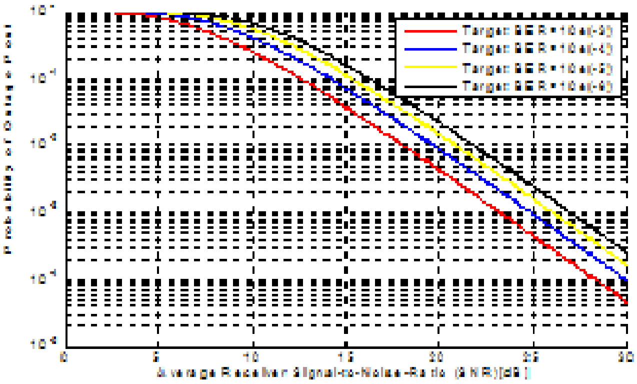

Figure (19) Outage probability for Nakagami fading (m=2)

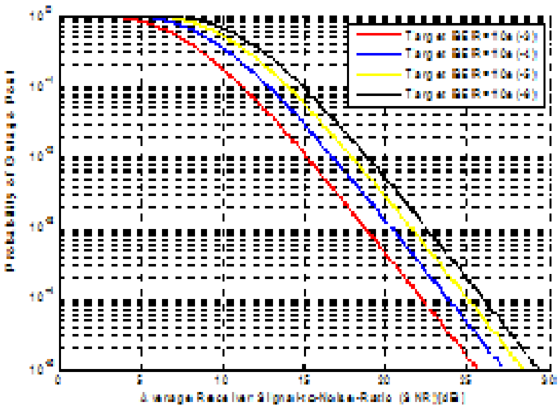

Figure (20) Outage probability for Nakagami fading (m=3)

Appendix (A)

Gamma function and Incomplete Gamma function

A.1 Gamma function

In mathematics, the gamma function (represented by the capital Greek letter Γ) is an extension of the factorial function, with its argument shift down by 1, to real and complex numbers. That is if n is a positive integer [15]:

$$\Gamma(n) = (n-1)! \tag{A.1}$$

The gamma function is defined for all complex numbers expcet the non-positive integers. For complex numbers with positive real part, it is defined via an improper integral that converges:

$$\Gamma(n) = \int_0^\infty e^{-t} t^{n-1}\, dt \tag{A.2}$$

The integral function is extended by analytic continuation to all complex numbers except the non-positive integers (where the function has simple poles), yielding the meromorphic function called the gamma function. The gamma function can be defined for negative value of the argument n by using the formula [16, 17]

$$\Gamma(n) = \frac{\Gamma(n+1)}{n} \tag{A.3}$$

Because the Gamma and factorial functions grow so rapidly for moderately large arguments, many computing environments include a function that returns the natural logarithm of the gamma function (often given the name lngamma in programming environments or gammaln in spreadsheets); this grows much more slowly, and for combinatorial calculations allows adding and subtracting logs instead of multiplying and dividing very large values.

Important Properties[16, 17]

1) $\Gamma(x+1) = x\Gamma(x)$
2) $\Gamma(x)\Gamma(1-x) = \pi/\sin\pi x$
3) $2^{2x+1}\,\Gamma(x)\Gamma\left(x+\frac{1}{2}\right) = \sqrt{\pi}\,\Gamma(2x)$ (duplication formula)
4) $\Gamma(x)\Gamma\left(x+\frac{1}{m}\right)\Gamma\left(x+\frac{2}{m}\right)\ldots\ldots\Gamma\left(x+\frac{m-1}{m}\right) = m^{1/2-mx}2x^{(m-1)/2}\Gamma(mx) \quad m = 1,2,3,\ldots.$

Special Value

1) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
2) $\Gamma\left(m+\frac{1}{2}\right) = \frac{1.3.5....(2m-1)}{2^m}\sqrt{\pi} \qquad m = 1,2,3,\ldots\ldots$
3) $\Gamma\left(-m+\frac{1}{2}\right) = \frac{(-1)^m 2^m}{1.3.5...(2m-1)}\sqrt{\pi} \qquad m = 1,2,3,\ldots\ldots$

Incomplete Gamma Function

$\Gamma(.,.)$is the complementary incomplete gamma function. The complementary incomplete gamma function is defined as

$$P(a,x) \equiv \frac{(a,x)}{\Gamma(a)} \equiv \frac{1}{\Gamma(a)}\int_0^x e^{-t}t^{a-1}\,dt, (a>0) \tag{A.4}$$

It has the limiting values

$$P(a,0) = 0 \quad and\ P(a,\infty) = 1 \tag{A.5}$$

The incomplete gamma function $P(a,x)$ is monotonic and (for a greater than one or so) rises from “near-zero” to “near-unity” in a range of x centered about $(a-1)$, and width about $\sqrt{a}$.
The complement of $P(a,x)$ is also confusingly called an incomplete gamma function [17],

$$Q(a,x) \equiv 1 - P(a,x) \equiv \frac{\Gamma(a,x)}{\Gamma(a)} \equiv \frac{1}{\Gamma(a)} \int_x^\infty e^{-t} t^{a-1} dt \qquad \text{(A.6)}$$

It has the limiting values

$$Q(a,0) = 1 \quad and \quad Q(a,\infty) \qquad \text{(A.7)}$$

Special Values [15, 16, 17]

1) $\Gamma(s,x) = (s-1)!\ e^{-s} \sum_{k=0}^{s-1} \frac{x^k}{K!}$ If s is a positive integer,
2) $\Gamma(s,0) = \Gamma(s), \Re(s) > 0,$
3) $\Gamma(1,s) = e^{-x},$
4) $(1-x) = 1 - e^{-x},$
5) $\Gamma(0,x) = -E_i(-x)\ for\ x > 0$,
6) $\Gamma(s,x) = x^s E_{1-s}(x),$
7) $\Gamma\left(\frac{1}{2},x\right) = \sqrt{\pi}\ erfc(\sqrt{x}),$
8) $\left(\frac{1}{2},x\right) = \sqrt{\pi}\ erfc(\sqrt{x}),$
9) $\Gamma(-s+k,x) = \frac{(-1)^{s-k}}{(s-k)!}[\Gamma(0,x) - e^{-x} \sum_{m=0}^{s-k-1} (-1)^m \frac{m!}{x^{m+1}}]\ \ [s-k \geq 1, k = 0,1, \ldots\ldots]$.

Appendix (B)
The Deviation the Capacity of Variable rate- Variable power

In this Appendix deviation of the capacityof Variable rate- Variable power is given [3, 4, 8]s

$$C = W \int_{\gamma_0}^{+\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma)\ \ d\gamma \qquad \text{(B.1)}$$

$p_\gamma(\gamma)$isgiven by[8]

$$p_\gamma(\gamma) = (\frac{m}{\overline{\gamma}})^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right)\ \ ,\gamma \geq 0 \qquad \text{(B.2)}$$

The optimal cutoff (γ_0)can be evaluated by the following equation

$$\int_{\gamma_0}^{+\infty} (\frac{1}{\gamma_0} - \frac{1}{\gamma})\ p_\gamma(\gamma)\ \ d\gamma = 1 \qquad \text{(B.3)}$$

Note that this expression depends only on the distribution (γ). The value for γ_0 cannot be solved for in closed form typical continuous pdf (γ) and thus must be found numerically [11]. Since no data are sentwhen $\gamma < \gamma_0$, the optimal policy suffers a probability of outage P_{out}, equal to the probability of no transmission, given by [8]

$$P_{out} = \int_0^{\gamma_0} p_\gamma(\gamma)\, d\gamma\ = 1 - \int_{\gamma_0}^{+\infty} p_\gamma(\gamma)\, d\gamma \qquad \text{(B.4)}$$

Substituting equation (B.2) in equation (B.3) yields

$$\int_{\gamma_0}^{+\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) (\frac{m}{\overline{\gamma}})^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right) d\gamma \qquad \text{(B.5)}$$

where$\Gamma(.,.)$ = the complementary incomplete gamma function.
Using the properties of incomplete gamma function in Appendix (A)) and let $x = m\frac{\gamma}{\bar{\gamma}}$

$$\int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)\left(\frac{m}{\bar{\gamma}}\right)^{m}\left(\frac{m}{\bar{\gamma}}\right)^{-1}\left(\frac{m}{\bar{\gamma}}\right)\frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)d\gamma = 1 \quad (B.6)$$

By arrangement equation (B.6) becomes

$$\int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)\left(m\frac{\gamma}{\bar{\gamma}}\right)^{m-1}\left(\frac{m}{\bar{\gamma}}\right)\frac{\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)}{\Gamma(m)}\,d\gamma = 1 \quad (B.7)$$

$$\Gamma(m,x) = \int_{\gamma_0}^{\infty}\left(m\frac{\gamma}{\bar{\gamma}}\right)^{m-1}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)d\gamma \quad (B.8)$$

From Appendix (A), equation (B.8) becomes

$$\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)\frac{m\Gamma(m,x)}{\bar{\gamma}\Gamma(m)} = 1 \quad (B.9)$$

By arrangement of the equation and using the properties of the complement incomplete gamma function equation (B.9) becomes

$$\frac{\Gamma(m,m\frac{\gamma_0}{\bar{\gamma}})}{\frac{\gamma_0}{\bar{\gamma}}} - m\Gamma\left(m-1,m\frac{\gamma_0}{\bar{\gamma}}\right) = \bar{\gamma}\,\Gamma(m) \quad (B.10)$$

In a special case of the Rayleigh fading channel (m=1), (B.10) reduces to

$$\frac{e^{-\gamma_0/\bar{\gamma}}}{\gamma_0/\bar{\gamma}} - E_1\left(\frac{\gamma_0}{\bar{\gamma}}\right) = \bar{\gamma} \quad (B.11)$$

where$E_1(\,.\,)$ = the exponential integral of first order defined by [13].

$$E_1 = \int_1^{\infty}\frac{e^{-xt}}{t}\,dt \quad ; \quad x \geq 0. \quad (B.12)$$

Substituting equation (B.2) into equation (B.1) given

$$C = W\int_{\gamma_0}^{+\infty}\log_2\left(\frac{\gamma}{\gamma_0}\right)\left(\frac{m}{\bar{\gamma}}\right)^{m}\frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)d\gamma \quad (B.13)$$

Let the integral $\mathcal{J}_n$be$\mathcal{J}_n(\mu) = \int_1^{+\infty} t^{n-1}\ln(t)\,e^{-\mu t}\,dt\,; \quad \mu > 0$ (B.14)

Let n = m, $\mu = -m\frac{\gamma}{\bar{\gamma}}$ and t = γ, equation (B.13) becomes

$$\mathcal{J}_m\left(-m\frac{\gamma}{\bar{\gamma}}\right) = \int_1^{+\infty}\gamma^{m-1}\ln(\gamma)\,e^{-(-m\frac{\gamma}{\bar{\gamma}})\gamma}\,d\gamma \quad (B.15)$$

Equation (B.15) is substitutedin equation (B.13), the channel capacity Ccan re-writtenas
$\frac{C}{W} = \frac{\log_2(e)}{\Gamma(m)}\left(\frac{m\gamma_0}{\bar{\gamma}}\right)^{m}\mathcal{J}_m\left(\frac{m\gamma_0}{\bar{\gamma}}\right)$(B.16)
The evaluation of $\mathcal{J}_m$for n a positive integer is derived in [13].
Using that result one can obtain the NMF channel capacity per bandwidth C/W[bits/sec/Hz] under the optimal power and the rate adaptation policy is given by

$$\frac{C}{W} = \log_2(e)\sum_{k=0}^{m-1}\frac{\Gamma(k,m\frac{\gamma_0}{\bar{\gamma}})}{k!} \quad (B.17)$$

This can also be written as [8]

$$\frac{C}{W} = \log_2(e)\left(E_1(m\gamma_0/\bar{\gamma}) + \sum_{k=1}^{m-1}\frac{\mathcal{P}_k(m\gamma_0/\bar{\gamma})}{k}\right. \quad (B.18)$$

where$\mathcal{P}_k(.)$denotes the Poisson distribution defined by,

$$\mathcal{P}_k(\mu) = e^{-u}\sum_{j=0}^{k-1}\frac{\mu^j}{j!} \quad (B.19)$$

For the special case of the Rayleigh fading channel, using equation (B.12) in equation (B.17) for $m = 1$, the optimal capacity per unit bandwidth reduces to the simple expression

$$\frac{C}{W} = \log_2(e)\left(\frac{e^{-\gamma_0/\bar{\gamma}}}{\gamma_0/\bar{\gamma}}\right) \quad \text{(B.20)}$$

Using equation (B.3) in the probability of the outage equation (B.4) yields

$$P_{out} = 1 - \frac{\Gamma(m, m\gamma_0/\bar{\gamma})}{\Gamma(m)} = 1 - \mathcal{P}_m(m\gamma_0/\bar{\gamma}) \quad \text{(B.21)}$$

Appendix (C)

The Deviation of the Capacity of Variable rate- constant power

Shannon capacity of a fading with receiver CSI (Channel Side Information) for an average power transmitter $\bar{S}$ is given as

$$C = \int_0^\infty B\log_2(1+\gamma)p_\gamma(\gamma)d\gamma \quad \text{(C.1)}$$

$p_\gamma(\gamma)$isgiven by [8]

$$p_\gamma(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right) \quad ,\gamma \geq 0 \quad \text{(C.2)}$$

Note that this formula is a probabilistic average, i.e. Shannon capacity is equal to Shannon capacity for an AWGN channel with SNR γ, averaged over the distribution of γ. That is why Shannon capacity is also called ergodic capacity [3]. The Shannon capacity of a channel defines its theoretical upper bound for the maximum rate of data transmission at an arbitrarily small BER, without any delay or complexity constraints. Therefore, the Shannon capacity represents an optimistic bound for practical communication schemes, and also severs as a benchmark against which to compare the spectral efficiency of all practical adaptive transmission schemes [15].

In fact, equation(C.1) represents the capacity of the fading channel without transmitter feedback (i.e. with the channel fade level known at the receiver only) [8].

Substituting equation (C.2) in equation (C.1) yields

$$C = W\int_0^\infty \log_2(1+\gamma)\left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)d\gamma \quad \text{(C.3)}$$

$$\frac{C}{W} = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\log_2(e)}{\Gamma(m)}\int_0^\infty \ln(1+\gamma)\,\gamma^{m-1}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)d\gamma \quad \text{(C.4)}$$

Defining the integral $\ell_n(\mu)$ [10] as

$$\ell_n(\mu) = \int_0^\infty t^{n-1}\ln(1+t)\,e^{-\mu t}dt \quad \text{(C.5)}$$

Let $n = m$ and $\mu = \frac{m}{\bar{\gamma}}$, and substitute equation (5) become

$$\ell_m\left(\frac{m}{\bar{\gamma}}\right) = \int_0^\infty \gamma^{m-1}\ln(1+\gamma)\,e^{-\left(\frac{m}{\bar{\gamma}}\right)}d\gamma \quad \text{(C.6)}$$

From [8]

$$\ell_m\left(\frac{m}{\bar{\gamma}}\right) = (m-1)e^{\left(\frac{m}{\bar{\gamma}}\right)}\sum_{k=1}^{m}\frac{\Gamma\left(-m+k,\frac{m}{\bar{\gamma}}\right)}{\left(\frac{m}{\bar{\gamma}}\right)^k} \quad \text{(C.7)}$$

Substituting equation (C.7) in equation (C.6) results in

$$\frac{C}{W} = \left(\frac{m}{\bar{\gamma}}\right)^{m} \frac{\log_2(e)}{\Gamma(m)}(m-1)!\ e^{\frac{m}{\bar{\gamma}}} \sum_{k=1}^{m} \frac{\Gamma(-m+k,\frac{m}{\bar{\gamma}})}{(\frac{m}{\bar{\gamma}})^{k}} \qquad \text{(C.8)}$$

By the arrangementof equation (C.8), one can rewrite C/W [bits/sec/Hz] to become

$$\frac{C}{W} = \log_2(e)e^{m/\bar{\gamma}} \sum_{k=0}^{m-1} \left(\frac{m}{\bar{\gamma}}\right)^{k} \Gamma\left(-k,\frac{m}{\bar{\gamma}}\right) \qquad \text{(C.9)}$$

One may also express equation (C.9) in term of the Poisson distribution as [13] resulting in

$$\frac{C}{W} = \log_2(e)(\mathcal{P}_m(-m/\bar{\gamma})\, E_1(m/\bar{\gamma}) + \sum_{k=1}^{m-1} \frac{\mathcal{P}_k(m/\bar{\gamma})\mathcal{P}_{m-k}(-m/\bar{\gamma})}{k} \qquad \text{(C.10)}$$

For the special case of Rayleigh fading channel (m=1), equation (C.10) reduces to

$$\frac{C}{W} = \log_2(e)e^{1/\bar{\gamma}}E_1(1/\bar{\gamma}) \qquad \text{(C.11)}$$