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Study Mass Parabola and Most Stable Isobar from Some Isobaric Nuclides

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Abstract. This study aims to determine most stable isobar from some isobaric elements with mass number (A= 50-65 & 180-195). This aim achieved by, firstly: plot mass parabolas for these isobaric family, second: calculated the atomic number for most stable isobar (Z_A) value. To plot the mass parabola, the binding energy (B.E) calculated from semi empirical formula for these isobars. The mass number (A) plotted as a function to the (Z_A) for each range; we get a linear relationship between them. An empirical formula for the most stable isobar has been developed from this linear dependence. From the results, we can see that mass parabolas for isobaric elements with odd mass number (A) are different from the mass parabolas of even mass number (A) isobars, so there is only one stable nuclides for odd (A) while for even (A) there is more than one stable nuclide..

Keywords: Binding Energy, Liquid Drop Model (LDM), Mass Parabolas, Isobaric nuclides.

INTRODUCTION:

The smallest part of the elements that is maintains basic properties are the atom. An atom consists of a small massive core named the nucleus, surrounded by orbiting electrons [1]

A nucleus consists of two types of separate nucleons having practically the similar mass; these are neutrons and the protons. So the binding energy of a nucleus is a dis continuous function of its mass [2].

Cables of nuclei taking same mass number (A) but different number of neutrons (N) and protons (Z) are called isobars. For example, the three nuclei 14C, 14N, and 14O are isobars with mass number (A) = 14[3].

Beta particles (β) are electrons which carry negative or positive charge (e- & e+). In the case of (β-) decay atomic number (Z) rises via one unit. But in the status of (β+) decay the atomic number (Z) will be reduction by one unit [1]. Some nuclei go through a radioactive change by taking an atomic electron, typically from the K shell, and emitting a neutrino in addition to reductions atomic number (Z) by one unit [4].

THEORETICAL PART

Bethe–Weizsäcker mass formula

Liquid Drop Model (LDM) was offered by Von Weizsäcker and then was extended by Bohr and Wheeler. Historically this model was the first model proposed to explain the different properties of the nucleus. The idea of considering the nucleus like a drop of liquid initially comes from considerations about its fullness properties and from the truth that the nucleus has a very low compressibility and a clear surface [5].

Liquid-drop model reach to theoretical calculations of the binding energies of nuclei in terms of (A) and (Z) as follow:

$$B.E(A,Z) = a_v A - a_s A2/3 - a_c Z(Z-1)A-1/3 - a_a(A-2Z)2A-1 + \delta A\text{-}1/2 + ɳ \text{----}(1)$$

Where: a_v, a_s, a_c, a_a, δ and ɳ are the energy constants that denote to the volume, surface, Coulomb, asymmetry, pairing and shell energy terms, respectively [6].

One set of this parameters is [7]: a_v=15.8Mev, a_s=18.3Mev, a_c=0.72Mev, a_a=23.2MeV.

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$$\text{And } \delta = \begin{cases} +11.2 \text{ MeV for (even N, even Z)}. \\ 0 \text{ for (even N odd Z, or even Z, odd N)}. \\ -11.2 \text{ MeV for (odd N, odd Z)}. \end{cases}$$

$$\text{And } \eta = \begin{cases} 3 \text{ Mev (N and Z} = \text{magic number).} \\ 2 \text{ Mev (N or Z} = \text{magic number and other is odd).} \\ 1 \text{ Mev (N or Z} = \text{magic number and other is even)}. \\ 0 \text{ (N and Z} \neq \text{magic number).} \end{cases}$$

Mass parabola

One of the highest uses of Bethe- Weizsäcker semi empirical mass formula is to determine the most stable isobar of a given mass number (A) against beta decay [8]. We can now define, for each mass number (A) value, the nucleus with the lowest mass (largest binding energy) which represents the most stable isobar, by solving the equation

$$\partial/(\partial Z)\ (B.E\ (Z,A)) = 0, \ldots\ldots \qquad \ldots..(2)$$

So the atomic number (Z_A) value for most stable isobar can be taken by [9].

$$Z_A = A/(2(1+1/(4)\ (a_c/a_a)\ A\text{^}(2/3))\ ) = Z_A = \ A/(2(\ 1 + 0.0077A\text{^}(2/3))\ )\ldots\ldots\ldots(3)$$

The isobars sited on the sides of the parabola are not stable, so to be more stable and lower on the parabola these nuclides will be decay, but the most stable nucleus is not usually located exactly at the bottom of the parabola. Nuclides on the low (Z) side of the parabolic decay by (β^{-}) emission toward the minimum (Z_A), but the nuclides on the great (Z) side of the minimum (Z_A) decay in the opposite direction toward the minimum, in this case the decay being either by (β^{+}) emission or electron capture (EC)[10].

RESULTS AND DISCUSSIONS

Binding energy (B.E) for nuclide with mass number (A= 50-65& 180-195) calculated by using Weizsäcker semi empirical formula eq. (1). The results of binding energy (B.E) which are ranging from (335-570) MeV for nuclides with mass number (A= 50-65) and (1434-1540) MeV for nuclides with mass number (A= 180-195) were plotted as a function to the atomic number (Z) for each isobar in isobaric family, so we get mass parabolas for different isobars as shown in the figures (1 &2). Here mass parabolas for odd and even mass number (A) are drawn separately.

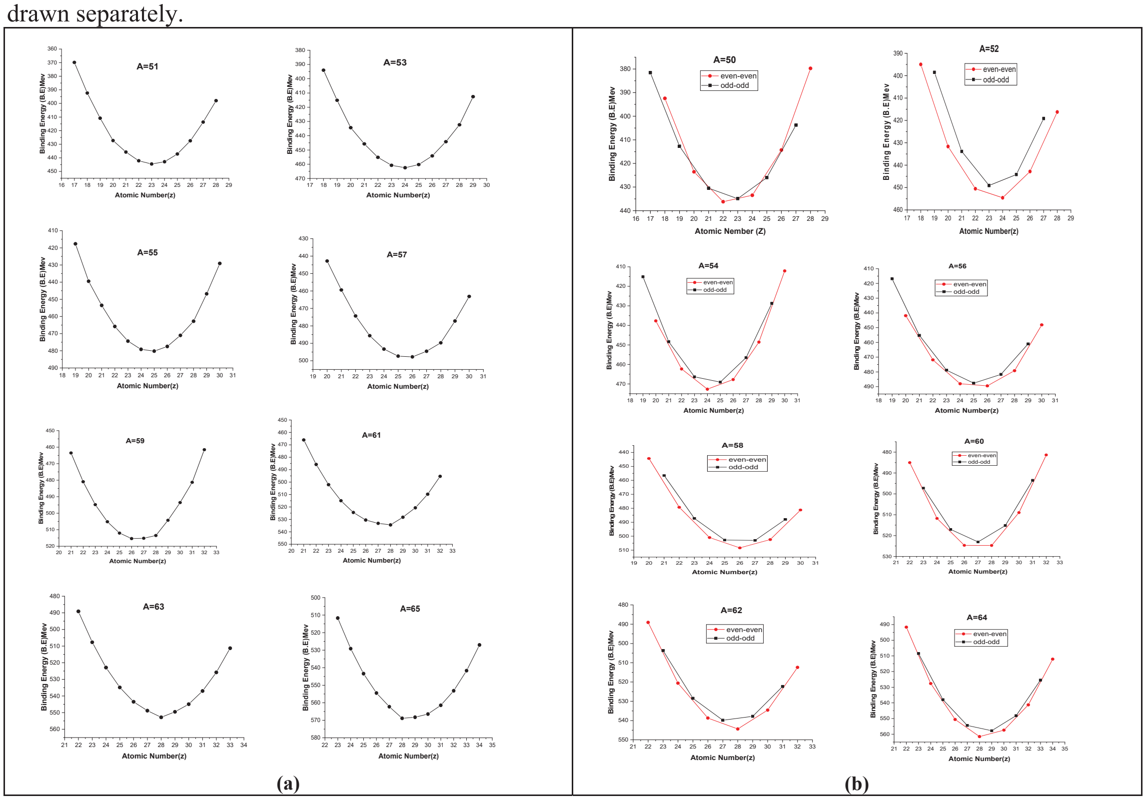
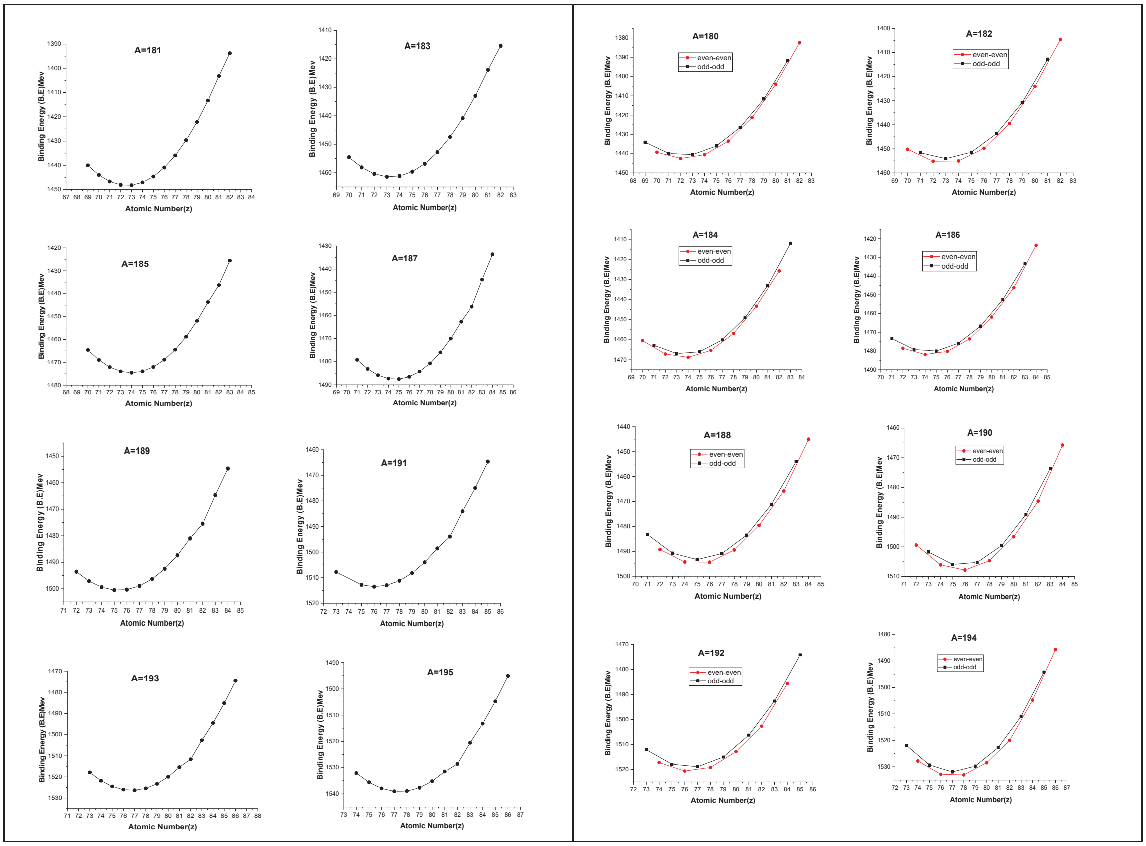


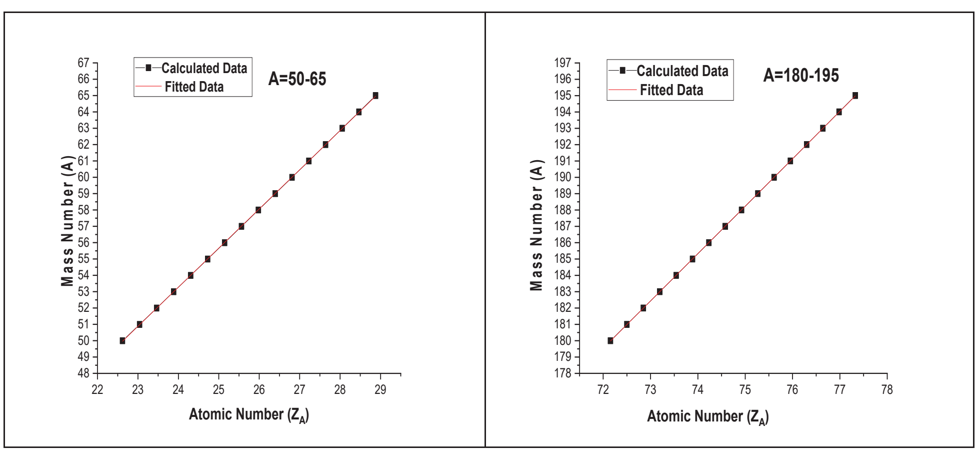
Figure.1 Mass parabolas for isobars with mass number (50-65) (a) odd (A) (b) even (A)



(a) (b)

Figure.2 Mass parabolas for isobars with mass number (A=180-195) (a) odd (A) (b) even (A).

The most stable isobar determined from equation (3) for the isobaric elements with mass number (A=50-65) and (A=180-195). Then the mass number plotted as a function to the atomic number of the most stable nuclei (Z_A) for two groups of isobars under this study as shown in figures (3-a & 3-b).



(a) (b)

Figure.3.Mass number (A) as a function to atomic number for most stable isobar (ZA) (a) (A=50-65) (b) (A=180-195).

These figures shows a linear relationship between mass number and most stable isobar (Z_A), so from this linear dependency of mass number and Z_A value, we have established an empirical formula to determine atomic number for most stable isobar (Z_A) eq. (1*) and eq. (2*) for isobars with mass number (A=50-65&) and (A=180-195) respectively.

$Z_A=(A+a)/b$......(1*)
Where:
a=4.23725 b=2.3964
And
$Z_A=(A+a)/b$………(2*)
Where:
a=29.26967 b=2.90006

The calculated values of atomic number (Z_A) for most stable isobar from an empirical formula eq. (1*) and eq. (2*) for nuclide with mass number (A=50-65 and A=180-195) are compared with their values which calculated from equation (3) as shown in tables (1&2) respectively.

Table 1. Mass Number (A), Atomic number for most stable isobar (Z_A) calculated from an empirical formula eq. (1*) and eq. (3) for nuclide with (A=50-65).

Mass number (A)	Atomic number for most stable isobar (Z_A) eq. (3)	Atomic number for most stable isobar (Z_A) eq. (1*)
50	22.61828	22.6328
51	23.04147	23.0501
52	23.46379	23.46739
53	23.88524	23.88468
54	24.30583	24.30197
55	24.72558	24.71927
56	25.14448	25.13656
57	25.56254	25.55385
58	25.97978	25.97114
59	26.3962	26.38844
60	26.81181	26.80573
61	27.22661	27.22302
62	27.64061	27.64031
63	28.05383	28.05761
64	28.46626	28.4749
65	28.87791	28.89219

Table 2. Mass Number (A), Atomic number for most stable isobar (Z_A) calculated from an empirical formula eq. (2*) and eq. (3) for nuclide with (A=180-195).

Mass number (A)	Atomic number for most stable isobar (Z_A) eq. (3)	Atomic number for most stable isobar (Z_A) eq. (2*)
180	72.15337	72.16046
181	72.50103	72.50528
182	72.84827	72.8501
183	73.1951	73.19492
184	73.54153	73.53974
185	73.88755	73.88456
186	74.23316	74.22938
187	74.57837	74.57421
188	74.92318	74.91903
189	75.26759	75.26385
190	75.6116	75.60867
191	75.95521	75.95349
192	76.29843	76.29831
193	76.64125	76.64313
194	76.98368	76.98795
195	77.32571	77.33277

From figure (1) we can show clearly that mass parabola for nuclide with mass number (A=50-56) are equivalent in their shape that's mean the number of isobars which decay by emission of (β^-) equals to the isobars which decay by emission of (β^+). While in figure (2) which represent mass parabola for nuclide with mass number (A=180-195) can see un equivalent mass parabola i.e. the number of isobars which decay by (β^+) emission greater than that the isobars which decay by (β^-) emission.

Figures (1&2) shows the mass parabola for many isobars which can be classified to two types depending on the value of mass number (A) odd or even. So figures (1a &2a) represent plotting of binding energy (B.E.)

versus atomic number (Z) for different odd isobars with mass number (A=50-65 &A=180-195) respectively. We can see from these figures that the isobars with odd value of mass number (A) have one mass parabola because the pairing term equals to zero in these isobars, unrelatedly with the type of nucleus is even-odd or odd-even. As well as, there is only one stable isobar which has highest value of binding energy (B.E.) which lies at or near the bottom of the parabola.

While the isobars with even value of mass number (A) have two mass parabola for each mass number because of the effect of pairing term (is positive for even-even nuclei and negative for odd-odd nucleus) in the binding energy formula, one of these mass parabola for odd-odd nuclei and second for even-even nuclide which lies below the parabola of odd-odd nuclei as displayed in figures (1b-2b), the reason of that is the value of binding energy for even-even nuclide greater than the value of binding energy of odd- odd nuclide for same mass number (isobar). That's mean nuclide with even mass number has more than one stable isobar.

Also we can show from figures (1&2) that the value of binding energy is increased by increasing the atomic number until reaching to the most stable isobar, and then decreased due to distance from the most stable isobar.

Finally, the most stable isobars which is determined in the lowest point of the mass parabola for each isobars in this study is in a good agreement with our results of most stable isobars (Z_A) from equation (3) and the results of our empirical formula prediction eq. (1*) and eq. (2*).

CONCLUSION

The binding energy of the nucleus depends essentially on the mass number (A). The nuclear stability of the isobars estimated by the mass parabolas. For odd mass number (A) nuclides only one parabolas but for even mass number (A) nuclides there are two parabolas. In two cases the most stable isobars found as the lowest point in the parabola, so there is only one stable isobar for odd mass number (A) and more than one stable isobar for nuclides with even mass number (A).

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