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Fibrewise soft ideal topological space

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Abstract. In this work we explain and discuss new notion of fibrewise topological spaces, calledfibrewise soft ideal topological spaces, Also, we show the notions of fibrewise closed soft ideal topological spaces, fibrewise open soft ideal topological spaces and fibrewise soft near ideal topological spaces.

Keywords. Soft set, soft continuous, soft ideal, fibrewise soft ideal topological spaces, fibrewise closed soft ideal topological spaces, fibrewise open soft ideal topological spaces, fibrewise soft near ideal topological spaces.

1. Introduction and Preliminaries.

To begin with our work in the type of fibrewise sets over a given set, called the base set. If the base set is denoted by$B$, then a fibrewise set over B consists of a set H together with a function$P : H \to B$, called the projection. For all point b of B the fibre over b is the subset $H_b = P^{-1}(b)$ of H; fibres may be empty because we do not require P to be surjective, in addition for all subset B^* of B we consider $H_{B^*} = P^{-1}(B^*)$ as a fibrewise set over B^* with the projection determined by P.Molodtsov [23] generalized with the introduction of soft sets the traditional concept of a set in the classical researches. With the introduction of the applications of soft sets [22], the soft set theory has been the research topic and has received attention gradually [5, 19, 24, 26]. The applications of the soft sets are redetected so as to develop and consolidate this theory, utilizing these new applications; auni-int decision-making method was established [7]. Numerous notions of general topology were involved in soft sets and then authors developed theories about soft topological spaces. Shabir and Naz [28] mentioned this term to define soft topological space. After that definition, I. Zorlutuna et al. [34], Aygunoglu et al. [6] and Hussain et al. [15] continued to search the properties of soft topological space.This topic has an excellent potential for applications in many branches of mathematics. In 1990, Jankovic and Hamlett [17] using a specific topological τ introduced another topological$\tau^*(I)$, which satisfies$\tau \subset \tau^*(I)$. In 1952, Hashimoto [12, 13] introducing the concept of ideal continuity. Then Jankovic and Hamlett [11] introduced I-open set in ideal topological spaces. Future, Abd El-Monsef [1] defined I-continuity for functions**.** The notion of soft ideal was first given by R. Sahin and A. Kucuk [27]. Kale and Guler [18] introduced the concept of soft ideal and discuss the properties thesoft ideal topological space. The purpose of this paper is introduced a new class of fibrewise topology called fibrewise soft ideal topological space are introduced and few of their properties are investigated, we built on some of the result in [2, 20, 29, 30, 31, 32, 33].

Definition 1.1. [16]Let$H$ and K be afibrewise sets over B, with projections $P_H : H \to B$ and$P_K : K \to B$, respectively, a function $\phi : H \to K$ issaid to befibrewise if $P_K \circ \phi = P_H$, in other words if $\phi(H_b) \subset K_b$for all point b of B.
Note that a fibrewise function $\phi : H \to K$ over Blimitedsby restriction, a fibrewise function $\phi_{B^*} : H_{B^*} \to K_{B^*}$ over B^* for all subset B^* of B .

Definition 1.2.[23] Let$H$ be an initial universe and E be a set of parameters. Assume that$P(H)$denote the power set of H and E be a non-empty subset of A. A pair (F, E) is called a soft set over H, where F is a function given by $F : E \to P(H)$. In other words, a soft set over H is a parameterized familyof subset of the universe H.For $\varepsilon \epsilon E$, $F(\varepsilon)$may be considered like the set of ε_approximate elements of the soft set (F, E). observe that the set of all soft sets over H will be denoted by $S(H)$.

Example 1.3. [23] Assume that we have six houses in the universe $H = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that $A = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The $e_i (i = 1, 2, 3, 4, 5)$ stand for the parameters ''expensive'', ''beautiful'', ''wooden'', ''cheap'', and ''in green surroundings'', respectively. Consider the functionF_A given by ''houses (.)''; (.) is to be filled in by one of the parameters $e_i \in A$. For instance,$F_E(e_1)$ means''houses (expensive)'', and its functional value is the set $\{h \in H : h$ is an expensive house$\}$. Assume that $E = \{e_1, e_3, e_4\} \subset A$ and $F_E(e_1) = \{h_1, h_4\}$, $F_E(e_3) = H$,and $F_E(e_4) = \{h_1, h_3, h_5\}$. Then, we can view the soft set F_E like consisting of the next collection of approximations:$F_E = \{(e_1, \{h_2, h_4\}), (e_3, H), (e_4, \{h_1, h_3, h_5\})\}$.

Definition 1.4.[22] A soft set (F, E) over H is said to be a null soft if $F(e) = \Phi$ for all $e \in E$ and this denoted by $\widetilde{\Phi}$. Also, (F, E) is said to be an absolute soft set if $F(e) = H$, for all $e \in E$ and this denoted by $\widetilde{H}$.

Definition 1.5.[22]
(a) The union of two soft sets of $(F, A)$and (G, B)over the common universe $H$is the soft set (L, C), where$C = A \cup B$and for all $e \in C$,

$$L(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

We write $(F, A)\ \widetilde{\cup}\ (G, B) = (L, C)$.
(b) The intersection (L, C) of two soft sets (F, A), (G, B) over a common universe H, denoted by $(F, A)\ \widetilde{\cap}\ (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$for all $e \in C$.

Definition 1.6. [28] Let τ be the collection of soft sets over H, then τ is said to be a soft topology on H if
(a) $\widetilde{\Phi}, \widetilde{H} \widetilde{\in} \tau$
(b) the union of any number of soft sets in τ belongs to τ
(c) the intersection of any two soft sets in τ belongs to τ.

The triplet $(H, \tau, E)$is called a soft topological space over H.The members of τ are called soft open sets in H. Also, a soft set (F, A) is called a soft closed if the complement $(F, A)^c$ belongs to τ.

Example 1.7.[28]Let $H = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{H}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over H,defined asthe following
$F_1(e_1) = \{h_1\}$, $F_2(e_2) = \{h_1\}$,
$F_2(e_1) = \{h_2, h_3\}$, $F_2(e_2) = \{h_1, h_2\}$,
$F_3(e_1) = \{h_1, h_2\}$, $F_3(e_2) = H$,
$F_4(e_1) = \{h_1, h_2\}$, $F_4(e_2) = \{h_1, h_3\}$.

Then τ defines a soft topology on $H$and hence $(H,\tau,E)$is a soft topological space over.

Definition 1.8. [25]

(a) A soft set (F,E) over $H$is said to be a soft element if $\exists e \in E$such that $F(e)$ is a singleton, say,$\{h\}$and $(e') = \Phi$, $\forall e'(\neq e) \in E$. Such a soft element is denoted by F_e^h. For simplicity of notation we denote such soft element as $\tilde{h}$. Let $SE(H,E)$ be the set of all soft elements of the universal set H.

(b) A soft set (F,E) is said to be a soft neighbourhood (briefly soft nbd) of the soft set (L,E) if there exists a soft set $(G,E) \tilde{\in} \tilde{\tau}$such that $(L,E) \tilde{\subseteq} (G,A) \tilde{\subseteq} (F,E)$. If $(L,E) = \tilde{h}$, then (F,E) is said to be a soft nbdof the soft element $\tilde{h}$. The soft neighbourhood system of soft element $\tilde{h}$, denoted by$N(\tilde{h})$, is the family of all its soft neighbourhood. The soft open neighbourhood system of soft element $\tilde{h}$, denoted by $V(\tilde{h})$, is the family of all its soft open neighbourhood.

Definition 1.9.[28]

Let $(H,\tilde{\tau},E)$ be a soft topological space and (F,E) be a soft set over H. Then, the soft closure of (F,E), denoted $\overline{(F,E)}$, is defined as the soft intersection of all soft closed super sets of (F,E).

Note that $\overline{(F,E)}$ is the smallest soft closed set that containing(F,E).

Definition 1.10.[8]

(a) Let $(H,\tilde{\tau},E)$ be a soft topological space over H and Kbe a non-empty subset of H, then the collection $\tau_K = \{\tilde{K} \tilde{\cap} (F,E): (F,E) \tilde{\in} \tilde{\tau}\}$ is called a soft subspace topology on K. Hence, (K,τ_K,E) is called a soft topological subspace of (H,τ,E).

(b) A soft basis of a soft topological space (H,τ,E) is a subcollection $\tilde{\mathcal{B}}$ of τ such that every element of τ can be expressed as the union of elements of $\tilde{\mathcal{B}}$.

Definition 1.8. [10] A soft ideal I is a nonempty collection of soft sets over H if

(a) If $(F,E) \tilde{\in} I$, $(G,E) \tilde{\subset} (F,E)$, then$(G,E) \tilde{\in} I$

(b) If$(F,E) \tilde{\in} I$, $(G,E) \tilde{\in} I$,then $(F,E) \tilde{\cup} (G,E) \tilde{\in} I$.

A soft topological space (H,τ,E) with a soft ideal $I$is said to be soft ideal topological space and denoted by (H,τ,E,I).

Definition 1.9. [10]Let(F,E)be a soft set in a soft ideal topological space (H,τ,E,I) and $(\cdot)^*$ be a soft operator from $S(H)$ to $S(H)$. Then the soft local function of (F,E) defined by $(F,E)^*(I,\tau) = \{\tilde{h}: (U,E) \tilde{\cap} (F,E) \tilde{\notin} I$ for every $(U,E) \tilde{\in} \tilde{V}(\tilde{h})$ denoted by $(F,E)^*$ simply. Moreover, the soft set operator Cl^* is said to be a soft $^*-$closure and is defined as $Cl^*(F,E) = (F,E) \tilde{\cup} (F,E)^*$ for a soft subset(F,E).

Definition 1.10. [21] Let H and K be two non-empty sets and E be the parameter set. Let$\{f_e : H \to K, e \in E\}$ be a collection of functions. Then a function $\tilde{f} : \mathrm{SE}(H,E) \to \mathrm{SE}(K,E)$ defined by $\tilde{f}(e_h) = e_{f_e(h)}$ is called a soft function, where $\mathrm{SE}(H,E)$ and $\mathrm{SE}(K,E)$ are sets of all soft elements of the soft sets $\tilde{H}$ and $\tilde{K}$ respectively.

Definition 1.11. [3]

(a) A soft subset (F,E) of an soft ideal topological space $(H,\tilde{\tau},E,I)$ is a soft I-open (briefly S.I-open) if $(F,E) \tilde{\subset} int(F,E)^*$.

(b) A soft subset (F,E) of a soft ideal topological space $(H,\tilde{\tau},E,I)$ is a soft I-closed (briefly S.I-closed) if it is complement is a soft I-open.

(c) A soft function $\phi : (H, \tilde{\tau}, E, I) \to (K, \tilde{\sigma}, L)$ is said to be an soft ideal continuous (briefly S.I-continuous) if the inverse image of every soft open set of K is a soft I-open set in H.
(d) An soft ideal topological space $(H, \tilde{\tau}, E, I)$ is said to be a soft near I-compact (briefly S.j.I-compact) if for every soft near ideal open caver (briefly S.j.I-open cover) $\{(W_i, E_i): i \in \Delta\}$ of H, there exists a finite subset Δ_o of Δ such that $\widetilde{H} - \{(W_i, E_i): i \in \Delta_o\} \in I$, where $j \in \{\alpha, S, P, b, \beta\}$.[4]

Lemma 1.12.[3]For any S.I-open set (F, E) of a space (H, τ, E, I), we have $(F, E)^* = (int(F, E)^*)^*$.

Definition 1.13.A subset $A$of an ideal topological space H is said to be
(a) α-I-open set [14] if $A \subset int(Cl^*(int(A)))$.
(b) pre-I-open set [1]if $A \subset int(Cl^*(A))$.
(c) sime-I-open set [14]if$A \subset Cl^*(int(A))$.
(d) b-open set [9] if$A \subset Cl^*(int(S)) \cup int(Cl^*(A))$.
(e) β-I-open set [14] if$A \subset Cl^*(int(Cl^*(A)))$.

Lemma 1.14. [3]For every soft function$\phi : (H, \tau, E, I) \to (K, \sigma, L)$, $\phi(I)$is an soft ideal continuous over$K$.

2- FIBERWISE SOFT IDEAL TOPOLOGICAL SPACES

During this section, we introduced a definition of fibrewise soft ideal topological space and its related properties.

Definition 2.1. Assume that (B, Ω, G) is a soft topology space the fibrewise soft ideal topology space (briefly, F.W.S.I-topological space) on a fibrewise set H over Bmeany any soft ideal topology space on H for which the projection function is S.I-continuous.

F.W.S. topological space and F.W.S.I-topological space this two concepts areThe following examples explain that.

Example 2.2. Let (H, τ, E, I) be a F.W.S.I-topological space over (B, Ω, G) given as follows : $H = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tau = \{\widetilde{\Phi}, \widetilde{H}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_2\})\}, \{(e, \{h_1, h_2, h_3\})\}\}$, $I = \{\widetilde{\Phi}, \{(e, \{h_1\})\}\}$, $B = \{a, b, c, d\}$, $G = \{g\}$, $\Omega = \{\widetilde{\Phi}, \widetilde{B}, \{(g, \{b, c, d\})\}\}$. Furthermore, assume that$f : H \to B$, $f(h_1) = a$, $f(h_2) = b$,$f(h_3) = c$, $f(h_4) = d$,$u : E \to G$, $u(e) = g$. Then the soft function $P_{fu} : (H, \tau, E, I) \to (B, \Omega, G)$ is soft I – continuous but is not soft continuous. Thus, (H, τ, E, I) is F.W.S.I-topological space but not F.W.S.topological space.

Example 2.3. Let(H, τ, E, I)be a F.W.S.I-topological space over (B, Ω, G) given asfollows: $H = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tau = \{\widetilde{\Phi}, \widetilde{H}, \{(e, \{h_4\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_1, h_3, h_4\})\}$,$I = \{\widetilde{\Phi}, \{(e, \{h_3\})\}, \{(e, \{h_4\})\}, \{(e, \{h_3, h_4\})\}\}$, $B = \{a, b, c, d\}$, $G = \{g\}$, $\Omega = \{\widetilde{\Phi}, \widetilde{B}, \{(g, \{a, c, d\})\}\}$. Furthermore, assume that$f : H \to B$, $f(h_1) = a$, $f(h_2) = b$,$f(h_3) = c$, $f(h_4) = d$,$u : E \to G$, $u(e) = g$. Then the soft function $P_{fu} : (H, \tau, E, I) \to (B, \Omega, G)$ is soft continuous but is not S.I-continuous. Thus, (H, τ, E, I) is F.W.S. topological space but not F.W.S.I-topological space.

Proposition 2.4. The following statements are equivalent.
(a) $(H, \tau, E, I)$is F.W.S.I-topological space over (B, Ω, G),
(b) For each soft element in (H, τ, E) and each soft open in (B, Ω, G)containing the image of soft element, there exist S.I-open of(H, τ, E, I) containing soft element such that the image of S.I-open containing in soft open,
(c) For each soft element in (H, τ, E) and each soft open in (B, Ω, G) containing the image of soft element, $(H_{(M,G)})^*$is nbd of soft element.

Proof.(a)⇔(b)Suppose that (H, τ, E, I) is F.W.S.I-topological space over (B, Ω, G), then there exist the projection function$P_{fu}: (H, \tau, E, I) \to (B, \Omega, G)$is S.I-continuous. To proof For each soft element in (H, τ, E) and each soft open in (B, Ω, G) containing the image of soft element, there exist S.I-open of

(H,τ,E,I) containing soft element such that the image of S.I-open containing in soft open, since $(M,G) \tilde{\in} \Omega$ containing $P_{fu}(\tilde{h})$, then by (a) $H_{(M,G)}$ is S.I-open in H. By taking $(F,E) = H_{(M,G)}$ which containing $P_{fu}(\tilde{h})$, thus $P_{fu}(F,E) \tilde{\subset} (M,G)$.
(b)⇒(c) Sincesoft open in (B,Ω,G) containing the image of soft element, then by (b) there exists (F,E) is S.I-open of (H,τ,E,I)containing $P_{fu}(\tilde{h})$, such that $P_{fu}(F,E) \tilde{\subset} (M,G)$. So$\tilde{h} \tilde{\in} (F,E) \tilde{\subset} int((F,E)^*) \tilde{\subset} int(H_{(M,G)})^* \tilde{\subset} (H_{(M,G)})^*$. Hence $(H_{(M,G)})^*$is anbd of $\tilde{h}$.
(c)⇒(a) Since soft open in (B,Ω,G) containing the image of soft element, then by (c) $(H_{(M,G)})^*$is a softnbd of$\tilde{h}$, then there exist a soft set $(F,E) \tilde{\in} \tau$ such that $\tilde{h} \tilde{\in} (F,E) \tilde{\subset} (H_{(M,G)})^*$, then $H_{(M,G)}$ is S.I-open, then P_{fu} is S.I-continuous, thus $(H,\tau,E,I)$is F.W.S.I-topological spaces.

Proposition 2.5.$(H,\tau,E,I)$is F.W.S.I-topological space over (B,Ω,G) if and only if the graph soft function $g: (H,\tau,E) \to (H,\tau,E) \times (B,\Omega,G)$, defined by $g(\tilde{h}) = (\tilde{h}, f(\tilde{h}))$, for each $\tilde{h} \tilde{\in} H$, is S.I-continuous.
Proof .(⇒) Suppose that (H,τ,E,I) is F.W.S.I-topological space over (B,Ω,G), then there exist the projection function $P_{fu} : (H,\tau,E, I) \to (B,\Omega,G)$ is S.I-continuous. For proof g is S.I-continuous, let $\tilde{h} \tilde{\in} H$ and $(W, E \times G)$ be any soft open of $H \times B$containing $g(\tilde{h}) = (\tilde{h}, f(\tilde{h}))$. Thus we have a basic soft open set $(F,E) \times (M,G)$ such that $g(\tilde{h}) = \left(\tilde{h}, f(\tilde{h})\right) \tilde{\in} (F,E) \times (M,G) \tilde{\subseteq} (W, E \times G)$. Since (H,τ,E,I) is F.W.S.I-topological space, then $P_{fu}$is S.I-continuous, we have a S.I-open set (L,E) of (H,τ,E) containing $(\tilde{h})$such that$P_{fu}(L,E) \tilde{\subset} (M,G)$. Because $(L,E) \tilde{\cap} (F,E)$is soft ideal open of(H,τ,E,I) and $(L,E) \tilde{\cap} (F,E) \tilde{\subset} (F,E)$ then $g\left((F,E) \tilde{\cap} (L,E)\right) \tilde{\subset} (F,E) \times (M,G) \tilde{\subset} (W, E \times G)$.This explains that g is S.I-continuous.
(⇐)assume that g is S.I-continuous. For proof (H,τ,E,I) is F.W.S.I-topological spaces i.e., the projection function$P_{fu} : (H,\tau,E, I) \to (B,\Omega,G)$ is S.I-continuous, let $\tilde{h} \tilde{\in} H$and (M,G) be any soft open set of (B,Ω,G)containing $P_{fu}(\tilde{h})$. Then $H \times (M,G)$ is soft open in $(H,\tau,E) \times (B,\Omega,G)$. Because g is S.I-continuous, we have(F,E) is S.I-open in (H,τ,E,I) containing $\tilde{h}$such that $g(F,E) \tilde{\subset} H \times (M,G)$. Hence, we get$P_{fu}(F,E) \tilde{\subset} (M,G)$.This explains that P_{fu} is S.I-continuous. Thus (H,τ,E,I) is F.W.S.I-topological space.
Proposition 2.6. Let(H,τ,E,I) be a F.W.S.I-topological space over (B,Ω,G). Then $(H_{B^*}, \tau_{B^*}, E_{B^*}, I_{B^*})$ is F.W.S. I-topological space over (B^*,Ω^*,G^*) for each open subspace $(B^*,\Omega^*,G^*)$of (B,Ω,G).
Proof. Suppose that (H,τ,E,I) is F.W.S.I-topological space over (B,Ω,G),then there exist the projection function $P_{fu} : (H,\tau,E,I) \to (B,\Omega,G)$ is S.I-continuous. To show that $(H_{B^*}, \tau_{B^*}, E_{B^*}, I_{B^*})$ is F.W.S.I-topological over (B^*,Ω^*,G^*)i.e.,the projection function$P_{B^*(fu)} : (H_{B^*}, \tau_{B^*}, E_{B^*}, I_{B^*}) \to (B^*,\Omega^*,G^*)$ is S.I-continuous. Assume(M,G) isa soft open subset of (B,Ω,G), then $(M,G) \tilde{\cap} (B^*,G^*)$is soft open in subspace (B^*,Ω^*,G^*) andso $H_{(M,G)} \tilde{\subset} int(H_{(M,G)})^*$, then $H_{(M,G)} \tilde{\cap} (H_{B^*}, E_{B^*}) \tilde{\subset} int\left(H_{(M,G)}\right)^* \tilde{\cap} (H_{B^*}, E_{B^*})$. This $(H_{B^*}, E_{B^*})_{(M,G)} = H_{(M,G)} \tilde{\cap} (H_{B^*}, E_{B^*}) \tilde{\subset} (H_{B^*}, E_{B^*})_{(M,G)} \tilde{\subset} int((H_{B^*}, E_{B^*}) \tilde{\cap} (H_{(M,G)}))^* = int((H_{B^*}, E_{B^*})_{(M,G)})^*$. This we have that $(H_{B^*}, E_{B^*})_{(M,G)}$ is S.I-open of$(H_{B^*}, E_{B^*}, I_{B^*})$. This shows that $P_{B^*(fu)}$ is S.I-continuous, then$(H_{B^*}, \tau_{B^*}, E_{B^*}, I_{B^*})$is F.W.S.I-topological space.

Proposition 2.7. Assume that (H,τ,E,I) is F.W.S. I-topological space over $(B,\Omega,G)$and for all member (H_i, E_i) of a soft open covering of (H,τ,E). Then $(H_{iB^*}, E_{iB^*}, I_{iB^*})$ is F.W.S.I-topological spaces over(B^*,Ω^*,G^*) for each open subspace $(B^*,\Omega^*,G^*)$of (B,Ω,G).

Proof. The proof is like to previous proposition.

Definition 2.8. Let $\phi : (H,\tau,E,I) \to (K,\sigma,L,J)$ be a function ϕ is called soft near irresolute(briefly, S. j-I-irresolute) if the inverse image of soft j-I-open set in K is soft j-I-open in H, where $j \in \{\alpha, S, P, b, \beta\}$.

Definition 2.9.A F.W.S.I-topological space (H,τ,E,I) over (B,Ω,G,J) is called fibrewise soft near ideal irresolute (briefly, F.W.S. I-irresolute) if the projection $P_{fu}$is soft j-I-irresolute where $j \in \{\alpha,S,P,b,\beta\}$.
Proposition 2.10. Let (H,τ,E,I) be a F.W.S.I-topological space over (B,Ω,G) and $(H_{(M,G)^*}) \tilde{\subseteq} (H_{(M,G)})^*$ for each soft subset $(M,G)$of (B,Ω,G). Then (H,τ,E,I) is F.W.S.I-irresolute.
Proof. Suppose that (H,τ,E,I) is F.W.S.I-topological space over (B,Ω,G), then there exist the projection function $P_{fu} : (H,\tau,E,I) \to (B,\Omega,G)$is S.I-continuous. For proof (H,τ,E,I) is F.W.S.I-irresolute, then there exist the projection function $P_{fu} : (H,\tau,E,I) \to (B,\Omega,G,J)$is soft I-irresolute. Let (M,G) be any S.I-open set of (B,Ω,G,J). By lemma (1.13), we have $(M,G)^* = (int(M,G)^*)^*$. Therefore, we have $H_{(M,G)^*}$=$H_{(int(M,G)^*)^*}$,such $(H_{(M,G)})^* \tilde{\subseteq} (H_{int(M,G)^*})^*$, $H_{(M,G)} \tilde{\subseteq} H_{int(M,G)^*}$.so$H_{(M,G)}$ is S.I-open in (H,τ,E,I),then P_{fu} is soft I-irresolute, thus (H,τ,E,I) is F.W.S.I-irresolute.
Remark 2.11. Assume that $\phi : (H,\tau,E,I) \to (K,\sigma,L)$ is afibrewise soft ideal function and $\psi : (K,\sigma,L,M) \to (Z,\vartheta,C)$ is fibrewise soft ideal function, where $(H,\tau,E,I),(K,\sigma,L,M),(Z,\vartheta,C,N)$ are F.W.S. I-topological space over (B,Ω,G) thenthe composition $\phi o \psi : (H,\tau,E,I) \to (Z,\vartheta,C)$ is need not fibrewise soft ideal function, in general, like shown by the next example.

Example 2.12. Let $H = Z = \{h_1,h_2,h_3\}$, $K = \{h_1,h_2,h_3,h_4\}$, $B = \{a,b,c\}$, $E = L = C = \{e\}$, $G = \{g\}$, with soft topologies, $\tau = \{\tilde{\Phi},\tilde{H},\{(e,\{h_1\})\}\}$,$\sigma = \{\tilde{\Phi},\tilde{K},\{(e,\{h_1,h_3\})\}\}$, $\vartheta = \{\tilde{\Phi},\tilde{Z},\{(e,\{h_3\})\},\{(e,\{h_2,h_3\})\}\}$,$\Omega = \{\tilde{\Phi},\tilde{B}\}$,$I = \{\tilde{\Phi},\{(e,\{h_3\})\}\}$, $M = \{\tilde{\Phi},\{(e,\{h_1\})\}\}$. Let$(H,\tau,E,I)$and (K,σ,L,M), (Z,ϑ,C,N) is F.W.S.I-topological spaces over (B,Ω,G). Define the identity function$f$from H, Z to B,let$U$ be a identify function from E, $C$to$G$ and define $q : K \to B$, $d : L \to G$ as: $q(h_1) = a$, $q(h_2) = q(h_4) = b$, $q(h_3) = c$, $d(e) = g$. Then the soft function $P_{H(fu)}: (H,\tau,E,I) \to (B,\Omega,G)$, $P_{K(qd)}: (K,\sigma,L,M) \to (B,\Omega,G)$ and $P_{z(tv)}: (Z,\vartheta,C,N) \to (B,\Omega,G)$are S.I-continuous. And define identity function $\phi : (H,\tau,E,I) \to (K,\sigma,L)$,$\psi : (K,\sigma,L,M) \to (Z,\vartheta,C)$ define as: $\psi(\tilde{h}_{1\tilde{b}}) = \tilde{h}_{1\tilde{b}}$, $\psi(\tilde{h}_{2\tilde{b}}) = \psi(\tilde{h}_{4\tilde{b}}) = \tilde{h}_{2\tilde{b}}$, $\psi(\tilde{h}_{3\tilde{b}}) = \tilde{h}_{3\tilde{b}}$ where$\tilde{b} \tilde{\in} B$. It clear that both ϕ and $\psi$are S.I-continuous function. However, the composition function $\phi o \psi$ is not S.I-continuous function because $\{(e,\{h_3\})\}$ is S. open in (Z,ϑ,C), but $(\phi o \psi)^{-1}\{(e,\{h_3\})\} = \{(e,\{h_3\})\}$ is not S.I-open in (H,τ,E,I).

Proposition 2.13. Let $\phi : (H,\tau,E,I) \to (K,\sigma,L)$be a fibrewise soft function, where(K,σ,L,M)aF.W.S.I-topological space over $(B,\Omega,G)$and (H,τ,E,I) has the induced F.W.S. I-topology,the follow are hold:

(a) If ϕ is soft continuous and for each F.W.S. I-topological space (Z,γ,C,N), a fibrewise soft function $\psi: (Z,\gamma,M,C) \to (H,\tau,E)$ is S.I-continuousthenthe composition $\phi \circ \psi: (Z,\gamma,C,N) \to (K,\sigma,L)$ is S.I-continuous.
(b) If ϕ is S.I-continuous and for each F.W.S. I-topological space (Z,γ,C,N), a fibrewise soft function $\psi: (Z,\gamma,M,C) \to (H,\tau,E,I)$ is soft I-irresolute then the composition $\phi \circ \psi: (Z,\gamma,C,N) \to (K,\sigma,L)$ is S.I-continuous.

Proof.(a)Assume that ψ is S.I-continuous. Let$\tilde{z} \tilde{\in} Z_{\tilde{b}}$, where $\tilde{b} \tilde{\in} B$ and (N,L) soft open set of $(\phi \circ \psi)(\tilde{z})$ =$\tilde{k} \tilde{\in} K_{\tilde{b}}$ in (K,σ,L). becauseϕ is soft continuous, $\phi^{-1}(N,L)$is a soft open set containing $\psi(\tilde{z}) = \tilde{h} \tilde{\in} H_{\tilde{b}}$in (H,τ,E). Becauseψ is S.I-continuous, then$\psi^{-1}(\phi^{-1}(N,L))$ is a S.I-open set containing $\tilde{z} \in Z_{\tilde{b}}$ in (Z,γ,C) and $\psi^{-1}(\phi^{-1}(N,L)$=$(\phi \circ \psi)^{-1}(N,L)$ is a S.I-open set containing $\tilde{z} \in Z_{\tilde{b}}$in (Z,γ,C,N).
(b)The proof is similar to the proof of (a).

3.Fibrewise Soft Ideal Closed and Soft Ideal Open Topological Spaces.

During this part, we explain the ideas of fibrewise soft ideal closed, soft ideal open topological spaces. Several topological properties on the obtained concepts are studied.

Definition 3.1.A function$\phi : (H,\tau,E,I) \to (K,\sigma,L)$ is called:

(a) Soft ideal open (briefly, S.I-open) function if the image of all S.I-open set in H is soft open set in K.
(b) Soft ideal closed (briefly, S. I-closed) function if the image of all S.I-closed set in H is soft closed set in K.

Definition 3.2. A F.W.S.I-topological space (H,τ,E,I) over (B,Ω,G) is called fibrewise soft ideal closed (briefly, F.W.S. I-closed) if the projection function is S. I-closed.
Definition 3.3. A F.W.S.I-topological space (H,τ,E,I) over (B,Ω,G) is called fibrewise soft ideal open (briefly, F.W.S. I-open) if the projection function is S.I-open.

F.W.S.I-open and F.W.S. open arethis two concepts are independent.

Example 3.4. Let (H,τ,E,I) bea F.W.S. I-topological space over (B,Ω,G) given likethe following:$H=\{h_1,h_2\}$, $E=\{e_1,e_2\}$,$\tau=\{\tilde{\Phi},\tilde{H},\{(e_1,\{h_2\})\}\}$, $I=\{\tilde{\Phi},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $B=\{a,b\}$, $G=\{g_1,g_2\}$, $\Omega=\{\tilde{\Phi},\tilde{B},\{(g_1,\{a\})\},\{(g_2,\{b\})\},\{(g_1,\{a\}),(g_2,\{b\})\}\}$. Also, let $f:H\to B$, $f(h_1)=a$, $f(h_2)=b$, $u:E\to G$, $u(e_1)=g_1$, $u(e_2)=g_2$. Then soft function $P_{fu}:(H,\tau,E,I)\to(B,\Omega,G)$ is S. I-pen but is not S. open. Thus, (H,τ,E,I) is F.W.S. I-open but not F.W.S.open.

Example 3.5. Let (H,τ,E,I)be a F.W.S. I-topological space over(B,Ω,G) given likethe following: $H=\{h_1,h_2\}$, $E=\{e_1,e_2\}$, $\tau=\{\tilde{\Phi},\tilde{H},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $I=\{\tilde{\Phi},\tilde{H},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $B=\{a,b\}$, $G=\{g_1,g_2\}$, $\Omega=\{\tilde{\Phi},\tilde{B},\{(g_1,\{a\})\},\{g_2,\{b\})\},\{(g_1,\{a\}),(g_2,\{b\})\}\}$. Also, let $f:H\to B$, $f(h_1)=a$, $f(h_2)=b$, $u:E\to G$, $u(e_1)=g_1$, $u(e_2)=g_2$. Then soft function $P_{fu}:(H,\tau,E,I)\to(B,\Omega,G)$ is S. open but is not S.I-open. Thus, (H,τ,E,I) is F.W.S. I-open but not F.W.S. open.

Proposition 3.6. Assume that (H,τ,E,I) is a F.W.S.I-open over (B,Ω,G)then for each soft element in (H,τ,E) and each soft nbd$(F,E)^*$ of the soft element, there exists soft open of (B,Ω,G) containing the image of soft element such that soft open containing in image of soft nbd
Proof. Suppose that (H,τ,E,I) is F.W.S.I-open, then there exist the projectionfunction$P_{fu}:(H,\tau,E,I)\to(B,\Omega,G)$is a S.I-open function. To proof for each soft element and each soft nbd(F,E) of the soft element, there exists soft open of (B,Ω,G) containing the image of soft element such that soft open containing in image of soft nbd, since for each soft element $\tilde{h}\,\tilde{\in}\,H$ and each soft nbd$(F,E)^*$of$\tilde{h}$, then there existsS.I-open(U,E)in (H,τ,E) such that $\tilde{h}\,\tilde{\in}\,(U,E)\,\tilde{\subset}\,(F,E)^*$. Since $P_{fu}$is S.I-open, $(M,G)=P_{fu}(U,E)$ is soft open of (B,Ω,G) and $P_{fu}(\tilde{h})\,\tilde{\in}\,(M,G)\,\tilde{\subset}\,P_{fu}(F,E)^*$.

Proposition 3.7. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G). then(H,τ,E,I) is F.W.S. I-open (res. F.W.S.I-closed) if and for all fibre soft $(H_{\tilde{b}},E_{\tilde{b}})$of (H,τ,E) and all S.I-open (res. S.I-open)subset (F,E) of $(H_{\tilde{b}},E_{\tilde{b}})$ in (H,τ,E), there is a softopen (soft closed) subset (F,G) of $\tilde{b}$where $(H_{(F,G)},E_{(F,G)})\,\tilde{\subset}\,(F,E)$.
Proof.(⇒)Suppose that (H,τ,E,I) is F.W.S.I-open (res. F.W.S.I-closed) then there existsthe projection function$P_{fu}:(H,\tau,E,I)\to(B,\Omega,G)$ isS.I-open(res. soft I-closed). Now, let $\tilde{b}\,\tilde{\in}\,B$ and (F,E)S.I-open(res. soft I-closed)subset of $(H_{\tilde{b}},E_{\tilde{b}})$ in (H,τ,E), then (H,E) _(F,E) is soft I-closed (res. S.I-open) in (H,τ,E), this implies $P_{fu}((H,E)-(F,E))$ issoft open (res. soft closed) in (B,Ω,E), let $(F,G)=(B,G)-P_{fu}((H,E)-(F,E))$, then (F,G)a soft open (res. soft closed) subset of $\tilde{b}$ in $(B,\Omega,G)$and$\left(H_{(F,E)},E_{(F,E)}\right)=H_{(F,G)}=(H,E)-H_{(P_{fu}((H,E)-(F,E))}\,\tilde{\subset}\,(F,E)$.
(⇐)Suppose that the assumption hold and there exist the projection function$P_{fu}:(H,\tau,E,I)\to(B,\Omega,G)$ is soft I-closed. Now, let (F,C) be a S. I-closed subset of (H,τ,E,I) and $\tilde{b}\,\tilde{\in}\,(B,G)-P_{fu}(F,C)$ and each S.I-open set (F,E) of fibre soft $(H_{\tilde{b}},E_{\tilde{b}})$ in (H,τ,E,I). By assumption we have a soft open (F,G) of $\tilde{b}$ such that $(H_{(F,G)},E_{(F,G)})\,\tilde{\subset}\,(F,E)$. It is easy to show that $(F,G)\,\tilde{\subset}\,(B,G)-P_{fu}(F,C)$,hence $(B,G)-P_{fu}(F,C)$ is soft open in (B,Ω,G) and this implies $P_{fu}(F,C)$ is softclosed in (B,Ω,G) and P_{fu} is soft I-closed.Thus, $(H,\tau,E,I)$is F.W.S. I-closed. For a S.I-open function, we can prove similarly.

Proposition 3.8. Assume that$\phi : (H,\tau,E,I) \to (K,\sigma,L)$is a fibrewise S.I-open function, where (K,σ,L,M)a F.W.S.I-topological space over (B,Ω,G)) and (H,τ,E,I) has the induced F.W.S. I-topologyand for each F.W.S. I-topological space (Z,γ,C,N), a fibrewise soft function $\psi: (Z,\gamma,C,N) \to (H,\tau,E)$ is S.I-open then the composition $\phi \circ \psi: (Z,\gamma,C,N) \to (K,\sigma,L)$ is S.I-open.
Proof. Assume that ψ is S.I-open. Let $\tilde{z} \tilde{\in} Z_{\tilde{b}}$, where $\tilde{b} \tilde{\in} B$ and (N,L)S.I-open set in (Z,γ,C,N). Since Ψ is S.I-open, $\psi(N,L)$ is a soft open set containing $\psi(\tilde{z}) = \tilde{h} \tilde{\in} H_{\tilde{b}}$ in (H,τ,E). Becauseϕ is S.I-open, then$\psi(\phi(N,L))$ is a soft open set containing $(\phi o \psi)(\tilde{z}) = \tilde{k} \tilde{\in} K_{\tilde{b}}$ in (K,σ,L) and $\psi(\phi(N,L)$=$(\phi o \psi)(N,L)$ is a soft open set containing $\tilde{k} \in K_{\tilde{b}}$in (K,σ,L).

4. Fibrewise soft near ideal topological spaces

Definition 4.1. The F.W.S.I-topological space (H,τ,E,I) over (B,Ω,G) is named fibrewise soft near I-compact (briefly, F.W.S. j-I-compact) if the soft ideal topological space (H,τ,E,I) is soft j-I-compact, where $\in \{\alpha,S,P,b,\beta\}$.

Remark 4.2. In F.W.S.I-topological space we work over at soft topological base space B, say. When B is a point-space the theory reduces to that of ordinary soft topology. A F.W.S.I-topological (resp., S.j-I-topological) space over B is just a soft ideal topological (resp., S. j-I-topological) space H together with a soft ideal continuous (resp., S.j-I-continuous) the projectionfunction$P_{fu} : (H,\tau,E,I) \to (B,\Omega,G)$. So the implication between F.W.S.I-topological spaces and the families of F.W.S.j-I-topological spaces are given in the following diagram where $j \in \{\alpha,S,P,b,\beta\}$.

F.W.S.I-topological space
⇓
F.W.S.α-I-topological space ⇒ F.W.S. S-I-topological space
⇓⇓
F.W.S. P-I-topological space ⇒F.W.S.b-I-topological space ⇒ F.W.S. β-I-topological space.

Example 4.3. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G) given like the following : $H = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tau = \{\tilde{\Phi},\tilde{H},\{(e,\{h_1\})\},\{(e,\{h_2\})\},\{(e,\{h_1,h_2\})\},\{(e,\{h_1,h_2,h_4\})\}\}$, $I = \{\tilde{\Phi},\{(e,\{h_1\})\},\{(e,\{h_3\})\},\{(e,\{h_1,h_3\})\}\}$, $B = \{a,b,c,d\}$, $G = \{g\}$, $\Omega = \{\tilde{\Phi}, \tilde{B}, \{(g, \{a,b\})\}\}$. Also, let $f: H \to B$, $f(h_1) = a$, $f(h_2) = b$,$f(h_3) = c$,$f(h_4) = d$,$u : E \to G$, $u(e) = g$. Then the soft function $P_{fu}: (H,\tau,E,I) \to (B,\Omega,G)$ is S.α-I-continuous but is not S.I-continuous. Thus, (H,τ,E,I) is F.W.S.α-I-topological space but not F.W.S.I-topological space.

Example 4.4. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G) given like the following : $H = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tau = \{\tilde{\Phi},\tilde{H},\{(e,\{h_1\})\},\{(e,\{h_2\})\},\{(e,\{h_1,h_2\})\},\{(e,\{h_1,h_2,h_4\})\}\}$, $I = \{\tilde{\Phi},\{(e,\{h_1\})\},\{(e,\{h_3\})\},\{(e,\{h_1,h_3\})\}\}$, $B = \{a,b,c,d\}$, $G = \{g\}$, $\Omega = \{\tilde{\Phi}, \tilde{B}, \{(g, \{a,b\})\}\}$. Also, let $f: H \to B$, $f(h_1) = a$, $f(h_2) = b$, $f(h_3) = c$,$f(h_4) = d$,$u : E \to G$, $u(e) = g$. Then the soft function $P_{fu}: (H,\tau,E, I) \to (B,\Omega,G)$ is S.α-I-continuous but is not S.I-continuous. Thus, (H,τ,E,I) is F.W.S.α-I-topological space but not F.W.S.I-topological space.

Example 4.5. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G) given like the following : $H = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tau = \{\tilde{\Phi},\tilde{H},\{(e,\{h_1\})\},\{(e,\{h_2,h_3\})\},\{(e,\{h_1,h_2,h_3\})\}\}$, $I = \{\tilde{\Phi},\{(e,\{h_1\})\},\{(e,\{h_4\})\},\{(e,\{h_1,h_4\})\}\}$, $B = \{a,b,c,d\}$, $G = \{g\}$, $\Omega = \{\tilde{\Phi}, \tilde{B}, \{(g, \{a,b,d\})\}\}$. Also, let $f: H \to B$, $f(h_1) = a$, $f(h_2) = b$, $f(h_3) = c$,$f(h_4) = d$, $u : E \to G$, $u(e) = g$. Then the soft function $P_{fu}: (H,\tau,E, I) \to (B,\Omega,G)$ is S.P-I-continuous but is not S.α-I-continuous. Thus, (H,τ,E,I) is F.W.S.P-I-topological space but not F.W.S.α-I-topological space.

Example 4.6. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G) given like the following : $H=\{h_1,h_2,h_3,h_4\}$, $E=\{e\}$, $\tau=\{\tilde{\Phi},\tilde{H},\{(e,\{h_2\})\},\{(e,\{h_1,h_3\})\},\{(e,\{h_1,h_2,h_3\})\}\}$, $I=\{\tilde{\Phi},\{(e,\{h_1\})\},\{(e,\{h_4\})\},\{(e,\{h_1,h_4\})\}\}$, $B=\{a,b,c,d\}$, $G=\{g\}$, $\Omega=\{\tilde{\Phi},\tilde{B},\{(g,\{a,b,d\})\}\}$. Also, let $f:H\rightarrow B$, $f(h_1)=a$, $f(h_2)=b$, $f(h_3)=c$, $f(h_4)=d$, $u:E\rightarrow G$, $u(e)=g$. Then the soft function $P_{fu}:(H,\tau,E,\ I)\rightarrow(B,\Omega,G)$ is S. S-I-continuous but is not S.α-I-continuous. Thus, (H,τ,E,I) is F.W.S.S-I-topological space but not F.W.S.α-I-topological space.

Example 4.7. Let (H,τ,E,I) be a F.W.S. I-topological space over (B,Ω,G) given like the following : $H=\{h_1,h_2,h_3,h_4\}$, $E=\{e\}$, $\tau=\{\tilde{\Phi},\tilde{H},\{(e,\{h_1\})\},\{(e,\{h_4\})\},\{(e,\{h_1,h_4\})\}\}$, $I=\{\tilde{\Phi},\{(e,\{h_3\})\}\}$, $B=\{a,b,c,d\}$, $G=\{g\}$, $\Omega=\{\tilde{\Phi},\ \tilde{B},\ \{(g,\{a,d\})\},\{(g,\{a\})\},\{(g,\{d\})\}\}$. Also, let $f:H\rightarrow B$, $f(h_1)=d$, $f(h_2)=b$, $f(h_3)=d$, $f(h_4)=b$, $u:E\rightarrow G$, $u(e)=g$. Then the soft function $P_{fu}:(H,\tau,E,\ I)\rightarrow(B,\Omega,G)$ is S.b-I-continuous but is not S. P-I-continuous. Thus, (H,τ,E,I) is F.W.S.b-I-topological space but not F.W.S.P-I-topological space.

Example 4.8. Let (H,τ,E,I) be a F.W.S.I-topological space over (B,Ω,G) given like the following : $H=\{h_1,h_2,h_3\}$, $E=\{e\}$, $\tau=\{\tilde{\Phi},\tilde{H},\{(e,\{h_1,h_2\})\}\}$, $I=\{\tilde{\Phi},\ \{(e,\{h_3\})\}\}$, $B=\{a,b,c\}$, $G=\{g\}$, $\Omega=\{\tilde{\Phi},\tilde{B},\ \{(g,\ \{a,b\})\}\}$. Also, let $f:H\rightarrow B$, $f(h_1)=a$, $f(h_2)=c$, $f(h_3)=b$, $u:E\rightarrow G$, $u(e)=g$. Then the soft function $P_{fu}:(H,\tau,E,\ I)\rightarrow(B,\Omega,G)$ is soft b-I-continuous but is not soft S-I-continuous. Thus, (H,τ,E,I) is F.W.S.b-I-topological space but not F.W.S.S-I-topological space.

Proposition 4.9. Let $\phi:(H,\tau,E,I)\rightarrow(K,\sigma,L)$be a fibrewise soft ideal function, where(K,σ,L,M)a F.W.S.j-I-topological space over(B,Ω,G) and(H,τ,E,I) has the induced F.W.S.j-I-topology. If ϕ is soft continuous and for each F.W.S. j-I-topological space (Z,γ,C,N), a fibrewise soft function $\psi:(Z,\gamma,M,C)\rightarrow(H,\tau,E)$ is soft j-I-continuous then the composition $\phi\circ\psi:(Z,\gamma,C,N)\rightarrow(K,\sigma,L)$ is soft j-I-continuous, where j ∈ {α, S, P, b, β}.
Proof . The proof is similar to that of Theorem 2.12.(a).

Proposition 4.10. Let(H,τ,E,I) be a F.W.S.j-I-topological space over (B,Ω,G). Then $(H_{B^*},\tau_{B^*},E_{B^*},I_{B^*})$ is F.W.S. j-I-topological space over (B^*,Ω^*,G^*) for each open subspace $(B^*,\Omega^*,G^*)$of (B,Ω,G)**,** where $j\in\{\alpha,S,P,b,\beta\}$.
Proof. The proof is similar to that of Theorem 2.6.

Proposition 4.11. Assume that (H,τ,E,I) is F.W.S.j-I-topological space over (B,Ω,G) if and only if the graph soft function $g:(H,\tau,E)\rightarrow(H,\tau,E)\times(B,\Omega,G)$, defined by $g(\tilde{h})=(\tilde{h},f(\tilde{h}))$, for every$\tilde{h}\tilde{\in}H$, is S. j-I-continuous, where $j\in\{\alpha,S,P,b,\beta\}$.
Proof. The proof is similar to that of Theorem 2.5.

Proposition 4.12. Let $\phi:(H,\tau,E,I)\rightarrow(K,\sigma,L,M)$be aS.j-I-irresolute fibrewisesurjective, where (H,τ,E,I) and (K,σ,L,M) are F.W.S.I-topological spaces over (B,Ω,G).If (H,τ,E,I) is F.W.S.j-I-compact then (K,σ,L,M) is so, where $j\in\{\alpha,S,P,b,\beta\}$.
Proof. Let $\phi:(H,\tau,E,I)\rightarrow(K,\sigma,L,M)$ be a S. j-I-irresoluteand (H,τ,E,I) is F.W.S.j-I-compact,then there exist the projection function$P_{H(fu)}:(H,\tau,E,I)\rightarrow(B,\Omega,G)$ is S. j-I-compact. For proof (K,σ,L,M) is F.W.S.j-I-compacti.e., the projection function$P_{K(qd)}:(K,\sigma,L,)\rightarrow(B,\Omega,G)$ is S.j-I-compact, since let $\{(W_i,E_i):i\in\Delta\}$ an S.I-open cover of K_b, where $b\in B$. Then $\phi^{-1}\{(W_i,E_i):i\in\Delta\}$ is S.I-open cover of $H_{\tilde{b}}$, where$\tilde{b}\in B$. There exists from the hypothesis a finite subset Δ_o of Δsuch that $\tilde{H}-\tilde{\cup}\{\phi^{-1}(W_i,E_i):i\in\Delta_o\}\in I$. Thus, $\phi\{\tilde{H}-\tilde{\cup}\{\phi^{-1}(W_i,E_i):i\in\Delta_o\}\in J$which explains that is $P_{K(qd)}$ soft j-I-compact. Thus (K,σ,L,M)F.W.S.j-I-compact, where $j\in\{\alpha,S,P,b,\beta\}$.

Definition 4.13 The F.W.S. topological space (H,τ,E,I) over(B,Ω,G) is called fibrewise soft near ideal connected (briefly F.W.S. j-I-connected) if $H_{\tilde{b}}$where$\tilde{b}\,\tilde{\in}\,B$is not the union of two disjoint non-empty S.j-I-open sets of (H,τ,E,I),where$j\in\{\alpha,S,P,b,\beta\}$.

Proposition 4.14. Assume that $\phi:(H,\tau,E,I)\to(K,\sigma,L,M)$ isaS.j-I-irresolute fibrewise surjection function, when (H,τ,E,I) also (K,σ,L,M) are F.W.S. topological spaces over (B,Ω,G). If (H,τ,E,I) is F.W.S. j-I-connected then so is (K,σ,L,M), where $j\in\{\alpha,S,P,b,\beta\}$.

Proof. Let (K,σ,L,M) be a F.W.S.I-topological space over (B,Ω,G), then there existsthe projection function $P_{K(qd)}:(K,\sigma,L,M)\to(B,\Omega,G)$ is S.I-continuous. To proof (K,σ,L,M) is F.W.S.j-I-connected, Suppose that (K,σ,L,M) is not F.W.S.j-I-connected, then there exist non-empty disjoint S.j-I-open subset (F,L), (G,L) of (K,σ,L,M) such that $(F,L)\,\tilde{\cup}\,(G,L)=K_{\tilde{b}}$, where $\tilde{b}\,\tilde{\in}\,B$. Since ϕ is soft j-I-irresolute, we have $\phi^{-1}(F,L)$, $\phi^{-1}(G,L)$ are non-empty disjoint in (H,τ,E,I). Moreover,$\phi^{-1}(F,L)\,\tilde{\cup}\,\phi^{-1}(G,L)=H_{\tilde{b}}$. This shows that (H,τ,E,I) is not F.W.S.j-I-connected. This is a contradiction and hence (K,σ,L,M) is F.W.S.j-I-connected, where $j\in\{\alpha,S,P,b,\beta\}$.

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