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SOME RESULTS OF (α, β) DERIVATIONS ON PRIME SEMIRINGS

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Abstract
This paper investigates the concept (*α, β*) derivation on semiring and extend a few results of this map on prime semiring. We establish the commutativity of prime semiring and investigate when (*α, β*) derivation becomes zero.

Keywords: Semirings, Prime Semirings, Semiprime Semirings, (*α, β*) Derivation.

بعض النتائج للمشتقات (α, β) على اشباه الحلقات الاولية

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الخلاصة
في هذا البحث درسنا مفهوم المشتقات (*α, β*) على اشباه الحلقات وقمنا بتوسيع بعض النتائج على اشباه الحلقات الاولية. و حصلنا على ابدالية اشباه الحلقات الاولية وكذلك متى تصبح المشتقات (*α, β*) صفرا عليها.

1. Introduction

The notation of semiring was first introduced by Vandiver in 1934, then many researchers had been studying diverse kinds of semirings , it's properties and different types of derivations on it. A nonempty set say S together with two binary operations (addition and multiplication), this triple is called semiring, if S with addition is a semigroup, S with multiplication is also semigroup and addition distributive with respect to multiplication on S [1]. The only difference between ring and semiring conditions is there´s no addition invertible elements in semirings but this property exist in rings since the set together with addition define a group. If we suppose S any semiring and $D: S \rightarrow S$ be a map defined on S, then D is called additive map if it preserves addition relation. Now, this additive map said to be derivation on S if $D(xy) = D(x)y + xD(y)$ for all x and y in S. Moreover (α, β) derivation introduced as d is derivation on S and α, β is two automorphisms on S such that $d(xy) = \alpha(x)d(y) + d(x)\beta(y)$ for all x and y in S [2]. We also used commutator which is defined in [3] as $[x, y] = xy - yx$ with $[x + y, z] = [x, z] + [y, z]$ and $[xy, z] = x[y, z] + [x, z]y$. We'll also present some necessary definitions for this paper in the preliminaries.

2. Preliminaries

Definition 2.1: - [4] A nonempty set S with the binary operation $*$ said to be semigroup iff $x * (y * z) = (x * y) * z$ for all $x, y, z \in S$.
Definition 2.2: - [4] A semigroup S called commutative iff $x * y = y * x$ for all $x, y \in S$.
Definition 2.3: - [4] A nonempty set S with two binary operation + and . is said to be a semiring iff the following conditions satisfied:-
1- $(S, +)$ Semigroup.
2- $(S, .)$ Semigroup.

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3- $x. (y + z) = x. y + x. z$ and $(y + z) .x = y. x + z. x$ for all $x, y, z \in S$.

Notation: Throughout this paper we shall assume that S contains 0 and 1.

Example 2.4: - [4] Let $B = \{0, 1\}$ and +, . defined on B by the tables below:

+	0	1
0	0	1
1	1	1

.	0	1
0	0	0
1	0	1

(B, +, .) is semiring.

Example 2.5: - [4] Let $Z_0^+ = \{x \in Z: x \geq 0\}$, + and . are usual addition and usual multiplication, then (Z_0^+, +, .) is semiring but not ring.

Example 2.6: - Let $S = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in Z_0^+ \}$ with usual matrices addition and multiplication of integers, then S is semiring.

Definition 2.7: - [5] A semiring S is called additively commutative iff $x + y = y + x$ for all $x, y \in S$, and called multiplicatively commutative iff $x. y = y. x$ for all $x, y \in S$. Also S is called commutative semiring iff it is both additively and multiplicatively commutative.

Definition 2.8: - [6] A semiring S is called additively cancellative iff $x + y = x + z$ implies $y = z$ for all $x, y, z \in S$, and it is called multiplicatively cancellative iff $x. y = x. z$ implies $y = z$ for all $x, y, z \in S$. Also S called cancellative semiring iff it is both additively cancellative and multiplicatively cancellative.

Definition 2.9: - [3] Let S be a semiring, the set $Z(S) = \{x \in S: x. y = y. x$, for all $x, y \in S\}$ is called the center of S.

Lemma 2.10:- If S is multiplicatively commutative then $Z(S) = S$.

Proof: - It´s clear that $Z(S) \subseteq S$, we need only to show that $S \subseteq Z(S)$.

Let $x \in S$, since S is multiplicatively commutative then $x. y = y. x$, for all $y \in S$, so $x \in Z(S)$.

We get $S \subseteq Z(S)$, then $Z(S) = S$.

Definition 2.11: - [4] Let (S, +, .) be a semiring, an element $0 \in S$ called zero of S iff $x + 0 = x = 0 + x$ for all $x \in S$, an element $1 \in S$ called identity of S iff $x.1 = x = 1.x$ for all $x \in S$.

Definition 2.12: - [1] Let (S, +, .) be a semiring and T nonempty proper subset of S, T is called subsemiring if it is semiring with + and . i.e. (T, +, .) is semiring itself.

Remark 2.13: - [2] If S is semiring with 0 and 1 then any subset of S which contain 0 and 1 is subsemiring of S.

Definition 2.14: - [7] Let (S, +, .) be a semiring and I a nonempty subset of S, if:-

1- $1 \notin I$.

2- $a + b \in S$ for all $a, b \in I$.

3- $r.a \in S$ for all $a \in I$ and $r \in S$.

Then I is called Left ideal of S. Right ideal defined similarly. If I is both left and right,then we call it an ideal.

Example 2.15: - Let Z_0^+ with usual addition and usual multiplication is semiring, $< 2 > = \{2n$: for some $n \in Z_0^+\}$ is an ideal of Z_0^+.

1- $1 \notin < 2 >$.

2- Since usual addition closed under Z_0^+ then it is closed under $< 2 >$.

3- Let $2n_1 \in < 2 >$ and $n \in z_0^+$ then $n. 2n_1 = (2.n)\ n_1 \in < 2 >$ since usual multiplication is associative.

So, $< 2 >$ is an ideal of Z_0^+ .

Definition 2.16:- [1] Let S be a semiring and I a nonzero ideal of S, the set $Z(I) = \{ a \in I : a. b = b. a, \forall b \in I\}$ called the center of I .

Lemma 2.17:- If I is commutative as semiring then $Z (I) = I$.

Proof: - Trivial.

Definition 2.18: - [8] A semiring S is called semiprime if whenever $x S x = 0$ implies $x= 0$ for all $x \in S$.

Definition 2.19: - [8] A semiring S is called Prime if whenever $x S y = 0$ implies either $x = 0$ or $y = 0$ for all $x, y \in S$.

Definition 2.20: - [8] A semiring S is called n-torsion free iff whenever $nx = 0$ then $x = 0$ for all $x \in S$, where $n \neq 0$.
Lemma 2.21:- Let S be a prime semiring and I a nonzero left (right) ideal of S, then $Z(I) \subseteq Z(S)$.
Proof: - Let $0 \neq a \in Z(I)$, since $Z(I) \subseteq I$, then $a \in I$.
Let $x \in S$, then $x\,a \in I$ (By definition of left ideal).
Since $a \in Z(I)$, we have $[xa, a] = x\,[a, a] + [x, a]\,a = 0$ then,

$$[x, a]\,a = 0 \text{ for all } x \in S \qquad \dots (1)$$

Replace x by xy in (1), where $y \in S$ we get $[xy, a]\,a = x\,[y, a]\,a + [x, a]\,ya = 0$.
By using (1) we get $[x, a]\,ya = 0$ for all $x, y \in S$ and for all $a \in I$.
Then $[x, a]\,S\,I = 0$.
By primness either $[x, a] = 0$ or $I = 0$.
Since I nonzero ideal of S then $[x, a] = 0$ for all $x \in S$ and $a \in I$
Then $Z(I) \subseteq Z(S)$.
Lemma 2.22: - Let S be a semiring and I a nonzero ideal of S, if I commutative as semiring then $I \subseteq Z(S)$ and if S prime then S is commutative.
Proof: - Since I commutative as semiring then $I = Z(I)$ by (Lemma 2.16)
By (lemma 2.21) we have $Z(I) \subseteq Z(S)$, then $I \subseteq Z(S)$.
Now, If S prime
Let $x, y \in S$ and $a \in I$
Then $a\,x \in Z(S)$
i.e. $[ax, y] = 0$ for all $y \in S$
$a\,[x, y] + [a, y]\,x = 0 \Rightarrow a\,[x, y] = 0$ for all $a \in I$
Then $I\,[x, y] = 0 \Rightarrow I\,S\,[x, y] = 0$
Now by primness of S we get either $I = 0$ or $[x, y] = 0$
Since I nonzero ideal then $[x, y] = 0$ for all $x, y \in S$, Then S commutative.
Definition 2.23: - [2] Let S be a semiring, a function f: $S \to S$ is called additive map if it is preserve addition relation i.e. $f(x + y) = f(x) + f(y)$ for all x, $y \in S$.
Definition 2.24: - [2] An additive map d: $S \to S$ is called derivation on S if $d(xy) = d(x)\,y + x\,d(y)$ for all $x, y \in S$
Definition 2.25: - [8] Let S be a semiring and α, β are two automorphisms of S, an additive map d: $S \to S$ is called (α, β) derivation on S if $d(xy) = \alpha(x)\,d(y) + d(x)\,\beta(y)$ for all $x, y \in S$.
Example 2.26: - [2] Let $S = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in Z_0^+\}$ with usual matrices addition and multiplication is semiring. Suppose α: $S \to S$ defined by $\alpha(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ and β: S→S defined by $\beta(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}$. Define a derivation d: $S \to S$ by $d(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, d is (α, β) derivation on S.
Since d is derivation on S (i.e. additive map), we will only check if $d(xy) = \alpha(x)\,d(y) + d(x)\,\beta(y)$.
Now, Let $x = \begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix}$ and $y = \begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix}$,
$d(x\,y) = \begin{pmatrix} 0 & a_1 b_2 + b_1 c_2 \\ 0 & 0 \end{pmatrix}$,
$\alpha(x)\,d(y) = \begin{pmatrix} 0 & a_1 b_2 \\ 0 & 0 \end{pmatrix}$ and $d(x)\,\beta(y) = \begin{pmatrix} 0 & b_1 c_2 \\ 0 & 0 \end{pmatrix}$.
Then $\alpha(x)\,d(y) + d(x)\,\beta(y) = \begin{pmatrix} 0 & a_1 b_2 + b_1 c_2 \\ 0 & 0 \end{pmatrix}$
We get, $d(xy) = \alpha(x)\,d(y) + d(x)\,\beta(y)$ then d is (α, β) derivation on S.
Lemma 2.27:- Let S be a prime semiring. Suppose that α and β are two automorphisms of S and d: $S \to S$ is (α, β) derivation such that for all $x \in S$, $a.\,d(x) = 0$ or $d(x).\,a = 0$, where $a \in S$, then either $a = 0$ or $d = 0$.
Proof:- Let $a.\,d(x) = 0$.
Since d is (α, β) derivation of S, then $d(xy) = \alpha(x)\,d(y) + d(x)\,\beta(y)$ for all $x, y \in S$.
Then $a.\,(\alpha(x)\,d(y) + d(x)\,\beta(y)) = a\,.\,\alpha(x)\,d(y) + a.\,d(x)\,\beta(y) = a\,.\,\alpha(x)\,d(y) = 0$.
Since α is automorphism of S we get, $a\,S\,d(y) = 0$.
By primness of S either $a = 0$ or $d(x) = 0$ for all $x \in S$. i.e. either $a = 0$ or $d = 0$.
Similarly for $d(x).\,a = 0$.

3. Results

Theorem 3.1:- Let S be a cancellative prime semiring. Suppose that α and β are two nonzero automorphisms of S and $d: S \longrightarrow S$ is (α, β) derivation. If d acts as homomorphism on S then $d = 0$ on S.

Proof: - Since d is (α, β) derivation of S, then

$$d(x\,y) = \alpha(x)\,d(y) + d(x)\,\beta(y) \text{ for all } x, y \in S \quad \ldots (1)$$

Since d acts as homomorphism on S, we get

$$d(x\,y) = d(x)\,d(y) \text{ for all } x, y \in S \quad \ldots (2)$$

From (1) and (2) we get:

$$\alpha(x)\,d(y) + d(x)\,\beta(y) = d(x)\,d(y) \text{ for all } x, y \in S \quad \ldots (3)$$

Replace x by $x\,r$ in (3), where $r \in S$, we get:

$\alpha(x\,r)\,d(y) + d(x\,r)\,\beta(y) = d(x\,r)\,d(y)$

Since d acts homomorphism on S and α, β automorphisms of S then

$$\begin{aligned}\alpha(x)\,\alpha(r)\,d(y) + d(x)\,d(r)\,\beta(y) &= d(x)\,d(r)\,d(y)\\ &= d(x)\,d(r\,y)\\ &= d(x)\,[\alpha(r)\,d(y) + d(r)\,\beta(y)]\\ &= d(x)\,\alpha(r)\,d(y) + d(x)\,d(r)\,\beta(y)\end{aligned}$$

Then by additively cancellative property we get:

$\alpha(x)\,\alpha(r)\,d(y) = d(x)\,\alpha(r)\,d(y)$

Now, by multiplicatively cancellative propertywe get:

$$\alpha(x) = d(x) \text{ for all } x \in S \quad \ldots (4)$$

Replace x by $x\,y$ in the above equation we get:

$\alpha(x\,y) = d(x\,y)$

Since α is automorphism of S then

$\alpha(x)\,\alpha(y) = d(xy)$

Since $\alpha = d$ from relation (4) then

$\alpha(x)\,d(y) = \alpha(x)\,d(y) + d(x)\,\beta(y)$

By additive cancellative property the result is:

$d(x)\,\beta(y) = 0$

By (Lemma 2.27) and since $\beta \neq 0$ then $d = 0$ on S.

Theorem 3.2: - Let S be a cancellative prime semiring. Suppose that α and β are two nonzero automorphisms of S and $d: S \longrightarrow S$ is (α, β) derivation such that α and β commute with d. If d acts as anti-homomorphism on S then $d = 0$ on S.

Proof: - since d is (α, β) derivation of S

$$\text{Then } d(x\,y) = \alpha(x)\,d(y) + d(x)\,\beta(y) \text{ for all } x, y \in S \quad \ldots (1)$$

Since d acts as anti-homomorphism on S

$$\text{Then } d(x\,y) = d(y)\,d(x) \text{ for all } x, y \in S \quad \ldots (2)$$

From (1) and (2) we get:

$$\alpha(x)\,d(y) + d(x)\,\beta(y) = d(y)\,d(x) \text{ for all } x, y \in S \quad \ldots (3)$$

Replace x by $x\,y$ in (3) we get:

$\alpha(x\,y)\,d(y) + d(x\,y)\,\beta(y) = d(x\,y)\,d(y)$

Since d acts homomorphism on S and α, β automorphisms of S, then

$$\begin{aligned}\alpha(x)\,\alpha(y)\,d(y) + d(y)\,d(x)\,\beta(y) &= d(y)\,d(x)\,d(y)\\ &= d(y)\,d(xy)\\ &= d(y)\,[\alpha(x)\,d(y) + d(x)\,\beta(y)]\\ &= d(y)\,\alpha(x)\,d(y) + d(y)\,d(x)\,\beta(y)\end{aligned}$$

Then by additively cancellative property we get:

$\alpha(x)\,\alpha(y)\,d(y) = d(y)\,\alpha(x)\,d(y)$

Now, since α commute with d and by multiplicatively cancellative property we get:

$$\alpha(y) = d(y) \text{ for all } y \in S \quad \ldots (4)$$

Replace y by $x\,y$ in the above equation we get:

$\alpha(x\,y) = d(x\,y)$

Since α is automorphism of S, then $\alpha(x)\,\alpha(y) = d(x\,y)$.

Since $\alpha = d$ from relation (4) then $\alpha(x)\,d(y) = \alpha(x)\,d(y) + d(x)\,\beta(y)$

By additive cancellative property the result is:

$d(x)\,\beta(y) = 0$
By (Lemma 2.27) and since $\beta \neq 0$ then $d = 0$ on S.
Lemma 3.3:- Let S be a semiring and I a nonzero ideal of S. Suppose that α, β are two automorphisms on S and d: $S \rightarrow S$ is (α, β) derivation such that d onto on I. If $[d(u), a] = 0$ for all $u \in I$ and $a \in S$, then $a \in Z(I)$.
Proof: - Since $[d(u), a] = 0$ for all $u \in I$, then $[d(I), a] = 0$ for all $a \in S$.
Since d is onto on I, then $d(I) = I$.
Then $[I, a] = 0$ for all $a \in S$.
Then $a \in Z(I)$.
Theorem 3.4: - Let S be a 2-torsion free prime semiring and I nonzero ideal of S. Suppose that α and β are two nonzero automorphisms of S and d: $S \longrightarrow S$ is (α, β) derivation on S such that d onto on I, and d commute with α and β. If $d(xy) = d(yx)$ for all $x, y \in I$ then S is commutative.
Proof: - For any element $c \in I$ such that $d(c) = 0$, We have $d(z\,c) = d(c\,z)$.
Let $c = [x, y]$ where $x, y \in I$.
Since d is (α, β) derivation on S then,
$\alpha(z)\,d(c) + d(z)\,\beta(c) = \alpha(c)\,d(z) + d(c)\,\beta(z)$
Since $d(c) = 0$ we get:
$d(z)\,\beta(c) = \alpha(c)\,d(z)$ … (1)
i.e. $[c, d(z)] = 0$ for all $z \in I$
By (Lemma 3.3) we get $c \in Z(I)$ for all $c \in I$
Then $[x, y] \in Z(I)$. That's mean:
$[a, [x, y]] = 0$ for all $a \in I$ … (2)
Replace y by $x\,y$ in (2) we get:
$[a, [x, x\,y]] = 0$
$[a, x]\,[a, y] = 0$ … (3)
Replace y by $y\,a$ in (3) we get:
$[a, x]\,[a, ya] = 0$
$[a, x]\,y\,[x, a] = 0$ for all $x, y, a \in I$
$[a, x]\,I\,[x, a] = 0$
$[a, x]\,S\,I\,[x, a] = 0$.
By primness of S, either $[a, x] = 0$ or $I\,[x, a] = 0$.
If $[a, x] = 0$ for all $a, x \in I$ then I is commutative.
If $I\,[x, a] = 0$, then $I\,S\,[x, a] = 0$.
By primness of S and since I is nonzero ideal we get $[x, a] = 0$ for all $x, a \in I$.
Then I is commutative.
By (Lemma2.22) we get S is commutative.
Lemma 3.5:- Let S be prime semiring. Suppose that α and β are two nonzero automorphisms of S and d: $S \longrightarrow S$ is (α, β) derivation on S. If d acts as homomorphism on I and $d = 0$ on I then $d = 0$ on S.
Proof: - Let $s\,v \in I$, where $s \in S$ and $v \in I$, then $d(s\,v) = 0$
Since d acts as homomorphism then,
$d(s)\,d(v) = 0$ … (1)
Since d is (α, β) derivation of S then,
$\alpha(s)\,d(v) + d(s)\,\beta(v) = 0$ … (2)
From (1) and (2) we get:
$d(s)\,d(v) = \alpha(s)\,d(v) + d(s)\,\beta(v)$
Since $d = 0$ on I then,
$D(s)\,\beta(v) = 0$ for all $s \in S$ and $v \in I$ … (3)
By (lemma 2.27) either $\beta(v) = 0$ for all $v \in I$ or $d(s) = 0$ for all $s \in S$
But β nonzero, then $d(s) = 0$ for all $s \in S$.
Then $d = 0$ on S.
Theorem 3.6: - Let S be a cancellative prime semiring. Suppose that α and β are two nonzero automorphisms of S and d: $S \longrightarrow S$ is (α, β) derivation on S. If d acts as homomorphism on I then $d = 0$ on S.
Proof

since d is (α, β) derivation of S

Then $d(uv) = \alpha(u)\, d(v) + d(u)\, \beta(v)$ for all $u, v \in I$ ... (1)

Since d acts as homomorphism on S

Then $d(u\, v) = d(u)\, d(v)$ for all u, v $\in I$ ... (2)

From (1) and (2) we get:

$\alpha(u)\, d(v) + d(u)\, \beta(v) = d(u)\, d(v)$ $u, v \in I$ ... (3)

Replace v by $v\, t$ in (3), where $t \in$ I, we get:

$\alpha(u)\, d(v\, t) + d(u)\, \beta(v\, t) = d(u)\, d(v\, t)$

Since d acts homomorphism on I and α, β are automorphisms of I then,

$\alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t) = d(u)\, d(v)\, d(t)$

$= d(u\, v)\, d(t)$

$= [\alpha(u)\, d(v) + d(u)\, \beta(v)]\, d(t)$

$= \alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t)$

Then by additively cancellative property we get:

$d(u)\, \beta(v)\, d(t) = d(u)\, \beta(v)\, \beta(t)$

Now, by multiplicatively cancellative propertywe get:

$d(t) = \beta(t)$ for all $t \in S$ … (4)

Substitute (4) in (3) we get:

$d(u)\, d(v) = \alpha(u)\, d(v) + d(u)\, d(v)$

By multiplicatively cancellative property the result is:

$\alpha(u)\, d(v) = 0$

Replace u by ur, where r $\in S$

$\alpha(ur)\, d(v) = 0$

$\alpha(u)\, \alpha(r)\, d(v) = 0$

Since α automorphisms of S then

$\alpha(u)\, S\, d(v) = 0$

By primness either $\alpha(u) = 0$ for all $u \in I$ or $d(v) = 0$ for all $u \in I$

But α is nonzero, then $d = 0$ on I.

By (Lemma 3.5) we get $d = 0$ on S.

Lemma 3.7:- Let S be a prime semiring. Suppose that α and β are two nonzero automorphisms of S and $d: S \longrightarrow S$ is (α, β) derivation on S. If d acts as anti-homomorphism on I and $d = 0$ on I then d = 0 on S.

Proof: **-** Let $s\, v \in I$, where $s \in S$ and $v \in I$, then $d(sv) = 0$

Since d acts as anti-homomorphism then,

$d(v)\, d(s) = 0$ … (1)

Since d is (α, β) derivation of S then,

$\alpha(s)\, d(v) + d(s)\, \beta(v) = 0$ … (2)

From (1) and (2) we get:

$d(v)\, d(s) = \alpha(s)\, d(v) + d(s)\, \beta(v)$

Since $d = 0$ on I then,

$d(s)\, \beta(v) = 0$ for all $s \in S$ and $v \in I$ … (3)

By (Lemma 2.27) either $\beta(v) = 0$ for all $v \in I$ or $d(s) = 0$ for all $s \in S$

But β nonzero, we get $d(s) = 0$ for all $s \in S$.

Then $d = 0$ on S.

Theorem 3.8: **-** Let S be a cancellative prime semiring. Suppose that α and β are two nonzero automorphisms of S and $d: S \longrightarrow S$ is (α, β) derivation on S such that d commute with α and β. If d acts as anti-homomorphism on I then $d = 0$ on S.

Proof: **-** Since d is (α, β) derivation of S

Then $d(u\, v) = \alpha(u)\, d(v) + d(u)\, \beta(v)$ for all $u, v \in I$ ... (1)

Since d acts as anti-homomorphism on S

Then $d(u\, v) = d(v)\, d(u)$ for all u, v $\in I$ ... (2)

From (1) and (2) we get:

$\alpha(u)\, d(v) + d(u)\, \beta(v) = d(v)\, d(u)$ $u, v \in I$ ... (3)

Replace v by $v\, t$ in (3), where $t \in I$, we get:

$\alpha(u)\, d(v\, t) + d(u)\, \beta(vt) = d(v\, t)\, d(u)$

Since d acts anti-homomorphism on I then

$$\begin{aligned} \alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t) &= d(t)\, d(v)\, d(u) \\ &= d(t)\, d(u\, v) \\ &= d(t)\, [\alpha(u)\, d(v) + d(u)\, \beta(v)]\, d(u) \\ &= d(t)\, \alpha(u)\, d(v) + d(t)\, d(u)\, \beta(v) \end{aligned}$$

Since α commute with d and by additively cancellative property we get:

$d(t)\, \beta(v)\, d(u) = d(u)\, \beta(v)\, \beta(t)$

Now, since β commute with d and by multiplicatively cancellative property we get:

$$d(t) = \beta(t) \quad \text{for all } t \in S \qquad \dots (4)$$

Substitute (4) in (3) we get:

$d(v)\, d(u) = \alpha(u)\, d(v) + d(v)\, d(u)$

By multiplicatively cancellative property the result is:

$\alpha(u)\, d(v) = 0$

By (Lemma 2.27) either $\alpha(u) = 0$ for all $u \in I$, or $d(v) = 0$ for all $v \in I$

But, α is nonzero, then $d(v) = 0$ for all $v \in I$,then $d = 0$ on I.

By (Lemma 3.7) we get $d = 0$ on S.

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