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Weakly and Strongly Forms of Fibrewise Fuzzy ω-Topological Spaces

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ABSTRACT

This paper is devoted to introduce weak and strong forms of fibrewise fuzzy ω-topological spaces, namely the fibrewise fuzzy θ-ω-topological spaces, weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω- topological spaces. Also, Several characterizations and properties of this class are also given as well. Finally, we focused on studying the relationship between weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω-topological spaces.

1. **Introduction and Preliminaries.**

In order to began the category in the classification of fibrewise (shortly., fw) sets on a given set, named the base set, which say B. A fw set on B consist of a set M with a function $p : M \rightarrow B$ that is named the projection(shortly., proj.). The fibre over b for every point b of B is the subset $M_b = p^{-1}(b)$ of M. Perhaps, fibre will be empty because we do not require p is surjective, also, for every subset B^* of B we considered $M_{B^*} = p^{-1}(B^*)$ as a fw set over B^* with the projection determined by p. The concept of fuzzy sets was introduced by Zadeh [18]. The idea of fuzzy topological spaces was introduced by Chang [6]. The concept of fuzzyω-continuity, fuzzy almost ω-continuous and fuzzy weakly ω-continuous in topological spaces was introduced by Gazwan [1]. In this paper, we introduce and study seven weak and strong forms of fibrewise fuzzy topological spaces, called fibrewise fuzzy ω-topological spaces, fibrewise fuzzy almost ω-topological spaces, fibrewise fuzzy almost weakly ω- topological spaces, fibrewise fuzzy weakly θ-ω-topological spaces, fibrewise fuzzy θ-ω-topological spaces, fibrewise fuzzy strongly θ-ω- topological spaces and fibrewise fuzzy almost strongly ω- topological spaces, we study their basic properties and we shall discuss relationships between weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω-topological spaces, we built on some of the result in [3,4,15,16,17].

Definition 1.1. [8] A mapping $\vartheta : M \rightarrow N$, where M and N are FW sets over B, with proj.'s $p_M : M \rightarrow B$ and $p_N : N \rightarrow B$, is said to be FW mapping (written as FW-M) if $p_N \circ \vartheta = p_M$, or $\vartheta(M_b) \subseteq N_b$, for all point $b \in B$.

Observe that a FW-M $\vartheta : M \rightarrow N$ over B limited by restriction, a FW-M $\vartheta : M_{B^*} \rightarrow N_{B^*}$ over B^* for all subset $B^* \subseteq B$.

Definition 1.2. [8] The fibrewise topology (written as FWT) on a FW set M over a topological space (B, σ) signify any topology on M for which the proj. p is continuous (written as FWTS).

Definition 1.3. [8] Let M and N be FWTS's over B, the FW-M $\vartheta : M \rightarrow N$ is said to be:
(a) continuous if $b \in B$ and for all point $m \in M_b$, the pre image of all open set of $\vartheta(m)$ is an open set of m.
(b) open if $b \in B$ and for all point $m \in M_b$, the image of all open set of m is an open set of $\vartheta(m)$.

Definition 1.4. [8] The FWTS (M, τ) over (B, σ) is said to be:
(a) FW closed (written as FWC) if the proj. p is closed mapping.
(b) FW open (written as FWO) if the proj. p is open mapping.

Definition 1.5. [18] assume that M is a nonempty set, a fuzzy set μ in M is a mapping $\chi_\mu : M \longrightarrow I$ where I is the closed unite interval $[0, 1]$ which is written as:

$\mu = \{(m, \chi_\mu(m)) : m \in M, 0 \leq \chi_\mu(m) \leq 1\}$, The family of each fuzzy subsets in M will be symbol by I^M thus is $I^M=\{\mu: \mu$ is fuzzy subset of $\mu\}$ and χ_μ is called the membership function.

Example 1.6. [12] We will suppose a possible membership function for the fuzzy set of real numbers close to zero as follows, $\chi_\mu: \mathbb{R} \longrightarrow [0,1]$, where

$$\chi_\mu(m) = \frac{1}{1+(m-10^2)}, \ \forall m \in \mathbb{R}$$

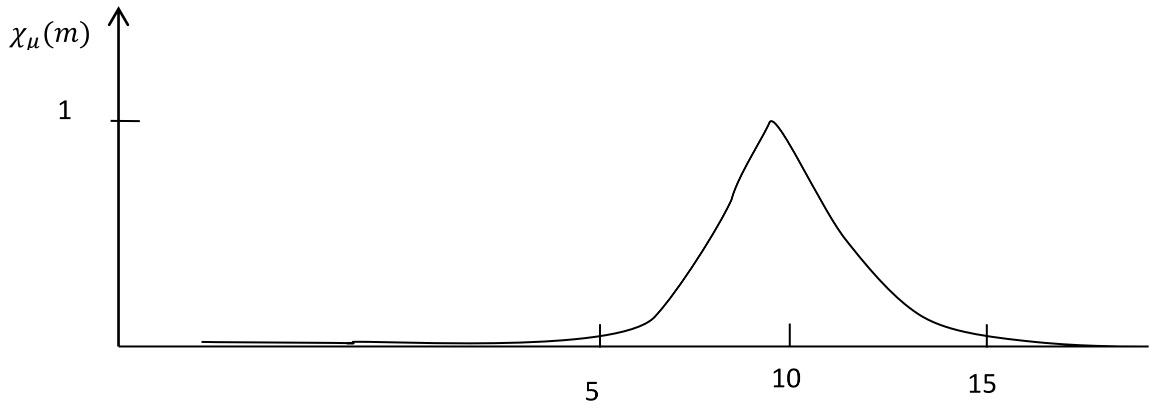


Figure -1

Definition 1.7. [18] A fuzzy set in M is empty denoted by $\bar{0}_\mu$, if its membership function is identically the zero function, i.e.,
$\bar{0}_\mu: M \to [0,1]$ s.t $\bar{0}_\mu(m) = 0$ $\forall m \in M$.

Definition 1.8. [18] A universal fuzzy set in M, denoted by $\bar{1}_\mu$, is a fuzzy set defined as $\bar{1}_\mu(m) = 1 \ \forall m \in M$.

Definition 1.9. [18] Let $\mu, \lambda \in I^M$. A fuzzy set μ is a subset of an fuzzy set λ, denoted by $\mu \le \lambda$ iff $\mu(m) \le \lambda(m)$, $\forall m \in M$.

Two fuzzy sets μ and λ are said to be equal ($\lambda = \mu$) if $\lambda(m) = \mu(m)$, $\forall m \in M$.

Definition 1.10. [18] Let λ and μ be fuzzy sets in M. Then, for all $m \in M$,
$\psi = \lambda \vee \mu \Leftrightarrow \psi(m) = max\{\lambda(m), \mu(m)\}$,
$\delta = \lambda \wedge \mu \Leftrightarrow \delta(m) = min\{\lambda(m), \mu(m)\}$,

$$\eta = \lambda^c \Leftrightarrow \eta(m) = 1 - \lambda(m).$$

More generally, for a family $\Lambda = \{\lambda_i \mid i \in I\}$ of fuzzy sets in M, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by

$$\psi(x) = sup_i\{\lambda_i(m) \mid m \in M\},$$
$$\delta(x) = inf_i\{\delta_i(m) \mid m \in M\}.$$

Definition 1.11. [6] A fuzzy topology is a family τ of fuzzy sets in M, which satisfies the following conditions:

(a) $\overline{0}, \overline{1} \in \tau$;
(b) If $\lambda, \mu \in \tau$, thus $\lambda \wedge \mu \in \tau$;
(c) If $\lambda_i \in \tau$ for all $i \in I$, thus $\vee_i \lambda_i \in \tau$.

(M, τ) is said to be fuzzy topological spaces and each member of τ is named fuzzy open set on M and its complement is fuzzy closed set.

Definition 1.12. [10] A fuzzy set on M is named a fuzzy point iff it takes the value 0 for each $y \in M$ except one, say, $m \in M$. If its value at m is r $(0 < r \leq 1)$ we denote thus fuzzy point by m_r, when the point m is named its support.

Definition 1.13. [6,14] Let μ be a fuzzy set and let(M, τ) be a fuzzy topological space. μ is a fuzzy neighborhood of a fuzzy point $\mathrm{m_r}$ if there exist a fuzzy open set ν since $r \leq \nu(\mathrm{m}) \leq \mu(\mathrm{m})$, $\forall\, m \in \mathrm{M}$.

Definition 1.14. [6] assume that (M, τ) is a fuzzy topological space as well $\mu \in I^M$. The fuzzy closure (fuzzy interior) of A is symbol by $cl(\mu)$ ($int(\mu)$) is defined by:

$$cl(\mu) = \wedge \{ \lambda^c \in \tau,\ \mu \leq \lambda\}$$

$$int(\mu) = \vee \{ \xi \in \tau;\ \xi \leq \mu\}.$$

Evidently, $cl((\mu)$ (resp., $int(\mu))$ is the smallest fuzzy closed (resp., largest fuzzy open) subset of M which contains (resp., contained in) μ. Note that μ is fuzzy closed (fuzzy open) iff $\mu = cl(\mu)$ (resp., $int(\mu)$).

Definition 1.15. [6] assume that $f : M \to N$ is a mapping. For a fuzzy set β in N and membership function β(n). The inverse image of β under f is the fuzzy set $f^{-1}(\beta)$ in M with membership function is denoted by the rule:

$$f^{-1}(\beta)(m) = \beta(f(m)),\ \forall m \in M. \quad (1)$$

For a fuzzy set λ in M, the image of λ under f is the fuzzy set $f(\lambda)$ in B with membership function $f(\lambda)(n)$, $n \in N$ is given by

$$f(\lambda)(n) = \begin{cases} \sup_{m \in f^{-1}(n)}\{(\lambda(m)\}, \text{ if } f^{1}(n) & \text{nonempty,} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Definition 1.16. [2] Assume that $\mathrm{m_r}$ is a fuzzy point and μ a fuzzy set in M. Then $\mathrm{m_r}$ is said to be in μ or (belong to μ) or ($\mathrm{m_r}$ content in μ) denoted $m_r \in \mu$ if and only if $r \leq \mu(\mathrm{m})$, for all $\mathrm{m} \in \mathrm{M}$

Definition 1.17. [9,19] The set $\{m: m \in M, \mu(m) > 0\}$ is called the support of μ and is denoted by Supp(μ)

Definition 1.18. [10] A fuzzy point m_r is said to be quasi-coincident with μ denoted by $m_r q\mu$ if there exist $m \in M$ such that $r + \mu(m) > 1$, if $\mathrm{m_r}$ is not quasi coincident with μ, then $r + \mu(m) \leq 1$ $\forall m \in M$ and denoted by $\mathrm{m_r}\ \tilde{q}\mu$.

Definition 1.19. [10] A fuzzy set μ in (M, τ) is called a "Q-neighborhood of m_λ " iff $\exists \nu \in \tau$ such that $m_\lambda q\nu < \mu$.

The family of all Q-nbhd's of m_λ is called the system of Q- nbhd's of m_λ.

Definition 1.20. [2] Fuzzy regular space if for each fuzzy point $\mathrm{m_r}$ in M and each fuzzy closed set F with $\mathrm{m_r}\tilde{q}F$ there exists fuzzy open μ, λ in M such that $r \leq \mu(\mathrm{m})$, $F(\mathrm{m}) \leq \lambda(\mathrm{m})$ $\forall$ m $\in$ M and $\mu\tilde{q}\lambda$.

Definition 1.21. [11] A fuzzy set μ is fuzzy θ-closed if μ = $cl_\theta(\mu)$ = $\{m_r$ fuzzy point in (M, τ) : $(cl\ (\nu))\ q\ \mu$, U is fuzzy open q-nbd. of $m_r\}$. The complement of fuzzy θ-closed called fuzzy θ-open set.

Definition 1.22. [14] Let μ be a fuzzy set in a fuzzy topological space (M, τ) is named a fuzzy uncountable iff supp(μ) is an uncountable subset of M.

Definition 1.23. [1] A fuzzy point m_r of a fuzzy topological space (M, τ) is named a fuzzy condensation point of μ on M if min$\{\mu(\mathrm{m}), \lambda(\mathrm{m})\}$is fuzzy uncountable for each fuzzy open set λ containing $\mathrm{m_r}$. And the set of all fuzzy condensation point of μ is denoted by Cond (μ)

Definition 1.24. [1] A fuzzy subset μ in a fuzzy topological space (M, τ) is called a fuzzy ω-closed set if it contains each its fuzzy condensation point. The complement fuzzy ω-closed sets are called fuzzy ω-open sets. And the family of all fuzzy ω-open (resp.fuzzy ω-closed) sets in a fuzzy topological space (M, τ) will be denoted by f.ω-open (resp. f.ω-closed).

Definition 1.25. [1] Assume that μ is a fuzzy set of a fuzzy topological space (M, τ) then The ω-closure of μ is symbol by $cl^\omega(\mu)$ and known that by $cl^\omega\mu(\mathrm{m})$ = $inf\{\ F(m) : \ F$ is a fuzzy ω-closed set in, $(m) \leq F(m)\}$.

Definition 1.26.[1] For a fuzzy topological space (M, τ) is named a fuzzy ω-regular space when all fuzzy ω-closed subset μ in M so well a fuzzy point $\mathrm{m_r}$ in M so that $\mathrm{m_r}\ \tilde{\mathrm{q}}\ \mu$, there exists two fuzzy ω-open sets λ and ν such that $r \leq \lambda(\mathrm{m})$, $\mu(\mathrm{m}) \leq \nu(\mathrm{m})$ and $\lambda\ \tilde{\mathrm{q}}\ \nu$

Definition 1.27. [6] A mapping $\phi: (M, \tau) \rightarrow (N, \Lambda)$ is said to be

(a) fuzzy continuous (briefly f. continuous) if the inverse image of every fuzzy open set of N is a fuzzy open set in M.

(b) fuzzy open (briefly f. open) map if the image of every fuzzy open set of M is a fuzzy open set in N.

(c) fuzzy close (briefly f. close) map if the image of every fuzzy close set of M is a f. close set in N.

Definition 1.28. [13] A mapping $\phi:(M,\tau)\rightarrow(N,\Lambda)$ is said to be Fuzzy θ-continuous (f.θ.continuous, for short) if for each fuzzy point m in (M,τ) and each fuzzy open q-nbd. $\mathcal{v}$ of $\phi(m)$, there exists fuzzy open q-nbd. $\mathcal{u}$ of m so that $p(cl(\mathcal{u}))\leq cl(\mathcal{v})$.

Definition 1.29. [1] A mapping $\phi:(M,\tau)\rightarrow(N,\Lambda)$ is said to be

(a) Fuzzy ω-continuous at a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$ there exists a fuzzy ω-open subset μ on M which contains m so that $\phi(\mu)\leq\lambda$ so well ϕ is called fuzzy ω-continuous if it is fuzzy ω-continuous at every fuzzy point.

(b) Fuzzy almost ω-continuous at a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$ there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu)\leq int(cl(\lambda))$ so well ϕ is named fuzzy almost ω-continuous if it is fuzzy almost ω-continuous at every fuzzy point.

(c) Fuzzy weakly ω-continuous at a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$ there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu)\leq cl(\lambda)$ so well ϕ is named fuzzy ω-continuous if it is fuzzy ω-continuous at every fuzzy point.

2- Weakly Fibrewise Fuzzy θ-ω-Topological Spaces

In this section, we study the weakly fibrewise fuzzy θ-ω-topological spaces and some theorems concerning them.

First, we introduced the following definition.

Definition 2.1. A mapping $\phi:(M,\tau)\rightarrow(N,\Lambda)$ is said to be fuzzy almost weakly θ-ω-continuous (briefly, f. almost weakly θ-ω-continuous) if in a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$ there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu)\leq cl(\lambda)$ so well ϕ is named fuzzy almost weakly ω-continuous if its fuzzy almost weakly ω-continuous at every fuzzy point.

Definition 2.2. A mapping $\phi:(M,\tau)\rightarrow(N,\Lambda)$ is said to be fuzzy θ-ω-continuous (briefly, f. θ-ω-continuous) at a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$ there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(cl^{\omega}(\mu))\leq cl(\lambda))$ as well ϕ is named fuzzy θ-ω-continuous if its fuzzy θ-ω-continuous at every fuzzy point.

Definition 2.3. A fuzzy set A is fuzzy θ-ω-closed if $A = cl^{\omega}_{\theta}(A) = \{p$ fuzzy point in (X,τ) : $(cl^{\omega}\ (U))\ q\ A$, U is fuzzy ω-open q-nbd. of $p\}$. The complement of fuzzy θ-ω-closed called fuzzy θ-ω-open set.

Definition 2.4. A mapping $\phi:(M,\tau)\rightarrow(N,\Lambda)$is said to be fuzzy weakly θ-ω-continuous (briefly, f. weakly θ-ω-continuous) if in a fuzzy point $m\in M$ when all fuzzy open subset λ in N contains $\phi(m)$

there exists a fuzzy θ-ω-open subset μ of M which contains m so that $\phi(\mu) \leq cl(\lambda)$ so well ϕ is named fuzzy weakly θ-ω-continuous if its fuzzy weakly θ-ω-continuous at every fuzzy point.

Definition 2.5. Let (B, σ) be a fuzzy topological space the fibrewise fuzzy ω-topological spaces, fibrewise fuzzy almost weakly ω-topological spaces, fibrewise fuzzy almost ω-topological spaces, fibrewise fuzzy weakly θ-ω-topological spaces and fibrewise fuzzy θ-ω-topological spaces (briefly, FWF ω-top. sp., FWF almost weakly ω-top. sp., FWF almost ω-top. sp., FWF weakly θ-ω-top. sp. and FWF θ-ω-top. sp.) on a fibrewise set M over B mean any fuzzy topology on M which of them the projection function p are fuzzy ω-continuous, fuzzy almost weakly ω-continuous, fuzzy almost ω-continuous, fuzzy weakly θ-ω-continuous and fuzzy θ-ω-continuous (briefly, f. ω-continuous, f. almost weakly ω-continuous, f. almost ω-continuous, f. weakly θ-ω-continuous and f. θ-ω-continuous).

Theorem 2.6. The FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp., then it is FWF almost ω-top. sp. .

Proof. Assume that (M, τ) is a FWF ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Assume that $m \in M_b$; $b \in B$ and, μ is a fuzzy open set contains $p(m)$ in B. Since p is f. ω-continuous, there is a f. ω-open set λ containing m so that $p(\lambda) \leq \mu$. Thus, $int(\mu) \leq \mu$ and $\mu \leq cl(\mu)$. Then, $int(\mu) \leq cl(\mu)$ and $int(int(\mu) \leq int(cl(\mu))$. It follows that, $p(\lambda) \leq int(cl(\mu))$. Therefore $p(\lambda) \leq int(cl(\mu))$. So, p is f. almost ω-continuous. Hence (M, τ) is FWF almost ω-top. sp. .

We can prove the same way by used property of fuzzy interior and fuzzy closure set.

The converses does not hold as we show by the following examples:

Example 2.7. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where

$\mu_1 = \{(a, 0.1)\}$

$\mu_2 = \{(b, 0.2)\}$

$\mu_3 = \{(a, 0.1), (b, 0.2)\}$

And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(z, 1)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = p(b) = p(c) = z$. Let $\lambda = \{(a, 0.1)\}$ fuzzy open in M and $\nu = \{(b, 0.2)\}$. Then, $p(cl\{(b, 0.2)\}) \leq cl\{(a, 0.1)\}$ but $p(\{(b, 0.2)\}) \nleq int(cl(\{(a, 0.1)\}$. Then, (M, τ) is FWF θ-ω-top. sp. but not FWF almost ω-top. sp. .

Example 2.8. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where

$\mu_1 = \{(a, 0.3), (b, 0), (c, 0.5)\}$

$\mu_2 = \{(a, 0.7), (b, 1), (c, 0.5)\}$

And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0), (y, 0.3), (z, 1)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\eta = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$ fuzzy θ-ω-open in M and

$\nu = \{(a, 0), (b, 0.3), (c, 0.5)\}$ is fuzzy open in B. Then, $p(\eta) \leq cl(\nu)$ but $p(cl(\eta)) \nleq cl(\nu)$. Then, (M, τ) is FWF weakly θ-ω-top. sp. but not FWF θ-ω-top. sp. .

Example 2.9. In Example 2.6, (M, τ) over (B, σ) is a FWF weakly θ-ω-top. sp., but is not FWF ω-top. sp.. Moreover, (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp., but is not FWF almost ω-top. sp.. Moreover, (M, τ) over (B, σ) is FWF almost weakly ω-top. sp., but is not FWF θ-ω-top. sp., and not FWF ω-top. sp.

Example 2.10. Let $= \{a, b\}$, $B = \{x, y\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.9), (b, 0.7)\}$
$\mu_2 = \{(a, 1), (b, 0.9)\}$
$\mu_3 = \{(a, 0.11), (b, 0.31)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.11), (y, 0.31)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$. Let $\eta = \{(a, 0.7), (b, 0.4)\}$ fuzzy ω-open in M and $\nu = \{(a, 0.11), (b, 0.31)\}$ is fuzzy open in B. Then, $p(\eta) \leq int(cl(\nu))$ but $p(\eta)) \nleq \nu$. Then, (M, τ) is FWF almost ω-top. sp. but not FWF ω-top. sp. . Moreover, (M, τ) is FWF almost weakly ω-top. sp. but not FWF almost ω-top. sp. .

Lemma 2.11. [1] A fuzzy topological space (M, τ) is fuzzy ω-regular if and only if for all fuzzy point m in M and all fuzzy ω-open μ containing m, there exists fuzzy ω-open set λ such that $m \in \lambda \leq cl^{\omega}(\lambda) \leq \mu$.

Theorem 2.12. Let (M, τ) be a fuzzy ω-regular space. The FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp., then it is FWF θ-ω-top. sp. .
Proof. Let (M, τ) be a FWF almost weakly ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. almost weakly ω-continuous. It suffices to demonstrate that p is f. θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ so well, μ is a fuzzy open set containing $p\,(m)$ in B. Since p is f. almost weakly ω-continuous, there exists is a f. ω-open set λ containing m such that $p(\lambda) \leq cl(\mu)$. Because (M, τ) is a fuzzy ω-regular space, by Lemma 2.12, there is η fuzzy ω-open in M_b, $b \in B$ so that $m \in \eta \leq cl^{\omega}(\eta) \leq \lambda$. Therefore, $p(cl^{\omega}(\eta)) \leq cl(\mu)$. Then, p is f. θ-ω-continuous. Then (M, τ) is FWF θ-ω-top. sp..

Corollary 2.13. Let (M, τ) be an fuzzy ω-regular space. The FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp. if and only if it is FWF θ-ω-top. sp. .

Theorem 2.14. Assume that $\phi : (M, \tau) \to (N, \Lambda)$ is a f. ω-continuous fibrewise surjection function, when (M, τ) so well (N, Λ) are FWF topological spaces on (B, σ). If (N, Λ) is a FWF almost weakly ω-top. sp., then (M, τ) is so.
Proof: Assume that $m \in M_b$, $b \in B$ and λ be a fuzzy open set containing $p_M(m)$ in B , since p_N is f. almost weakly ω-continuous, there exists is a fuzzy open set μ containing $\phi(m)$ in N_b, $b \in B$ such that $p_N(\mu) \leq cl(\lambda)$. Since ϕ is f. ω-continuous, then for each $m \in M_b$,

$b \in B$ and each fuzzy open set μ of $\phi(m) = n \in N_b$ in N, there exists a f. ω-open η of m in M_b, $b \in B$ such that $\phi(\eta) \leq \mu$. Thus, $p_N(\phi(\eta)) \leq p_N(\mu)$. And, $p_M = (p_N \circ \phi)_\eta \leq p_N(\eta)$. Then, $p_M(p_N \circ \phi)_\eta \leq cl(\lambda)$. Thus, p_M f. almost weakly ω-continuous. Hence, (M, τ) is FWF almost weakly ω-top. sp..

Theorem 2.15. Let (M, τ) be a fuzzy ω-regular space. The FWF topological space (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp., then it is FWF ω-top. sp..

Proof. Assume that (M, τ) is a FWF weakly θ-ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Let $m \in M_b$; $b \in B$ and, μ be a fuzzy open set containing $p(m) \in B$. Where M is a f. ω-regular space there is a fuzzy open set $\mu_1 \in M_b$ so that $p(m) \in \mu_1$. And, $cl(\mu_1) \leq \mu$ where p is f. weakly θ-ω-continuous, there is an f. ω-open set λ containing m such that $p(\lambda) \leq cl(\mu_1)$. It follows that, $p(\lambda) \leq \mu$. Therefore, p is f. ω-continuous. Thus, (M, τ) is FWF ω-top. sp. .

Corollary 2.16. Assume that (M, τ) is a fuzzy ω-regular space. The FWF topological space (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF ω-top. sp. .

Theorem 2.17. The FWF topological space (M, τ) over (B, σ) is fuzzy ω-regular space. If (M, τ) is FWF θ-ω-top. sp., then it is FWF almost ω-top. sp. .

Proof. Assume that (M, τ) is a FWF θ-ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Let $m \in M_b$; $b \in B$ and, μ be a fuzzy open set containing $p(m)$ in B. Because p is θ-ω-continuous, there exists is a f. ω-open set η containing m such that $p(cl^\omega(\eta)) \leq cl(\mu)$. Because $int(cl(\mu) \leq cl(\mu)$, then $p(cl^\omega(\eta)) \leq int(cl(\mu)) \leq cl(\mu)$, then $p(cl^\omega(\eta)) \leq cl(\mu)$. Also (M, τ) is f. ω-regular space, there exists is a f. ω-open set η_1 in M_b such that $m < \eta_1$. Also, $cl(\eta_1) \leq \eta$. Thus, $p(cl^\omega(\eta_1)) \leq p(\eta)$ and $int(cl(\mu) \leq cl(\mu)$. It follows, $p(\eta) \leq int(cl(\mu))$. So, p is f. almost ω-continuous. Thus (M, τ) is FWF almost ω-top. sp..

Corollary 2.18. The FWF topological space (M, τ) over (B, σ) is fuzzy ω-regular space. Then (M, τ) is FWF θ-ω-top. sp. if and if it is FWF almost ω-top. sp..

Theorem 2.19. Assume that (B, σ) is a fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp., then it is FWF ω-top. sp. .

Proof. Assume that (M, τ) is a FWF almost weakly ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. almost weakly ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Let $m \in M_b$; $b \in B$ and, μ be a fuzzy open set containing $p(m) \in B$. Where B is a f. ω-regular space there is a fuzzy open set μ_1 in B so that $p(m) \in \mu_1$. So well, $cl(\mu_1) \leq \mu$ since p is f. almost weakly ω-continuous, there exists is a f. ω-open set λ containing m such that $p(\lambda) \leq cl(\mu_1)$. It follows that, $p(\lambda) \leq \mu$. Therefore, p is f. ω-continuous. Then (M, τ) is FWF ω-top. sp..

Corollary 2.20. Assume that (B, σ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp. if and only if it is FWF ω-top. sp..

Theorem 2.21. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp., then it is FWF θ-ω-top. sp. .

Proof. Assume that (M, τ) is a FWF weakly θ-ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \rightarrow (B, \sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ and, μ is an fuzzy open set containing $p\ (m) \in B$. Where M is an f. ω-regular space there is a fuzzy open set $\mu_1 \in M_b$ so that $p\ (m) \in \mu_1$. And, $cl(\mu_1) \leq \mu$ where p is f. weakly θ-ω-continuous, there is f. ω-open set λ containing m such that $p(\lambda) \leq cl(\mu)$. It follows that, $p(cl^{\omega}(\lambda)) \leq cl(\ \mu)$. Therefore, p is f. θ-ω-continuous. Thus, (M, τ) is FWF θ-ω-top. sp. .

Corollary 2.22. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF θ-ω-top. sp..

Theorem 2.23. Assume that (B, σ) is a fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., then it is FWF ω-top. sp..

Proof. Assume that (M, τ) is a FWF almost ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \rightarrow (B, \sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Assume that $m \in M_b$; $b \in B$ and, μ is an fuzzy open set containing $p(m) \in B$. Since p is f. almost ω-continuous, there exists is an f. ω-open set λ contains m such that $p(\lambda) \leq int(\ cl(\mu))$. Because $int(cl(\mu)) \leq cl(\mu)$. Then $(\lambda) \leq int(cl(\mu)) \leq cl(\mu)$. Thus, $p(\lambda) \leq cl(\mu)$, and B is a f. ω-regular space there exists is a f. ω-open set λ_1 in M_b such that $m \in \lambda_1$. And, $cl(\mu_1) \leq \mu$. Therefore, $p(\lambda) \leq cl(\mu_1) \leq \mu$. It follows that, $p(\lambda) \leq \mu$. Thus, p is f. ω-continuous. Then (M, τ) is FWF ω-top. sp. .

corollary 2.24. Assume that (B, σ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp. if and only if it is FWF ω-top. sp..

Theorem 2.25. Assume that (B, σ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF weakly θ-ω-top. sp., then it is FWF almost ω-top. sp..

Proof. Assume that (M, τ) is a FWF weakly θ-ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \rightarrow (B, \sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Let $m \in M_b$; $b \in B$ and, μ be a fuzzy open set containing $p\ (m) \in B$. Where B is f. ω-regular space, there is a fuzzy open set $\mu_1 \in B$ so that $\lambda(m) \in \mu_1$ and $cl(\mu_1) \leq \mu$. Because p is weakly θ-ω-continuous, there is a f. ω-open set λ contains m so that $p\ (\lambda) \leq cl(\mu_1)$. Where, $int(cl(\mu) \leq cl(\ \mu)$, then $p\ (\lambda) \leq int(cl(\mu)) \leq cl(\mu)$, Therefore $p(\lambda) \leq int(cl(\mu))$. So, p is f. almost ω-continuous on M. Then (M, τ) is FWF almost ω-top. sp. .

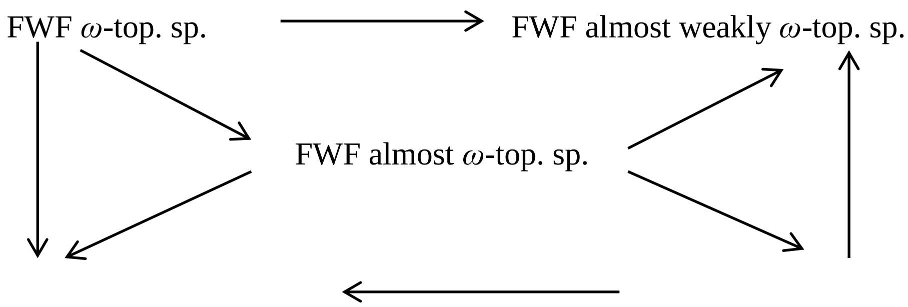
Corollary 2.26. Assume that (B,σ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF almost ω-top. sp..

Theorem 2.27 The FWF topological space (M,τ) over (B,σ) is FWF θ-ω-top. sp. Iff the graph fuzzy mapping $g:(M,\tau) \to (M,\tau) \times (B,\sigma)$, knowledge before $g(m) = (m \ \ ,p(m))$, for every $m \in M$ is a f. θ-ω-continuous

Proof. Necessity. Let g be an f. θ-ω-continuous. It suffices to demonstrate that (M,τ) is a FWF θ-ω-top. sp. over (B,σ), i.e. the proj. $p:(M,\tau) \to (B,\sigma)$ is f. θ-ω-continuous. Let $m \in M_b$, $b \in B$ and λ be a fuzzy open set containment $p\ (m)$. Thus, $M\ \times\ \lambda$ is an fuzzy open set of $M \times B$ containing $g(m)$. Because g is θ-ω-continuous, there is f. ω-open set η contains m so that $g\left(cl^{\omega}(\eta)\right) \leq cl(M \times \lambda) = M \times cl(\lambda)$. Therefore, $p(cl^{\omega}(\eta)) \subseteq cl(\lambda)$. Then, p is f. θ-ω-continuous. Then, (M,τ) is FWF θ-ω-top. sp. .
Sufficiency. Assume that (M,τ) is a FWF θ-ω-top. sp. over (B,σ), then the proj. $p:(M,\tau) \to (B,\sigma)$ f. θ-ω-continuous. It suffices to demonstrate that g is f. θ-ω-continuous. Let $m \in M_b$; $b \in B$ and μ be a fuzzy open set of $M\ \times\ B$ containing $g\ (m)$, there exists a fuzzy open sets $\eta_1 \leq\ M$. And, $\lambda \leq B$ such that $g\ (m) = (m,p(m))\ <\ \eta_1 \times \lambda \leq \mu$. Because p is f. θ-ω-continuous, there is f. ω-open η_2 so that $p(cl^{\omega}(\eta_2)) \leq\ cl(\mu)$. Assume $\eta =\ \eta_1 \wedge \eta_2$. Then, η is f. ω-open in M. Therefore, $g(cl^{\omega}(\eta)) \leq cl(\eta_1) \times p(cl^{\omega}(\eta_2)) \leq cl(\eta_1) \times cl(\lambda) \leq cl(\mu)$. Then , g is θ-ω-continuous.

Theorem 2.28. Assume that (M,τ) is a FWF topological space over (B,σ) so well (B,σ) is a fuzzy ω-regular space. The following properties are equivalent:
(a) FWF weakly θ-ω-top. sp..
(b) FWF ω-top. sp..
(c) FWF almost ω-top. sp..
(d) FWF θ-ω-top. sp..
(e) FWF almost ω-top. sp..
Proof: The proof follows directory from by Theorems 2.15, 2.6, 2.17, and 2.25.

Remark 2.29. The relation between FWF weakly ω-top. sp.is given by the following figure:



FWF weakly θ-ω-top. sp. FWF θ-ω-top. sp.

Figure 2

3. Strongly fibrewise fuzzy θ-ω-topological spaces

In this part, we study the strongly fibrewise θ-ω-continuous topological spaces and some theorems concerning them.

Definition 3.1. A function $\phi: (M,\tau) \rightarrow (N,\Lambda)$ is called fuzzy almost strongly ω-continuous (shortly., f. almost strongly ω-continuous) when if every $m \in M$ so well all fuzzy open set μ in B contains $\phi(m)$, there is a fuzzy ω-open subset λ so that $\phi(cl(\lambda)) \leq int(cl(\mu))$.

Definition 3.2. A function $\phi: (M,\tau) \rightarrow (\mathrm{N},\Lambda)$ is named fuzzy strongly θ-ω-continuous (briefly f. strongly θ-ω-continuous) when if every $m \in M$ so well all fuzzy open set μ in B contains $\phi(m)$, there is a fuzzy ω-open subset λ such that $\phi(cl^{\omega}(\lambda)) \leq \mu$.

Definition 3.3. The FWF topological space (M,τ) over (B,σ) is named a FWF strongly θ-ω-top. sp. (resp., FWF almost strongly ω-top. sp.) if the proj. function p is f. strongly θ-ω-continuous mapping (resp., f. almost strongly ω-continuous) mapping.

The converses does not hold as we show by next examples:

Example 3.4. Assume $M = \{a,b,c\}$, $B = \{x,y,z\}$**,** $\tau = \{\bar{0},\bar{1},\mu_1,\mu_2\}$ where

$\mu_1 = \{(a,0.3),(b,0),(c,0.5)\}$

$\mu_2 = \{(a,0.7),(b,0.7),(c,0.5)\}$

So well assume that $\sigma = \{\bar{0},\bar{1},\lambda\}$, where $\lambda = \{(x,0.6),(y,0.7),(z,0.5)\}$ is the fuzzy topologies on set M and B respectively and let the projection function $p: (M,\tau) \rightarrow (B,\sigma)$ be the fuzzy function as $(a) = p(b) = y$, $p(c) = z$. Let $\eta = \{(a,0.5),(b,0.5),\ (c,0.5)\}$ fuzzy ω-open of M so well $\nu = \{(a,0.6),(b,0.7),(c,0.5)\}$ is fuzzy open of B. Thus, $p(\eta) \leq (\nu)$ but $p(cl^{\omega}(\eta)) \nleq (\nu)$. Then, (M,τ) is FWF ω-top. sp. but not FWF strongly θ-ω-top. sp..

Theorem 3.5. Assume that (B,σ) is a fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp., then it is FWF strongly θ-ω-top. sp..

Proof. Assume that (M,τ) is a FWF ω-top. sp. over (B,σ), then the proj. $p: (M,\tau) \rightarrow (B,\sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set containing $p\ (m) \in B$. Since B is a fuzzy regular space there is a fuzzy open set μ, such that $p(m) \in \mu \leq cl(\mu) \leq \lambda$ since p is f. ω-continuous. Thus, M_{μ} is a f. ω-open set so well, $M_{cl(\mu)}$ is a f. ω-closed. Assume $\xi = M_{\mu}$. Then, $m \in M_{\mu} \leq M_{cl(\mu)}$, ξ is a f. ω-open. Also $cl^{\omega}(\xi) \leq M_{cl(\mu)}$, we have $p(cl^{\omega}(\xi)) \leq cl(\mu) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then, $(M,\tau\ )$ is FWF strongly θ-ω-top. sp. .

Corollary 3.6. Assume that (B,σ) is a fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp. .

Example 3.7. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0},\overline{1},\mu_1,\mu_2\}$ where
$\mu_1 = \{(a,0.1),(b,0.2),(c,0.5)\}$
$\mu_2 = \{(a,0.4),(b,0.3),(c,0.5)\}$
And let $\sigma = \{\overline{0},\overline{1},\lambda\}$, where $\lambda = \{(x,0.3),(y,0.4),(z,0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p:(M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\eta = \{(a,0.2),(b,0.1),(c,0.3)\}$ fuzzy ω-open of M so well $\nu = \{(x,0,3),(y,0.4),(z,0.5)\}$ is fuzzy open of B. Thus, $p(\eta) \leq (\nu)$ but $p(cl(\eta)) \nleq int(cl(\nu))$. Then, (M,τ) is FWFω-top. sp. but not FWF almost strongly ω-top. sp..

Theorem 3.8. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp., then it is FWF almost strongly ω-top. sp..
Proof. Assume that (M,τ) is a FWF ω-top. sp. over (B,σ), then the proj. $p:(M,\tau) \to (B,\sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. almost strongly ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set contains $p\,(m)$ in B. Since p is f. ω-continuous, there is a f. ω-open set μ contains m in M so that $p(\mu) \leq \lambda$. And, $\lambda \leq cl(\lambda)$. Thus, $p(\mu) \leq cl(\lambda)$. Since M is f. ω-regular, there is a f. ω-open set $\mu_1 \in M$ so that $m \in \mu_1$ and, $cl(\mu_1) \leq \mu$. Thus, $p(cl(\mu_1)) \leq p(\mu)$. And, $p(\mu) \leq cl(\lambda)$ then, $int(cl((\lambda)) \leq cl(\lambda)$. It follows that, $(cl(\mu_1)) \leq int(cl(\lambda))$. Therefore, p is f. almost strongly ω-continuous. Thus, (M,τ) is FWF almost strongly ω-top. sp..

Corollary 3.9. Assume that (M,τ) is a fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp. if and only if it is FWF almost strongly ω-top. sp..

Example 3.10. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0},\overline{1},\mu_1,\mu_2\}$ where
$\mu_1 = \{(a,0.3),(b,0,4),(c,0.5)\}$
$\mu_2 = \{(a,0.2),(b,0.2),(c,0.5)\}$
And let $\sigma = \{\overline{0},\overline{1},\lambda\}$, where $\lambda = \{(x,0.2),(y,0.2),(z,0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p:(M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Then, (M,τ) is FWF θ-ω-top. sp. but not FWF strongly θ-ω-top. sp..

Theorem 3.11. Assume that (B,σ) is a fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF θ-ω-top. sp., then it is FWF strongly θ-ω-top. sp..
Proof. Assume that (M,τ) is a FWF θ-ω-top. sp. over (B,σ), then the proj. $p:(M,\tau) \to (B,\sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set contains $p\,(m)$ in B. Because p is f. θ-ω-continuous, there is a f. ω-open set μ contains m in M so that $p(cl^{\omega}(\mu)) \leq cl(\lambda)$ since B is f. regular, there exists is a fuzzy open set $\aleph$ such that $(m) \in \aleph \leq cl(\aleph) \leq \lambda$. Then,

$(cl^{\omega}(\mu)) \leq cl(\aleph) \leq \lambda$. Therefore, $p(cl(\mu \quad)) \leq \lambda$. Thus, p is f. strongly θ-ω-continuous. Then (M,τ) is FWF strongly θ-ω-top. sp..

Corollary 3.12. Let (B,σ) be a fuzzy regular space. The FWF topological space (M,τ) over (B,σ) is FWF θ-ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp..

Example 3.13. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\bar{0},\bar{1},\mu_1,\mu_2\}$ where
$\mu_1 = \{(a,0.3),(b,0,4),(c,0.5)\}$
$\mu_2 = \{(a,0.3),(b,0.3),(c,0.5)\}$
And let $\sigma = \{\bar{0},\bar{1},\lambda\}$, where $\lambda = \{(x,0.3),(y,0.3),(z,0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p:(M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Then, (M,τ) is FWF θ-ω-top. sp. but not FWF almost strongly ω-top. sp..

Theorem 3.14. Assume that (B,σ) is a fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF θ-ω-top. sp., then it is FWF almost strongly ω-top. sp..
Proof. Assume that (M,τ) is a FWF θ-ω-top. sp. over (B,σ), then the projection $p:(M,\tau) \to (B,\sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set contains $p\ (m) \in B$. Since p is f. θ-ω-continuous, there exists is an f. ω-open set μ contains $m \in M$ such that $p(cl^{\omega}(\mu)) \leq cl(\lambda)$. Since B is f. ω-regular, there exists is a fuzzy open set $\lambda_1$in B such that $p(m) \in \lambda_1$ so well $cl(\lambda_1) \leq \lambda$. Thus, $(cl(\lambda_1)) \leq cl(\lambda_1)$. It follows that, $p(cl(\mu)) \leq int(cl(\lambda_1))$. Then, p is f. almost strongly ω-continuous. Then (M,τ) is FWF strongly θ-ω-top. sp..

Example 3.15. Let $M = \{a,b\}$, $B = \{x,y,z\}$, $\tau = \{\bar{0},\bar{1},\mu_1,\mu_2,\mu_3\}$ where
$\mu_1 = \{(a,0.6),(b,0,7)\}$
$\mu_2 = \{(a,1),(b,0.9)\}$
$\mu_2 = \{(a,0.2),(b,0.3)\}$
And let $\sigma = \{\bar{0},\bar{1},\lambda\}$, where $\lambda = \{(x,0.2),(y,0.3)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p:(M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, let $\eta = \{(a,0.5),(b,0.5)\}$ fuzzy ω-open in M. Then, (M,τ) is FWF almost ω-top. sp. but not FWF almost strongly ω-top. sp..

Theorem 3.16. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp., then it is FWF almost strongly ω-top. sp..
Proof. Assume that (M,τ) is a FWF almost ω-top. sp. over (B,σ), then the proj. $p:(M,\tau) \to (B,\sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. almost strongly ω-continuous. Assume that $m \in M_b$; $b \in B$ so well, λ is a fuzzy open set containing $p\ (m) \in B$. Since p is f. almost ω-continuous. There is a f. ω-open set μ containing m of M so that $p(\mu) \leq int(cl(\lambda))$. Since M is fuzzy ω-regular. There is a f. ω-open set $\mu_1 \in M$ so that $m \in \mu_1$ so well, $cl(\mu_1) \leq \mu$. Thus, $(cl(\mu_1)) \leq p(\mu)$. where, $p(cl(\mu_1)) \leq p(\mu) \leq$

$int(cl(\lambda))$. It follows that, $(cl(\mu_1)) \leq int(cl(\lambda))$. Therefore, p is f. almost strongly ω-continuous. Then (M,τ) is FWF almost strongly ω-top. sp..

Corollary 3.17. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp. if and only if it is FWF almost strongly ω-top. sp..

Lemma 3.18. Assume that $\phi : (M,\tau) \rightarrow (N,\Lambda)$ is a f. strongly θ-ω-continuous fibrewise surjection function, since (M,τ) so well (N,Λ) are FWF topological spaces over (B,σ).Just as (N,Λ) is a FWF top. sp., so (M,τ) is FWF strongly θ-ω-top. sp..

Theorem 3.19 The FWF topological space (M,τ) over (B,σ) is FWF strongly θ-ω-top. sp. and (M,τ) is a fuzzy ω-regular iff the graph fuzzy mapping $g: (M\ ,\tau) \rightarrow (M,\tau) \times (B,\sigma)$, knowledge before $g(m) = (m, p(m))$, for all $m \in M$ is a f. strongly θ-ω-continuous.

Proof. By Lemma 3.17. Then, (M,τ) is FWF strongly θ-ω-top. sp. if the graph mapping g is f. strongly θ-ω-continuous. It follows that, M is fuzzy regular. To prove conversely. Assume that (M,τ) is a FWF strongly θ-ω-top. sp. over (B,σ), then the proj. $p: (M,\tau) \rightarrow (B,\sigma)$ f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $g\ (m)$ in $M \times B$, there exists fuzzy open sets μ_1 in M also ν in B such that $g\ (m) = (m, p(g)) \in \mu_1 \times \nu \leq \lambda$. Because p is f. strongly θ-ω-continuous, there is μ_2 is f. ω-open so that $p(cl^{\omega}(\mu_2)) \leq \lambda$. Because M is a f. ω-regular and, $\mu_1 \wedge \mu_2$ is f. ω-open, there is μ f. ω-open such that $m \in \mu \leq cl^{\omega}(\mu) \leq \mu_1 \wedge \mu_2$ by Lemma 2.12. Therefore, $g(cl^{\omega}(\mu)) \leq \mu_1 \times p\ (cl^{\omega}(\mu_2)) \leq \mu_1 \times \nu \leq \lambda$. Then, g is f. strongly θ-ω-continuous.

Example 3.20. In Example 3.14. Then, (M,τ) is FWF almost ω-top. sp. but not FWF strongly θ-ω-top. sp..

Theorem 3.21. Assume that (M,τ) is a fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp., then it is FWF strongly θ-ω-top. sp..

Proof. Assume that (M,τ) is a FWF almost ω-top. sp. over (B,σ), then the proj. $p: (M,\tau) \rightarrow (B,\sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m) \in B$. where p is f. almost ω-continuous. There is a f. ω-open set μ containing $m \in M$ so that $p(\mu) \leq int(cl(\lambda))$. Where M is fuzzy ω-regular. There is a f. ω-open set $\mu_1 \in M$ such that $m \in \mu_1$ so well, $cl(\mu_1) \leq \mu$. Thus, $(cl(\mu_1)) \leq p(\mu)$. Then, $int(cl(\lambda)) \leq cl(\lambda))$. It follows that, $p(cl(\mu_1)) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then (M,τ) is FWF strongly θ-ω-top. sp..

Corollary 3.22. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp., if and only if it is FWF strongly θ-ω- top. sp..

Theorem 3.23. Assume that (M, τ) is an FWF topological space over (B, σ) so well (B, σ) is a fuzzy ω-regular space. The following properties are equivalent:

(a) FWF almost strongly θ-ω-top. sp..

(b) FWF ω-top. sp..

(c) FWF almost ω-top. sp..

(d) FWF θ-ω-top. sp..

Proof. The proof follows directory from by Theorems 3.4, 3.6, 3.10, 3.12 and 3.13.

Remark 3.24. The relation between FWF strongly ω-top. sp. is given by the following figure:

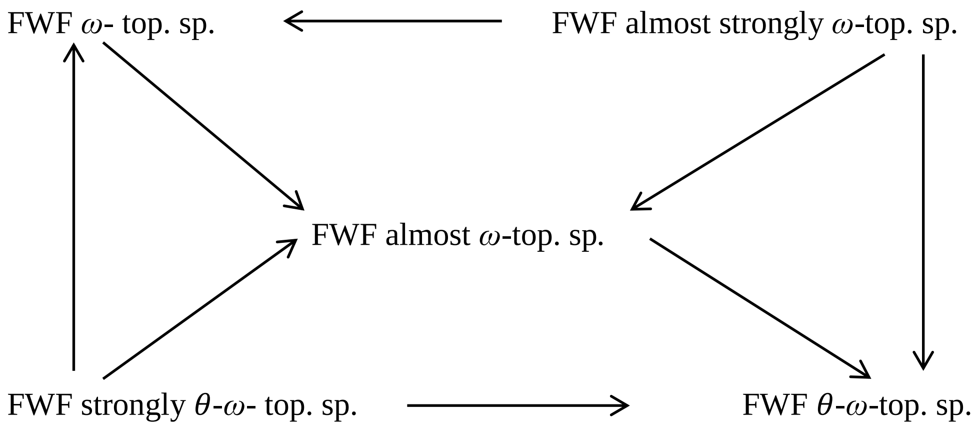


Figure 3

4. Relationship between Weak and Strong Forms of Fibrewise Fuzzy ω-Topological Spaces

In this section, we study the relation between FWF weakly θ-ω-top. sp. and FWF strongly θ-ω-top. sp. and the some theorems concerning them.

Definition 4.1. A mapping $\phi : (M, \tau) \rightarrow (\mathrm{N}, \Lambda)$ are said to be fuzzy almost weakly (resp., fuzzy almost strongly) continuous (briefly, f. almost weakly and f. almost strongly) continuous if for each $m \in M$ and each fuzzy open neighborhood (resp., fuzzy open set) λ of N containing $\phi(m)$, there exists a f. ω-open neighborhood (resp., f. ω-open set) μ of M so that $\phi\ (int\ (cl(\mu)) \leq \lambda$ (resp., $\phi(\mu) \leq cl(\lambda)$, $\phi(cl(\mu)) \leq \lambda$.

Definition 4.2. A mapping $\phi : (M, \tau) \rightarrow (\mathrm{N}, \Lambda)$ is said to be f. super (resp., f. weakly, f. strongly) ω-continuous if for each $m \in M$ and each fuzzy open (resp., fuzzy regular open) set λ of N containing $\phi(m)$, there is a fuzzy open set μ of M so that $\phi\ (\mu) \leq cl(\lambda)$ (resp., $\phi(cl(\mu)) \leq \lambda)$.

Definition 4.3. A mapping $\phi : (M,\tau) \to (\mathrm{N},\Lambda)$ is called fuzzy weakly θ-continuous (briefly, f. weakly θ-continuous) if for each $m \in M$ and each fuzzy open λ of B containing $p(m)$, there exists a fuzzy open set μ of M such that $p\,(\mu) \leq cl(\lambda)$.

Definition 4.4. The FWF topological space (M,τ) over (B,σ) is named a FWF super ω-top. sp. (resp., FWF weakly ω-top. sp., FWF strongly ω-top. sp., FWF almost strongly ω-top. sp., FWF almost weakly ω-top. sp., FWF weakly θ-top. sp.) if the projection function p is fuzzy super ω-continuous mapping (resp., f. weakly ω-continuous, f. strongly ω-continuous, f. almost strongly ω-continuous, f. almost weakly ω-continuous, f. weakly θ-continuous) mapping.

The relation between FWF weakly and FWF strongly ω-top. sp. given by the following figure

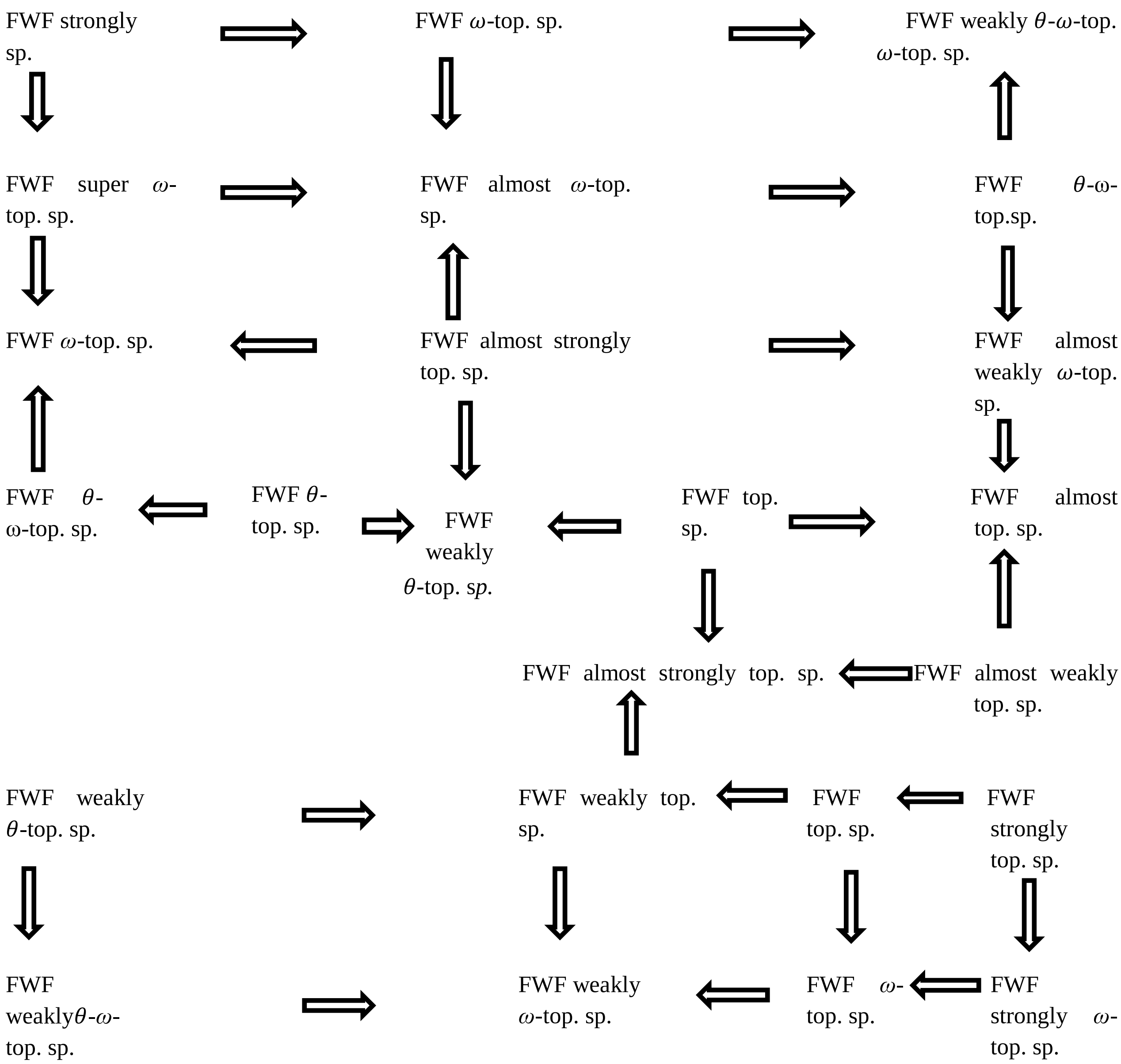


Figure 4

The following examples show that these implications are not reversible:

Example 4.5. Assume $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.3), (b, 0,4), (c, 0.5)\}$
$\mu_2 = \{(a, 0.2), (b, 0.2), (c, 0.5)\}$
$\mu_3 = \{(a, 0.5), (b, 0.6), (c, 0.5)\}$
So well assume that $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.5), (y, 0.6), (z, 0.5)\}$ is the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, $p(c) = z$. let $\eta = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$ fuzzy ω-open in M. Then, (M, τ) is FWF super ω-top. sp. but not FWF strongly θ-ω-top. sp..

Theorem 4.6. Assume that (M, τ) is a fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF super ω-top. sp., then it is FWF strongly θ-ω-top. sp..

Proof. Assume that (M, τ) is a FWF super ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. super ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ so well, λ is a fuzzy open set containing $p\ (m) \in B$. Because of p is a f. super ω-continuous, there exists is a fuzzy regular open set μ containing m, such that $p(\mu) \leq \lambda$. Because $int(cl(\lambda)) \leq cl(\lambda)$, then $p(\mu) \leq int(cl\ \ (\lambda)) \leq cl(\lambda)$. Then, $p\ (\mu) \leq cl(\lambda)$. And, M is a fuzzy regular space, there is an fuzzy open set ν so that $m \in \nu \leq cl(\nu) \leq \mu$. since, $p(cl(\nu)) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then (M, τ) is FWF strongly θ-ω-top. sp..

Corollary 4.7. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF super ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp..

Example 4.8. Let $M = \{a, b\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.7), (b, 0,6)\}$
$\mu_2 = \{(a, 0.7), (b, 0.9)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.7), (y, 0.6)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, let $\eta = \{(a, 0.5), (b, 0.5)\}$ fuzzy ω-open in M. Then
, (M, τ) is FWF ω-top. sp. but not FWF super ω-top. sp..

Theorem 4.9. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp., then it is FWF super ω-top. sp..

Proof. Assume that (M, τ) is a FWF ω-top. sp. over (B, σ), then the proj. $p: (M, \tau) \to (B, \sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. super ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m)$ in B. Because of p is a f. ω-continuous, there is a fuzzy ω-open set μ contains m, so that $p(\mu) \leq \lambda$, also $int(\ cl(\mu)) \leq cl(\mu)$. Then, $p(int(\ cl(\mu)) \leq p\ (cl(\mu))$. And, M is a fuzzy regular space. There is an fuzzy open set μ_1

such that $\in \mu_1 \leq cl(\mu_1) \leq \mu$. Thus, $p(int(cl(\mu)) \leq p\,(cl(\mu_1))$ so well, $p(\mu) \leq \lambda$. Then, $p(int(cl(\mu)) \leq \lambda$. It follow that, p is f. super ω-continuous. Then (M,τ) is FWF super ω-top. sp..

Corollary 4.10. Assume that (M,τ) is an fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp. if and only if it is FWF super ω-top. sp..

Example 4.11. For an fuzzy topological space $(M,\tau) = (B,\sigma)$ Let $\sigma = \tau = \{\bar{0}, \bar{1}, \mu: \frac{1}{3} \leq \mu(m) \leq \frac{2}{3}$, for some fixed element m of M and $\mu(m) = 0$, otherwise$\}$. Assume that (M,τ) is a FWF topological space over (B,σ) also assume that the projection function $p: (M,\tau) \to (B,\sigma)$ is the fuzzy function as the identity maps. Then, (M,τ) is FWF top. sp. but not FWF strongly top. sp. .

Theorem 4.12. Assume that (M,τ) is an fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF top. sp. , then it is FWF strongly top. sp. .
Proof. Assume that (M,τ) is a FWF top. sp. over (B,σ), then the proj. $p: (M,\tau) \to (B,\sigma)$ f. continuous. It suffices to demonstrate that p is f. strongly continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\,(m)$ in B. Because of p is a f. continuous, there is a fuzzy open set μ contains m, so that $p(\mu) \leq \lambda$, where M is fuzzy regular space, there is a fuzzy open set $\mu_1 \in M$ such that $m \in \mu_1$ also, $cl(\mu_1) \leq \mu$. Thus, $p(cl(\mu_1)) \leq p(\mu)$. Then, $p(cl(\mu_1)) \leq \lambda$. Therefore, p is f. strongly continuous. Then (M,τ) is FWF strongly compact.

Corollary 4.13. Assume that (M,τ) is an fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp., if and only if it is FWF strongly compact.

Theorem 4.14. Let (B,σ) be a fuzzy regular space. The FWF topological space (M,τ) over (B,σ) is FWF weakly top. sp., then it is FWF top. sp. .
Proof. Assume that (M,τ) is a FWF weakly top. sp. over (B,σ), then the proj. $p: (M,\tau) \to (B,\sigma)$ f. weakly continuous. It suffices to demonstrate that p is f. continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\,(m) \in B$. Where B is fuzzy regular, there is a fuzzy open set $\lambda_1 \in B$ so that $p(m) \in \lambda_1$ also, $cl(\lambda_1) \leq \lambda$. Because p is weakly continuous, there exists is a fuzzy open set μ containing m in M so that $p(\mu) \leq cl(\lambda_1)$. Thus, $p(\mu) \leq \lambda$. It follows that, p is f. continuous. Then, (M,τ) is FWF top. sp. .

Corollary 4.15. Assume that (B,σ) is an fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly top. sp. if and only if it is FWF top. sp. .

Example 4.16. Let $= \{a,b\}$, $B = \{x,y\}$, $\tau = \{\bar{0}, \bar{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.60), (b, 0.60)\}$
$\mu_2 = \{(a, 1), (b, 0.9)\}$
$\mu_3 = \{(a, 0.11), (b, 0.31)\}$

And let $\sigma = \{\bar{0}, \bar{1}, \lambda\}$, where $\lambda = \{(x, 0.11), (y, 0.31)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$. Let $\eta = \{(a, 0.7), (b, 0.4)\}$ fuzzy ω-open in M also $\nu = \{(a, 0.11), (b, 0.31)\}$ be an fuzzy open of B. Thus, $p(\eta) \leq int(cl(\nu))$ but $(int\ cl(\eta)) \nleq \nu$. Then, (M, τ) is FWF almost ω-top. sp. but not FWF super ω-top. sp..

Definition 4.17. [5] A fuzzy topological space (M, τ) is called a fuzzy semi-regular space iff the collection of all fuzzy regular open sets of M forms a base for fuzzy topology τ.

Theorem 4.18 assume that (M, τ) and (B, σ) are an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., then it is FWF super ω-top. sp..

Proof. Assume that (M, τ) is a FWF almost ω-top. sp. over (B, σ), then the projection $p: (M, \tau) \rightarrow (B, \sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. super ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m)$ in B. Because of p is f. almost ω-continuous, there exists is a f. ω-open set μ containing m. For each fuzzy regular open set λ of B contains $p(m)$ so that $p(\mu) \leq \lambda$. Thus, $(\ \mu) \leq (int(cl(\lambda))$. Because the space M is fuzzy semi-regular space, There exists is a fuzzy open set $\mu_1 \in M$ so that $m \in\ \mu_1$ also, $\lambda \leq\ int(cl(\lambda)) \leq \mu$. Thus, $(\lambda) \leq p(int(\ \ cl(\lambda))) \leq p(\mu)$. Also, $p(\mu) \leq int(cl(\ \mu))$. Thus, $p(int(cl(\lambda))) \leq p(\mu) \leq int(cl(\lambda)\ \ )$. So well, the space B is fuzzy semi-regular space, there exists is a fuzzy open set λ_1 in B such that $p(m) \in \lambda_1$ then, $\mu \leq int(cl(\mu)) \leq \lambda$. Thus, $p(\mu) \leq p(int(cl(\mu)))$. It follows that, $p(int(cl(\mu))) \leq \lambda$. Then, p is f. super ω-continuous. Hence (M, τ) is FWF super ω-top. sp..

corollary 4.19. Let (M, τ) and (B, σ) be a fuzzy regular space. The FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp. if and only if it is FWF super ω-top. sp..

Example 4.20. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\bar{0}, \bar{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.1), (b, 0.2), (c, 0.5)\}$
$\mu_2 = \{(a, 0.4), (b, 0.3), (c, 0.5)\}$
And let $\sigma = \{\bar{0}, \bar{1}, \lambda\}$, where $\lambda = \{(x, 0.3), (y, 0.4), (z, 0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p: (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\nu = \{(x, 0{,}3), (y, 0.4), (z, 0.5)\}$ is fuzzy open in B. Then, $p(\mu_1) \leq cl(\nu)$ but $p(cl(\mu_1)) \nleq int(cl(\nu))$. Then, (M, τ) is FWF almost weakly top. sp. but not FWF almost strongly top. sp. .

Theorem 4.21. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp., then it is FWF almost strongly top. sp. .

Proof. Assume that (M, τ) is a FWF almost weakly top. sp. over (B, σ), then the proj. $p: (M, \tau) \rightarrow (B, \sigma)$ f. almost weakly continuous. It suffices to demonstrate that p is f. almost strongly continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m)$ in B. Because of p is f. almost weakly continuous, $m\ \in M_b$, $b \in B$ for each open set $\lambda$of B

containing $p(m)$ there is a fuzzy open set μ contains m so that $p(\mu) \leq cl(\lambda)$. Because the space M is a fuzzy regular space, there is a fuzzy open set $\mu_1 \in M$ such that $m \in \mu_1$ also $cl(\mu_1) \leq \mu$, so $p(cl(\mu_1)) \leq p(\mu)$. Also, $p(\mu) \leq cl(\lambda)$. Then, $p(cl(\mu_1)) \leq cl(\lambda)$ also, $int(cl(\lambda_1)) \leq cl(\lambda_1)$. Then, $(cl(\lambda_1)) \leq int(cl(\lambda_1))$. It follows that, p is f. almost strongly continuous. Hence (M,τ) is FWF almost strongly top. sp. .

Corollary 4.22. Assume that (M,τ) is an fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost weakly ω-top. sp. if and only if it is FWF almost strongly top. sp. .

Theorem 4.23. Assume that (M,τ) is a FWF topological space over (B,σ) also (B,σ) is a fuzzy regular space. The following properties are equivalent:

(a) FWF strongly top. sp. .
(b) FWF top. sp. .
(c) FWF weakly top. sp. .

Proof. The proof follows directory from by Theorems 4.12, 2.16.

Definition 4.24. [4] Let M and B be an fuzzy spaces are called fuzzy homeomorphic denoted by $M \cong B$ if there exists a fuzzy homeomorphism on M to B.

Theorem 4.25 The FWF topological space (M,τ) over (B,σ) is FWF strongly top. sp. Also (M,τ) is a fuzzy regular, so the graph fuzzy function $g: (M,\tau) \to (M,\tau) \times (B,\sigma)$, defined by $g(m) = (m, p(m))$, for each $m \in M$ is a f. strongly continuous.

Proof: Assume that (M,τ) is a FWF strongly top. sp. over (B,σ), then the proj. $p: (M,\tau) \to (B,\sigma)$ f. strongly continuous mapping. Let $m \in M_b$, $b \in B$ and μ be a fuzzy open set of $M \times B$ containing $p(m)$. There exists fuzzy open sets $\xi_1 \in I^M$ and $\lambda \in I^B$ so that $g(m) = (m, p(m)) < \xi_1 \times \lambda \leq \mu$. Where p is f. strongly continuous also, M is fuzzy regular space, there is an fuzzy open set ξ containing m in M so that $cl(\xi) \leq \xi_1$ also $p(cl(\xi)) \leq \lambda$. Therefore, $p(cl(\xi)) \leq \xi_1 \times \lambda \leq \mu$. Then, p is f. strongly continuous. Thus, the mapping $g = id_M \triangle p : (M,\tau) \to (M,\tau) \times (B,\sigma)$ maps fuzzy homeomorphically onto the graph $g(m)$ which is fuzzy closed subset of $M \times B$, so p is f. continuous and because M is an fuzzy regular , then $M \times B$ is fuzzy regular, by Theorem 4.24. Hence, $g: M \to M \times B$ is f. strongly continuous mapping.

Theorem 4.26. Assume that (M,τ) is a FWF topological space over (B,σ) also (B,σ) is a fuzzy regular space. The following properties are equivalent:

(a) FWF almost strongly θ-ω-top. sp..
(b) FWF ω-top. sp..
(c) FWF almost ω-top. sp..
(d) FWF θ-ω-top. sp..
(e) FWF almost weakly ω-top. sp.

Proof. The proof follows directory from by Theorems 3.6, 2.15, 3.16.

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