Open Access

Baghdad Science Journal **Vol.13(3)2016**

DOI: http://dx.doi.org/10.21123/bsj.2016.13.3.0599

Some Results on Weak Essential Submodules

Muna Abbas Ahmed

Department of Mathematics, College of Science for Women, University of Baghdad, Iraq-Baghdad

E-mail:math.200600986@yahoo.com

Received 26/3 /2015
Accepted 25/11 /2015

Abstract

Throughout this paper R represents commutative ring with identity and M is a unitary left R-module. The purpose of this paper is to investigate some new results (up to our knowledge) on the concept of weak essential submodules which introduced by Muna A. Ahmed, where a submodule N of an R-module M is called weak essential, if N ∩ P ≠ (0) for each nonzero semiprime submodule P of M. In this paper we rewrite this definition in another formula. Some new definitions are introduced and various properties of weak essential submodules are considered.

Key words: Semiprime submodule, Essential submodules, Weak essential submodules, Weak uniform modules, Fully semi-semiprime modules and Fully essential* modules.

Introduction:

Let R be a commutative ring with identity and let M be a unitary left R-module. Assume that all R-modules under study contain semiprime submodules. It is well known that a submodule N of M is called essential, if whenever N ∩ L = (0), then L = (0) for each nonzero submodule L of M [1] and [2].]. A proper submodule P of M is called prime, if whenever rm ∈ P for r ∈R and m ∈M, then either m ∈ P or r ∈ $(P\underset{R}{:}M)$ [3]. A nonzero submodule N of M is called semi essential, if N ∩ P ≠ (0) for each nonzero prime submodule P of M [4]. This paper consists of three sections; in section 1, we give some remarks and examples, and discuss the transitivity property of weak essential submodules. In section 2, we introduce some new results (up to our knowledge) on the concept of weak essential submodules. Section 3, is devoted to the study ascending and descending chain conditions on weak essential submodules.

Muna in [5] introduced the concept of weak essential submodules as a generalization of the class of essential submodules. A proper submodule N of M is called semiprime , if for each r∈R and x∈M with r^k x∈L, then rx∈L [6]. Equivalently, if r^2 x ∈ L, then rx∈L [7]. And a submodule N of M is called weak essential, if N ∩ L ≠ (0), for each nonzero semiprime submodule L of M. Muna saw in [5] that the class of weak essential submodules lies between the class of essential submodules and the class of semi essential submodules.

In this work we give some new results (up to our knowledge) about this class of submodules.

Firstly we rewrite the definition of weak essential submodules in another formula. In fact we did not find any reasonable reason to exclude the zero submodule from this definition. We find it may be useful in some cases instead of the origin formula.

Definition (1): A submodule N of an R-module M is called weak essential, if whenever $N \cap P = (0)$, then $P = (0)$ for every semiprime submodule P of M.

We see that in order to add other results for weak essential submodules, it must be necessary giving some other simple remarks about this class of submodules as well as the remarks which were mentioned in [5].

Remarks (2):

1. When a submodule N of an R-module M is nonzero in the Def (1), then N is a weak-essential submodule if $N \cap P \neq (0)$ for each semiprime submodule P of M, and this is the same definition which is mentioned in [5].

2. Every module is a weak essential submodule in itself.

3. In the concept of the essential submodules, (0) is an essential submodule of an R-module M if and only if M = (0). But in the concept of weak essential submodules this statement is not satisfying. In fact $(0) \leq_{weak} (0)$, but sometimes (0) may be weak essential submodule in a nonzero module, for example $(\bar{0})$ is a weak essential submodule of the Z-module, Z_5, and in other examples such as Z-module Z, we note that (0) is not weak essential submodule.

4. If $(0) \neq M$, and the only semiprime submodule in M is zero, then $(0) \leq_{weak} M$.

5. The sum of two weak essential submodules is also weak essential submodule.

Proof (5): Let M be an R-module and let L and K be two weak essential submodules of M. Note that $L \leq L+K$, since $L \leq_{weak} M$, so by [5, Rem(1.5)(2)], $L+K \leq_{weak} M$.

6. Let M be an R-module, and let $N \leq M$. Then for each R-module M' and for each homomorphism $f: M \to M'$ with $\ker f \cap N \neq (0)$, implies that $N \leq_{weak} M$.

Proof (6): Let P be a nonzero semiprime submodule of M, and let $\pi: M \to \frac{M}{P}$ be the natural epimorphism. By assumption $\ker \pi \cap N \neq (0)$. But $\ker \pi = P$, then $P \cap N \neq (0)$, hence $N \leq_{weak} M$.

Proposition (3): Let $f: M \to M'$ be an isomorphism. If $N \leq_{weak} M$, then $f(N) \leq_{weak} M'$.

Proof: Let P be a nonzero semiprime submodule of M'. Since f is an epimorphism, and $\ker f = (0) \subseteq P$, then $f^{-1}(P)$ is a semiprime submodule of M [7, P. 49 Prop (2.1)]. But $N \leq_{weak} M$, then $N \cap f^{-1}(P) \neq (0)$, On the other hand f is a monomorphism thus $f(N) \cap P \neq (0)$.

In the following proposition we prove the transitive property for nonzero submodules. Before that we need the following Lemma which appeared in [7, Prop (1.11), p.48].

Lemma (4): If P is a semiprime submodule of C and B is a submodule of an R-module C, such that $B \nleq P$, then $P \cap B$ is a semiprime submodule in B.

Proposition (5): Let C be an R-modules, and let A, B be submodules of C such that $(0) \neq A \leq B \leq C$. If $A \leq_{weak} B$ and $B \leq_{weak} C$, then $A \leq_{weak} C$.

Proof: Let P be a semiprime submodule of C such that $A \cap P = (0)$. Note that $(0) = A \cap P = (A \cap P) \cap B = A \cap (P \cap B)$. But P is a semiprime submodule of C, so we have two cases. If $B \leq P$ then $(0) = A \cap (P \cap B) = A \cap B$, hence $A \cap B = (0)$, but $A \leq B$, so $A \cap B = A$, which is implies that $A = (0)$. But this is a contradiction with our assumption. Thus $B \nleq P$, and by Lemma (4), $P \cap B$ is a semiprime submodule of B. But $A \leq_{weak}$

B, therefore P ∩ B = (0), and since B $\leq_{weak}$ C, then P = (0), that is A $\leq_{weak}$ C.

Remark (6): The condition A ≠ (0) in Prop (1.5) is necessary. In fact in the Z-module Z_8, $(\bar{0})$ is a weak essential submodule of $\{\bar{0}, \bar{4}\}$ and $\{\bar{0}, \bar{4}\}$ is a weak essential submodule of Z_8, but $(\bar{0})$ not weak essential in Z_8.

The converse of Prop (5) is not true in general, as the following example shows.

Example (7): Consider the Z-module, Z_{36}, the submodule $(\overline{18})$ is a weak essential submodule of Z_{36}. But $(\overline{18})$ is not weak essential submodule of $(\bar{2})$.

1.Other results on weak essential submodules

In this section we introduce other properties of weak essential submodules. We start by the following definition which is analogue of that in [8].

Definition (1.1): A nonzero R-module is called fully essential*, if every nonzero weak essential submodule of M is an essential submodule of M.

It is clear that every fully essential* module is a fully essential module, since every weak essential submodule is a semi-essential submodule [5].

Recall that an R-module M is called fully semiprime, if every proper submodule of M is a semiprime submodule [9].

Before giving the following proposition, we need to introduce the following lemma.

Lemma (1.2): Let A and B be submodules of an R-module M such that A ≤ B. If A is a semiprime submodule of M, then A is a semiprime submodule in B.

Proof: It is clear.

Proposition (1.3): Let M be a fully semiprime R-module, and let N ≤ M. Then N $\leq_{weak}$ L if and only if N $\leq_e$ L for every submodule L of M.

Proof: ⇒) Let L be a submodule of M and let A be a submodule of L such that N ∩ A = (0), since M is a fully semiprime module then both of N and A are semiprime submodules of M, and by Lemma (1.2), N is a semiprime submodule of L. But N is a weak essential submodule of L, therefore A = (0), that is N is an essential submodule of L.

⇐) It is clear.

Corollary (1.4): If M is a fully semiprime module, then every nonzero weak essential submodule of M is an essential submodule of M.

Corollary (1.5): Every fully semiprime module is a fully essential* module.

Recall that a nonzero R-module M is called weak uniform if every nonzero R-submodule of M is a weak essential. A ring R is called weak uniform if R is a weak uniform R-module, [5].

Proposition (1.6): Let M be an R-module, then M is uniform module if and only if M is weak uniform and fully essential* module.

Proof: ⇒) It is obvious.

⇐) Let (0) ≠ N ≤ M. Since M is a weak uniform module, then is a weak essential submodule of M. But M is a fully essential* module, therefore N is an essential submodule of M, and we are done.

Corollary (1.7): Let M be a fully semiprime R-module, then a nonzero module M is a uniform if and if M is a weak uniform module.

The following theorem gives the hereditary of "fully essential* property" between the ring R and the module M which defined on R.

Theorem (1.8): Let M be a nonzero finitely generated, faithful and multiplication R-module. Then M is a fully essential* module if and only if R is a fully essential* ring.

Proof: **⇒)** Assume that M is a fully essential* module, and let I be a nonzero weak essential ideal of R, then IM is a submodule of M say N. Since M is a finitely generated, faithful and multiplication module so by [5,Th

(3.6)], N is a weak essential submodule of M. Since I ≠ (0) and M is a faithful module, then N ≠ (0). But M is a fully essential* module, therefore N is an essential submodule of M. Since M is a faithful and multiplication module, thus I is an essential ideal of R [10, Th (2.13)], that is R is a fully essential* ring.

⇐) Suppose that R is a fully essential* ring and let $(0) \neq N \leq_{weak} M$. Since M is a multiplication module, then there exists a weak essential ideal I of R such that N = IM [5]. By assumption I is an essential ideal of R. But M is a finitely generated faithful and multiplication module, then N is an essential submodule of M [10, Th (2.13)], and we are done.

The following proposition deals with the direct sum of weak essential submodules.

<u>Proposition (1.9):</u> Let $M = M_1 \oplus M_2$ be a fully semiprime R-module where M_1 and M_2 are submodules of M, and let $(0) \neq K_1 \leq M_1$ and $(0) \neq K_2 \leq M_2$. Then $K_1 \oplus K_2$ is a weak essential submodule of $M_1 \oplus M_2$ if and only if K_1 is a weak essential submodule of M_1 and K_2 is a weak essential submodule of M_2.

<u>Proof:</u> ⇒) Since M is a fully semiprime module, then by Cor (1.4), $K_1 \oplus K_2$ is an essential submodule of $M_1 \oplus M_2$, and by [11], K_1 is an essential submodule of M_1 and K_2 is an essential submodule of M_2. But every essential submodule is a weak essential, therefore $K \leq_{weak} M_1$.

⇐) It follows similarly

In the following proposition we give another case for the direct sum of weak essential submodules.

<u>Proposition (1.10):</u> Let $M = M_1 \oplus M_2$ be an R-module where M_1 and M_2 are submodules of M, and let $K_1 \leq M_1$ and $K_2 \leq M_2$. If $K_1 \oplus K_2$ is a weak essential submodule of $M_1 \oplus M_2$, then K_1 is a weak essential submodule of M_1, provided that every semiprime submodule of M_1 is a semiprime submodule of M.

<u>Proof:</u> Let P_1 is a semiprime submodule of M_1 such that $K_1 \cap P_1 = (0)$. By using some properties in set theory, we can easily show that $(K_1 \oplus K_2) \cap P_1 = (0)$. But $K_1 \oplus K_2 \leq_{weak} M$ and by assumption P_1 is a semiprime submodule of M, Thus $P_1 = (0)$.

Let us introduce the following definition.

<u>Definition (1.11)</u>: Let M be an R-module and let N be a submodule of M. A semiprime submodule L of M is called weak-relative intersection complement of N in M, if whenever $N \cap P = (0)$, where P is a semiprime submodule of M, such that $L \subseteq P$, then L = P. In other words L is a maximal submodule with the property $N \cap L = (0)$.

<u>Remark (1.12):</u> It is well known that every submodule of an R-module has a relative complement [1, P.17]. We verify by example that not every submodule has a weak-relative intersection complement, for example; the submodule $(\bar{2})$ of Z_4-module Z_4 hasn't weak-relative intersection complement, since there exists only one submodule $(\bar{0})$ of Z_4 such that $(\bar{2}) \cap (\bar{0}) = (\bar{0})$, and $(\bar{0})$ is not semiprime submodule of Z_4 as Z_4, i.e. $(\bar{0})$ is not semiprime ideal of the ring Z_4. In fact $(\bar{0})$ is not the only nilpotent ideal in the ring Z_4, so by [1, P.2], $(\bar{0})$ is not nilpotent ideal of Z_4.

Muna in [5] showed by example that the intersection of two weak essential submodules need not be weak essential submodule, and she satisfied that under certain condition, see [5, Prop (1.6)]. In this work we give a different condition.

<u>Proposition (1.13):</u> Let M be an R-module and let N_1 and N_2 be a weak essential submodules of M such that $N_1 \cap N_2 \neq (0)$ and all semiprime submodules of N_1 are semiprime submodules of M, then $N_1 \cap N_2 \leq_{weak}$ M.

Proof: Let P be a semiprime submodule of M such that $(N_1 \cap N_2) \cap P = (0)$. This implies that $N_2 \cap (N_1 \cap P) = (0)$. If $N_1 \leq P$, then we have a contradiction with the assumption, thus $N_1 \nleq P$. By Lemma (1.4), $N_1 \cap P$ is a semiprime submodule of N_1. Since $N_2 \leq_{weak}$ M and by our assumption $N_1 \cap P$ is a semiprime submodule of M, we have $N_1 \cap P = (0)$. But $N_1 \leq_{weak}$ M, therefore P = (0), hence $N_1 \cap N_2 \leq_{weak}$ M.

Note: The condition "all semiprime submodules of N_1 are semiprime submodules of M" in Prop (1.13), can also be applied for N_2.

Proposition (1.14): Let M be an R-module and N_1 and N_2 are weak essential submodule of M such that $N_2 \cap P$ is a semiprime submodules of M for all semiprime submodule P of M, then $N_1 \cap N_2 \leq_{weak}$ M.

Proof: Let P be a semiprime submodule of M such that $(N_1 \cap N_2) \cap P = (0)$. This implies that $N_1 \cap (N_2 \cap P) = (0)$. Since $N_2 \cap P$ is a semiprime submodule of M and $N_1 \leq_{weak}$ M, then $N_2 \cap P = (0)$. But $N_2 \leq_{weak}$ M, thus P = (0).

As a generalization of the result in [11, Prop (5.21), P.75], we give the following proposition.

Proposition (1.15): Let N be a nonzero R-module of M, and let N' be a nonzero semiprime submodule of M. If N' is a weak relative intersection complement of N in M, then $\frac{N \oplus N'}{N'}$ is a weak essential submodule of $\frac{M}{N'}$.

Proof: Let g: M $\to \frac{M}{N'}$ be a natural epimorphism, and let N' be a weak relative complement of N in M. Let $\frac{K}{N'}$ be a nonzero semiprime submodule of $\frac{M}{N'}$ such that $\frac{N \oplus N'}{N'} \cap \frac{K}{N'} = (0)$. By [7], $g^{-1}(\frac{K}{N'})$ is a semiprime submodule of M [5, P.216], put $g^{-1}(\frac{K}{N'})$ = P for some semiprime submodule P of M, then g(P) $= \frac{K}{N'}$. Thus $\frac{N \oplus N'}{N'} \cap \frac{K}{N'} = (0)$, this implies that $\frac{(N \oplus N') \cap K}{N'} = (0)$ hence $(N \oplus N') \cap K = N'$. By modular law $N \cap K \subseteq N'$, that is $N \cap K \subseteq N' \cap N$. Since N' is a weak relative complement of N in M, then N' is the maximal submodule with the property $N \cap N' = (0)$. It follows that $N \cap K = (0)$, and by maximality of N' we get K = N', therefore $\frac{K}{N'} = (0)$. That is $\frac{N \oplus N'}{N'}$ is a weak essential submodule of $\frac{M}{N'}$.

We need the following definition which appeared in [12].

Definition (1.16): Let M be an R-module and $N \leq M$. If there exists a semiprime submodule of M containing N, then the intersection of all semiprime submodule of M containing N is called semi-radical of N, and it is denoted by S- rad N. If there is no semiprime submodule of M containing N, then we say that S- rad N = M, in particular S-rad M = M.

Proposition (1.17): Let M be an R-module and let $(0) \neq N \leq M$. If N' is a weak relative complement of N in M, and N' ≤ S-rad(M), then $N \oplus N' \leq_{weak}$ M.

Proof: Consider the natural epimorphism π: M $\to \frac{M}{N'}$. Since N' is a weak relative complement of N in M, so by Prop (1.15), $\frac{N \oplus N'}{N'} \leq_{weak} \frac{M}{N'}$. But ker π = N' and N' ≤ S-red(M), then by [5,Prop (2.3)(2)], $\pi^{-1} (\frac{N \oplus N'}{N'}) \leq_{weak}$ M. Hence $N \oplus N' \leq_{weak}$ M.

Recall that an R-module M is called multiplication, if for each submodule N of M, there exists an ideal I of R such that N=IM [13].

Proposition (1.18): Let M be a faithful and multiplication module such that M satisfies the condition (*), and let I, J be ideals of R. If IM $\leq_{weak}$ JM, then I $\leq_{weak}$ J, where:

Condition (*): For any two ideals L and K of R, if L is a semiprime ideal of K, then LM is a semiprime submodule of KM.

Proof: Let P be a semiprime ideal of J such that I ∩ P = (0), then IM ∩ PM = (0). . Since M is a faithful and multiplication, therefore IM ∩ PM = (0) [10, Th (1.7)]. By condition (*), PM is a semiprime submodule of JM. But IM $\leq_{weak}$ JM, then PM = (0). Since M is a faithful module so P = (0), thus I $\leq_{weak}$ J.

Note that the condition (*) which mentioned in Prop (1.18) is not hold in general, as shown in the following example.

Example (1.19): The Z_4-module, Z_4 is not satisfying the condition (*), since there exists a prime ideal I = $\{\overline{0}, \overline{2}\}$ of the ring Z_4, with IZ_4 not prime submodule of Z_4. In fact IM = $\{\sum a_i m_j \mid a_i \in \text{I and } m_j \in Z_4\}$ = $(\overline{0})$ is not prime submodule of Z_4.

The converse of Prop (1.18) is true without using the condition (*), but we need to add another condition as the following proposition shows.

Proposition (1.20): Let M be a finitely generated, faithful and multiplication R-module. If I $\leq_{weak}$ J then IM $\leq_{weak}$ JM for every ideals I and J of R.

Proof: Let P be a semiprime submodule of JM such that IM ∩ P = (0). Since M is a multiplication and faithful module, then P = EM for some semiprime ideal E of R [14, Prop (2.5), P.36]. So IM ∩ EM = (0), this implies that (I ∩ E) M = (0). Since M is a faithful module, then I ∩ E = (0). On the other hand since EM ≤ JM and M is a finitely generated, faithful and multiplication module so by [10, Th (3.1)] E ≤ J. But E is a semiprime ideal of R, then by Lemma (1.2), E is a semiprime ideal of J. Since I is a weak essential ideal of J, then E = (0), and hence P = (0). That is IM $\leq_{weak}$ JM.

From Prop (1.18) and Prop (1.20) we have the following theorem.

Theorem (1.21): Let M be a finitely generated, faithful and multiplication module such that M satisfies the condition (*). Then I $\leq_{weak}$ J if and only if IM $\leq_{weak}$ JM for every two ideals I and J of R.

It is well known that If a ring R has only one maximal ideal I, then I is an essential ideal of R if and only if I ≠ (0). In the following proposition we generalize one direction of this statement for essential (hence weak essential) submodules.

Proposition (1.22): if M is a nonzero multiplication module with only one nonzero maximal submodule N, then N is an essential (hence weak essential) submodule.

Proof: It is clear.

Remark (1.22): In [8, Prop (1.6), P. 7], if an R-module M is finitely generated, then every proper submodule of M is contained in a maximal submodule of M. If we use this statement and replace the condition "nonzero multiplication module" in Prop (1.22) by the condition "finitely generated module", then we get the same result.

2.Modules with ACC (DCC) on weak essential submodules

Recall that an R-module M called satisfies ACC (DCC), if each ascending (descending) condition of submodules of M is finite [2]. In this section we study this property on a special class of submodules which is the class of weak essential submodules. We study the hereditary property for this definition between M and it's submodules, and between M and the ring R which defined on it. We start by the following definition.

Definition (2.1): An R-module M is called satisfied the ascending chain condition (ACC) on weak essential submodules if each ascending chain of weak essential submodules $N_1 \subseteq N2 \subseteq ... \subseteq Nn \subseteq$ ...is finite. And M is called satisfied descending chain condition (DCC) on weak essential submodules if each descending chain of weak essential submodules $N_1 \supseteq N2 \supseteq ... \supseteq Nn \supseteq$ … is finite.

The proof of the following remark is obvious so it is omitted.

Remark (2.2):
Let M be an R-module and let N be an R-submodule of M such that N $\subseteq$ Srad(M). If M satisfies ACC (DCC) on weak essential R-submodules, then M/N satisfies ACC(DCC) on weak essential submodules.

Proposition (2.3):
Let M be an R-module, then M satisfies ACC on weak essential submodules if each weak essential submodule of M is finitely generated.

Proof: Let $N_1 \subseteq N_2 \subseteq ... \subseteq N_n \subseteq$ ... be an ascending chain of weak essential submodules of M. Put $\sum_{i\in I} N_i$ = N. By Rem (2)(5), N is a weak essential R-submodule of M. By assumption N is finitely generated, therefore there exists a finite set α of the index I such that $\sum_{\alpha\in I} N_i$ = N. Hence the chain is finite. Similarity for satisfying DCC on weak essential submodules.

The following theorem gives the hereditary property for the ACC (DCC) between R-module M and R itself.

Theorem (2.4): Let M be a finitely generated faithful multiplication R-module. Then M satisfies ACC (DCC) on weak essential R-submodules if and only if R satisfies ACC (DCC) on weak essential ideals.

Proof: We will prove the hereditary property between R-module M and R, for satisfying ACC on weak essential submodules, and similarity for the case DCC. Let $E_1 \subseteq E_2 \subseteq ... \subseteq E_n \subseteq$ ...be an ascending chain of weak essential ideals of R. Then $E_1M \subseteq E_2M \subseteq ... \subseteq E_nM \subseteq$ ...is an ascending chain of weak essential submodules of M [5]. Since M satisfies ACC on semi- essential submodules, then there exists a positive integer n such that $E_nM = E_{n+1}M = \dots$ But M is a finitely generated faithful multiplication R-module, then En=En+1=... [10]. Hence R satisfies ACC on weak essential ideals.

Conversely; let $N_1 \subseteq N_2 \subseteq ... \subseteq N_n \subseteq$ ...be an ascending chain of weak essential submodules of M. Since M is multiplication, then Ni = E_i M for some weak essential ideals E_i of R,for each i=1,2,3,...,n,... [5].Thus $E_1M \subseteq E_2M \subseteq ... \subseteq E_nM \subseteq$ ..., and since M is a finitely generated faithful multiplication R-module, then $E_1 \subseteq E_2 \subseteq ... \subseteq E_n \subseteq$ ... is an ascending chain of weak essential ideals of R [10]. But R satisfies ACC on weak essential ideals, thus there exists a positive integer n such that En=En+1=... .Hence $E_nM= E_{n+1}M$=... .Therefore M satisfies ACC on weak essential submodules.

Theorem (3.5): Let M be a finitely generated faithful multiplication R-module, then the following statements are equivalent.

1. M satisfies ACC (DCC) on weak essential R-submodules.
2. R satisfies ACC (DCC) on weak essential ideals.
3. S=End(M) satisfies ACC (DCC) on weak essential ideals, where S is the endomorphism ring of homomorphism.
4. M satisfies ACC (DCC) on weak essential R-submodules as an S-module.

Proof:

(1) ⇔ (2) By Th (2.4).

(2) ⇔ (3) Since M is a finitely generated faithful multiplication R-module, then R ≅ S [14]. Thus R satisfies ACC (DCC) on weak essential ideals if and only if S satisfies ACC (DCC) on weak essential ideals.

(3) ⇔ (4) By Th (2.4) and R ≅ S.

(1) ⇔ (4) By [15], R ≅ S. Therefore M satisfies ACC (DCC) on weak essential submodules as an S-module.

Reference:

[1] Goodearl, K. R. 1972. Ring Theory, Marccl Dekker, New York.

[2] Kasch, F. 1982. Modules and Rings. London: Academic Press.

[3] Saymeh, S. A. 1979. On Prime R-Submodules, Univ. Ndc. Tucuma'n Rev. Ser. A29, 129-136.
[4] Mijbass, A. S. and Abdullah, N. K. 2009. Semi-Essential Submodules and Semi-Uniform Modules. J. of Kirkuk University-Scientific studies, 4 (1): 48-58.
[5] Ahmed, M. A. 2010. Weak Essential Submodules, Um-Salama Science Journal, 23 (3): 214-221.
[6] Dauns, J. and McDonald, B.R. 1980. Prime Modules and One-Sided Ideals in The Ring Theory and Algebra III, Proceedings of the Third Oklahoma conference:301-344.
[7] Athab, E. A. 1996. Prime and Semiprime Submodules, M. SC. Thesis, university of Baghdad.
[8] Ahmed, M. A. and Dakheel, S. O. 2015. S-Maximal Submodules, J. of Baghdad for Science, 12(1): 210-220.
[9] Behboodi, M; Karamzadeh, O. A. S and Koohy. H. 2004. Modules Whose Certain Submodule are Prime, Vietnam J. of Mathematics, 32(3): 303-317.
[10] El-Bast, Z. A. and Smith, P. F. 1988. Multiplication Modules, Comm. In Algebra, 16: 755-779.
[11] Anderson, F. W. and Fuller, K. R. 1992. Rings and Categories of Modules, Springer-Verlag, New York., Academic Press Inc. London.
[12] Tavallaee, H. A. and Varmazyar, R. 2008. Semi-Radical of Submodules in Modules, IUST International Journal of Engineering Science, 19(1-2): 21-27.
[13] Barnard, A. 1981. Multiplication Modules, J. Algebra, 71: 174-178.
[14] Ahmed, A. A. 1992. On Submodules of Multiplication Modules, M.Sc. Thesis, Univ. of Baghdad.
[15] Naoum, A. G. 1990. On The Ring of Endomorphism of Finitely Generated multiplication modules, Periodica Mathematica Hungarica, 21(3): 249-255.

بعض النتائج عن المقاسات الجزئية الجوهرية الضعيفة

منى عباس أحمد

قسم الرياضيات ، كلية العلوم للبنات، جامعة بغداد

الخلاصة:

لتكن R حلقة ابدالية ذات عنصر محايد، وليكن M مقاساً أحادياً ايسر على R. هدفنا في هذا البحث هو التقصي عن بعض النتائج الجديدة حول المقاسات الجزئية الجوهرية الضعيفة. حيث يقال للمقاس الجزئي N من M بأنه شبه جوهري ضعيف، إذا كان $N \cap P \neq 0$ لكل مقاس جزئي شبه أولي غير صفري P من M. تم إعطاء بعض التعاريف الجديدة ذات العلاقة بهذا المفهوم، كما قدمنا أيضاً عدد من القضايا والخواص الجديدة (على حد علمنا) لهذا النوع من المقاسات الجزئية.

الكلمات المفتاحية: المقاسات الجزئية شبه الأولية، المقاسات الجزئية الجوهرية، المقاسات الجزئية الجوهرية الضعيفة، المقاسات المنتظمة الضعيفة، المقاسات الأولية والمقاسات الجوهرية المتكاملة من النمط *.