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## Fibrewise Pairwise Soft Separation Axioms

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**Abstract:** The main idea of this research is to study fibrewise pairwise soft forms of the more important separation axioms of ordinary bitopology named fibrewise pairwise soft  $T_0$  spaces, fibrewise pairwise soft  $T_1$  spaces, fibrewise pairwise soft  $R_0$  spaces, fibrewise pairwise soft Hausdorff spaces, fibrewise pairwise soft functionally Hausdorff spaces, fibrewise pairwise soft regular spaces, fibrewise pairwise soft completely regular spaces, fibrewise pairwise soft normal spaces and fibrewise pairwise soft functionally normal spaces. In addition we offer some results concerning it.

**Keywords:** Fibrewise bitopological spaces, Fibrewise pairwise soft  $T_0$  spaces, Fibrewise pairwise soft  $T_1$  spaces, Fibrewise pairwise soft  $R_0$  spaces, fibrewise pairwise soft Hausdorff spaces, Fibrewise pairwise soft regular spaces and fibrewise pairwise soft normal spaces.

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### 1. Introduction

In order to began the category in the classification of fibrewise (shortly.,  $fw$ ) sets on a given set, named the base set, which say  $B$ . A  $fw$  set on  $B$  consist of a set  $M$  with a function  $P : M \rightarrow B$  that is named the projection. The fibre over  $b$  for every point  $b$  of  $B$  is the subset  $M_b = P^{-1}(b)$  of  $M$ . Perhaps, fibre will be empty because we do not require  $P$  is surjective, also, for every subset  $B^*$  of  $B$  we considered  $M_{B^*} = P^{-1}(B^*)$  as a  $fw$  set over  $B^*$  with the projection determined by  $P$ . The another notation  $M|B^*$  is some time convenient. We considered the Cartesian product  $B \times T$ , for every set  $T$ , like a  $fw$  set  $B$  by the first projection.

**Definition 1.1.** [9] If  $M$  and  $N$  are  $fw$  sets on  $B$ , with projections  $P_M : M \rightarrow B$  and  $P_N : N \rightarrow B$ , respectively, a function  $\Phi : M \rightarrow N$  is said to be  $fw$  function if  $P_N \circ \Phi = P_M$ , or  $\Phi(M_b) \subset N_b$  for every point  $b$  of  $B$ .

Observe that a  $fw$  function  $\Phi : M \rightarrow N$  over  $B$  limited by restriction, a  $fw$  function  $\Phi_{B^*} : M_{B^*} \rightarrow N_{B^*}$  over  $B^*$  for every subset  $B^*$  of  $B$ .

**Definition 1.2.** [9] Let  $(B, \gamma)$  be a topological space. The  $fw$  topology on a  $fw$  set  $M$  over  $B$  mean any topology on  $M$  for which the projection  $P$  is continuous.

**Definition 1.3.** [9] The  $fw$  function  $\Phi : M \rightarrow N$ , where  $M$  and  $N$  are  $fw$  topological spaces over  $B$  is named:



- (a) Continuous if for every point  $m \in M_b$ ;  $b \in B$ , the inverse image of every open set of  $\Phi(m)$  is an open set of  $m$ .
- (b) Open if for every point  $m \in M_b$ ;  $b \in B$ , the direct image of every open set of  $m$  is an open set of  $\Phi(m)$ .

**Definition 1.4.** [9] The  $fw$  topological space  $(M, \tau)$  over  $(B, \gamma)$  is named  $fw$  closed, (resp.,  $fw$  open) if the projection  $P$  is closed (resp., open).

In the year 1999, Russian researcher Molodtsov [12], started the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. Many research studied the theory of soft sets as Maji, Biswas and Roy in 2003 [11], D. Chen in 2005 [6]. Topological structures of soft set have been studied by some authors in recent years, Naim Cagman et al. in 2011 [5] and M. Shabir et al. in 2011 [16]. The soft set theory has a rich potential for applications in some directions.

**Definition 1.5.** [12] Let  $M$  be an initial universe and  $E$  be a set of parameters. Let  $\mathbb{P}(M)$  denote the power set of  $M$  and  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is named a soft set over  $M$ , where  $F$  is a function given by  $F : A \rightarrow \mathbb{P}(M)$ . In other words, a soft set over  $M$  is a parameterized family of subset of the universe  $M$ . For  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

Note that the set of all soft sets over  $M$  will be denoted by  $S(M, E)$ .

**Definition 1.6.** [11] A soft set  $(F, E)$  on  $M$  is named to be a null soft if  $F(e) = \phi$  for all  $e \in E$  and this denoted by  $\tilde{\phi}$ . Also,  $(F, E)$  is said to be an absolute soft set if  $F(e) = M$  for each  $e \in E$  and this denoted by  $\tilde{M}$ .

**Definition 1.7.** [16] The difference of two soft sets  $(F, E)$  and  $(G, E)$  over the common universe  $M$ , denoted by  $(F, E) - (G, E)$  is the soft set  $(H, E)$  where for all  $e \in E$ ,  $H(e) = F(e) - G(e)$ .

**Definition 1.8.** [11]

- (a) The union of two soft sets of  $(F, A)$  and  $(G, B)$  over the common universe  $M$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ .

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write  $(F, A) \tilde{\cup} (G, B) = (H, C)$ .

- (b) The intersection of two soft sets of  $(F, A)$  and  $(G, B)$  over the common universe  $M$  is the soft set  $(H, C)$ , where  $C = A \cap B$  and for all  $e \in C$ , we have  $H(e) = F(e) \cap G(e)$ . We write  $(F, A) \tilde{\cap} (G, B) = (H, C)$ .

**Definition 1.9.** [1] The complement of a soft set  $(F, E)$ , denoted by  $(F, E)^c$ , is defined by  $(F, E)^c = (F^c, E)$ ;  $F^c : E \rightarrow \mathbb{P}(M)$  is a function given by  $F^c(e) = M - F(e)$  for all  $e \in E$ .  $F^c$  is called the soft complement function of  $F$ . Clearly,  $(F^c)^c$  is the same as  $F$  and  $((F, E)^c)^c = (F, E)$ .

**Definition 1.10.** [16] Let  $\tilde{\tau}$  be the collection of soft sets over  $M$ , then  $\tilde{\tau}$  is said to be a soft topology on  $M$  if

- (a)  $\tilde{\phi}, \tilde{M} \in \tilde{\tau}$
- (b) the union of any number of soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$
- (c) the intersection of any two soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .

The triplet  $(M, \tilde{\tau}, E)$  is called a soft topological space over  $M$ . The members of  $\tilde{\tau}$  are called soft open sets in  $M$ . Also, a soft set  $(F, A)$  is named a soft closed if the complement  $(F, A)^c$  belongs to  $\tilde{\tau}$ .

**Definition 1.11.** [14]

- (a) A soft set  $(F, E)$  over  $M$  is said to be a soft element if  $\exists e \in E$  such that  $F(e)$  is a singleton, say,  $\{m\}$  and  $F(e') = \phi$ , for all  $(e' \neq e) \in E$ . Such a soft element is denoted by  $\tilde{m}$ .
- (b) A soft set  $(F, E)$  is said to be a soft neighbourhood (shortly., soft nbd) of the soft set  $(H, E)$  if there exists a soft set  $(G, E) \in \tilde{\tau}$  where  $(H, E) \subseteq (G, E) \subseteq (F, E)$ . If  $(H, E) = \tilde{m}$ , then  $(H, E)$  is said to be a soft nbd of the soft element  $\tilde{m}$ .

**Definition 1.12.** [16]

Let  $(M, \tilde{\tau}, E)$  be a soft topological space and  $(F, E)$  be a soft set over  $M$ . Then, the soft closure of  $(F, E)$ , denoted  $\overline{(F, E)}$ , is defined as the soft intersection of all soft closed super sets of  $(F, E)$ .

Note that  $\overline{(F, E)}$  is the smallest soft closed set that containing  $(F, E)$ .

**Definition 1.13.** [5]

- (a) Let  $(M, \tilde{\tau}, E)$  be a soft topological space over  $M$  and  $N$  be a non-empty subset of  $M$ . Then, the collection  $\tilde{\tau}_N = \{(G, K) \cap (F, E) : (F, E) \in \tilde{\tau}\}$  is called a soft subspace topology on  $N$ . Hence,  $(N, \tilde{\tau}_N, K)$  is called a soft topological subspace of  $(M, \tilde{\tau}, E)$ .
- (b) A soft basis of a soft topological space  $(M, \tilde{\tau}, E)$  is a subcollection  $\tilde{\mathcal{B}}$  of  $\tilde{\tau}$  such that every element of  $\tilde{\tau}$  can be expressed as the union of elements of  $\tilde{\mathcal{B}}$ .

**Definition 1.14.** [13] Let  $M$  and  $N$  be two non-empty sets and  $E$  be the parameter set. Let  $\{f_e : M \rightarrow N, e \in E\}$  be a collection of functions. Then a function  $\tilde{f} : SE(M, E) \rightarrow SE(N, E)$  defined by  $\tilde{f}(e_m) = e_{f_e(m)}$  is called a soft function, where  $SE(M, E)$  and  $SE(N, E)$  are sets of all soft elements of the soft sets  $\tilde{M}$  and  $\tilde{N}$  respectively.

**Definition 1.15.** [4] Let  $S(M, E)$  and  $S(N, K)$  be families of soft sets. Let  $f : M \rightarrow N$  and  $u : E \rightarrow K$  be mappings. Then a mapping  $P_{fu} : S(M, E) \rightarrow S(N, K)$  is defined as:

(1) Let  $(F, A)$  be a soft set in  $S(M, E)$ . The image of  $(F, A)$  under  $P_{fu}$ , written as  $P_{fu}(F, A) = (f_{pu}(F), u(A))$ , is a soft set in  $S(N, K)$  such that

$$P_{fu}(F)(k) = \begin{cases} \bigcup_{e \in u^{-1}(k) \cap A} f(F(e)), & u^{-1}(k) \cap A \neq \phi \\ \phi & \text{otherwise} \end{cases}$$

for all  $k \in K$ .

(2) Let  $(G, B)$  be a soft set in  $S(N, K)$ . The inverse image of  $(G, B)$  under  $P_{fu}$ , written as

$$P_{fu}^{-1}(G)(e) = \begin{cases} f^{-1}(G(u(e))), & u(e) \in B \\ \phi, & \text{otherwise} \end{cases}$$

for all  $e \in E$ .

The soft mapping  $P_{fu}$  is called surjective if  $f$  and  $u$  are surjective. The soft mapping  $P_{fu}$  is called injective if  $f$  and  $u$  are injective.

**Definition 1.16.** [3] A soft function  $\Phi : (M, \tilde{\tau}, E) \rightarrow (N, \tilde{\sigma}, L)$  is said to be soft continuous (shortly., s-continuous) if the inverse image of every soft open set of  $N$  is a soft open set in  $M$ .

**Definition 1.17.** [3] A function  $\Phi : (M, \tilde{\tau}, E) \rightarrow (N, \tilde{\sigma}, L)$  is said to be

- (a) Soft open (shortly., s-open) if the image of every soft open set in  $M$  is soft open set in  $N$ ,
- (b) Soft closed (shortly., s-closed) if the image of every soft closed set in  $M$  is soft closed set in  $N$ .

**Definition 1.18.** [15] If  $(M, E)$  is a soft set, then the soft set  $\Delta_{(M, E)} = \{(m_e, m_e) : m_e \in (M, E)\}$  is called as the diagonal of  $(M, E) \times (M, E)$ . Here  $\Delta_{(M, E)} = (\Delta_M, \Delta_E)$  is defined by  $((m, m), (e, e)) = (m, m)_{(e, e)} = (m_e, m_e)$ .

**Definition 1.19.** [3] Let  $(F, E)$  and  $(G, L)$  be two soft sets over  $M$ , then the Cartesian product of  $(F, E)$  and  $(G, L)$  is defined as,  $(F, E) \times (G, L) = (H, E \times L)$  where  $H : E \times L \rightarrow \mathbb{P}(E \times L)$  and  $H(e, l) = F(e) \times G(l)$ , where  $(e, l) \in E \times L$ .

$$\text{i.e., } H(e, l) = \{(m_i, m_j) : m_i \in F(e) \text{ and } m_j \in G(l)\}.$$

The Cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets. The Cartesian product  $(F_1, E) \times (F_2, E) \times \dots \times (F_n, E)$  of the nonempty soft sets  $(F_1, E), (F_2, E), \dots, (F_n, E)$  is the soft set of all ordered n-tuple  $(m_1, m_2, \dots, m_n)$ , where  $m_i \in F_i(e)$ .

**Definition 1.20.** [2] Let  $\{(M_r, \tilde{\tau}_r, E_r)\}_{r \in S}$  be a family of soft topological spaces. Then, the initial soft topology on  $M = \prod_{r \in S} M_r$  generated by the family  $\{(f_r, u_r)\}_{r \in S}$  is called product soft topology on  $M$ . (Here,  $(f_r, u_r)$  is the soft projection function from  $M$  to  $M_r, r \in S$ ).

The product soft topology is denoted by  $\prod_{r \in S} (M_r, \tilde{\tau}_r, E_r)$ .

**Definition 1.21.** [21]. A family  $C$  of soft sets is a cover of a soft set  $(F, E)$  if  $(F, E) \tilde{\subseteq} \tilde{\cup} \{(F_i, E) \tilde{\in} C, i \in I\}$ . It is a soft open cover if each member of  $C$  is a soft open set. A subcover of  $C$  is a subfamily of  $C$  which is also a cover.

The bitopological space study was first created by Kelly [10] in 1963 and after that a large number of researches have been completed to generalize the topological ideas to bitopological setting. In this research  $(M, \tau_1, \tau_2)$  and  $(N, \sigma_1, \sigma_2)$  (or briefly.,  $M$  and  $N$ ) always mean bitopological spaces on which no separation axioms are supposed unless clearly stated. By  $\tau_i$ -open (resp.,  $\tau_i$ -closed), we shall mean the open (resp., closed) set with respect to  $\tau_i$  in  $M$ , where  $i = 1, 2$ .  $A$  is open (resp., closed) in  $M$  if it is both  $\tau_1$ -open (resp.,  $\tau_1$ -closed) and  $\tau_2$ -open (resp.,  $\tau_2$ -closed). As well as, we built on some of the result in [17, 18, 19, 20].

**Definition 1.22.** [10] A triple  $(M, \tau_1, \tau_2)$  where  $M$  is a non-empty set and  $\tau_1$  and  $\tau_2$  are topologies on  $M$  is named bitopological space.

**Definition 1.23.** [10] A function  $\varphi : (M, \tau_1, \tau_2) \rightarrow (N, \sigma_1, \sigma_2)$  is said to be  $\tau_i$ -continuous (resp.,  $\tau_i$ -open,  $\tau_i$ -closed), if the function  $\varphi : (M, \tau_i) \rightarrow (N, \sigma_i)$  is continuous (resp., open, closed).  $\varphi$  is named continuous (resp., open, closed) if it is  $\tau_i$ -continuous (resp.  $\tau_i$ -open,  $\tau_i$ -closed) for every  $i = 1, 2$ .

**Definition 1.24.** [8] Let  $(M, \tilde{\tau}_1, E)$  and  $(M, \tilde{\tau}_2, E)$  be the two different soft topologies on  $M$ . Then  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called a soft bitopological space.

**2. Fibrewise Pairwise Soft  $T_0$ , Pairwise Soft  $T_1$  and Pairwise Soft Hausdorff spaces**

The concepts of soft near open sets have an important role in fibrewise pairwise soft separation axioms. By using these concepts we can construct many several fibrewise soft pairwise separation axioms. At this time gave concepts of fibrewise pairwise soft  $T_0$  and soft  $T_1$  spaces like in the following.

**Definition 2.1.** Assume that  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Now  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called fibrewise pairwise soft  $T_0$  (briefly, *fw-ps $T_0$* ) if whenever  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ , there exists either a  $\tilde{\tau}_i$ - soft open set  $(F_1, E)$  from  $\tilde{m}_1$  that does not include on  $\tilde{m}_2$  on  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  otherwise  $\tilde{\tau}_j$ - soft open set  $(F_2, E)$  of  $\tilde{m}_2$  which does not contain  $\tilde{m}_1$  in  $M$ , anywhere  $i, j=1, 2, i \neq j$ .

**Example 2.2.** Suppose that  $M = \{m_1, m_2\}, B = \{a, b\}, E = \{e_1, e_2\}, D = \{d_1, d_2\}$  and  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is a *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Define  $f : M \rightarrow B$  and  $u : E \rightarrow D$ , such that  $f(m_1) = \{a\}, f(m_2) = \{b\}, u(e_1) = \{d_1\}, u(e_2) = \{d_2\}$ . Then  $\tilde{\tau}_1 = \{\tilde{\Phi}, \tilde{M}, (F_1, E), (F_2, E)\}, \tilde{\tau}_2 = \{\tilde{\Phi}, \tilde{M}, (F_3, E), (F_4, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$  are soft sets over  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , defined as follows:

$(F_1, E) = \{(e_1, \{m_1\})\}, (F_3, E) = \{(e_2, \{m_2\})\}$   
 $(F_2, E) = \{(e_1, \{m_1\}), (e_2, \{m_2\})\}, (F_4, E) = \{(e_1, \{m_1\}), (e_2, \{m_2\})\}$   
 $\tilde{\gamma}_1 = \{\tilde{\Phi}, \tilde{B}, (L, D)\}, \tilde{\gamma}_2 = \{\tilde{\Phi}, \tilde{B}, (O, D)\}, (L, D) = \{(d_1, \{a\})\}, (O, D) = \{(d_2, \{b\})\}$  and then the projection function  $P_{fu} : M \rightarrow B$  be a s-continuous, then there exists soft open  $\{(e_1, \{m_1\})\} \in \tilde{\tau}_1, \{(e_2, \{m_2\})\} \in \tilde{\tau}_2$  such that  $\tilde{m}_1 \in \{(e_1, \{m_1\})\}, \tilde{m}_2 \notin \{(e_1, \{m_1\})\}$  and  $\tilde{m}_2 \in \{(e_2, \{m_2\})\}, \tilde{m}_1 \notin \{(e_2, \{m_2\})\}$ . Also,  $M$  is *fw-ps  $T_0$*  space

**Remark 2.3.**

- (a)  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps $T_0$*  space iff each fiber pairwise soft  $M_{\tilde{b}}$  is  $T_0$  space.
- (b) Soft subspaces of *fw-ps  $T_0$*  spaces are *fw-ps  $T_0$*  spaces.
- (c) The *fw-sbt* products of *fw-ps  $T_0$*  spaces with a group fibrewise pairwise soft projections are *fw-ps  $T_0$*  spaces.

Of course one can express a fibrewise pairwise soft type of the  $T_1$  (briefly, *fw-ps  $T_1$* ) space in a like fashion. Assume that  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Then  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called *fw-ps  $T_1$*  if whenever  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ , there is a  $\tilde{\tau}_i$ - soft open sets  $(F_1, E), (F_2, E)$  in  $M$  where  $\tilde{m}_1 \in (F_1, E), \tilde{m}_2 \notin (F_2, E)$  and  $\tilde{m}_1 \notin (F_2, E), \tilde{m}_2 \in (F_2, E), i, j=1, 2, i \neq j$ . On the other hand it turns out that there is no real use for this in what we are working to do. In its place we make some use of added axiom "The axiom is that all  $\tilde{\tau}_i$ - softopen set contains the closure of each of its points", also

use the term pairwise soft  $R_0$  space. This is exact for pairwise soft  $T_I$  spaces of course also for pairwise soft regular spaces. Searching of it equally a feeble pattern of patterns pairwise soft regularity. Such as, soft indiscrete spaces are pairwise soft  $R_0$  spaces. The fibrewise pairwise soft type of the  $R_0$  axiom is like the following.

**Definition 2.4.** The *fw-sbts*  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is called fibrewise pairwise soft  $R_0$  (briefly, *fw-ps  $R_0$* ) if for all soft point  $\tilde{m} \in \tilde{E}(M_{\tilde{b}}, E)$ , anywhere  $\tilde{b} \in \tilde{E}(B, D)$ , and all  $\tilde{\tau}_i$ -soft open set  $(F, E)$  from  $\tilde{m}$  in  $M$ , there is a soft nbd  $(H, D)$  from  $\tilde{b}$  in  $(B, D)$  so as the  $\tilde{\tau}_i$ -soft closure of  $\{\tilde{m}\}$  in  $M_{(H, D)}$  is contained in  $(F, E)$  (i.e.,  $M_{(H, D)} \tilde{\cap} \tilde{\tau}_i\text{-Cl}\{\tilde{m}\} \subset \tilde{E}(F, E)$ ), where  $i, j=1, 2, i \neq j$ .

Such as,  $B \times T$  is *fw-ps  $R_0$*  space for each pairwise soft  $R_0$  spaces  $T$ , where  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ ,  $(T, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is a *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ .

**Remark 2.5.**

- (a) The nbds. of  $\tilde{m}$  are provided by a fibrewise basis it is enough if the condition in Definition (3.1.3) is satisfied for each fibrewise soft basic nbds.
- (b) Condition  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps  $R_0$*  space over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ , now  $(M_{B^*}, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E_{B^*})$  is *fw-ps  $R_0$*  space over  $B^*$  for all subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $B$ .

The soft subspaces from *fw-ps  $R_0$*  spaces are *fw-ps  $R_0$*  spaces. In fact we have.

**Proposition 2.6.** Assume that  $\phi : M \rightarrow M^*$  is a s-continuous fibrewise embedding, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $M^*$  is *fw-ps  $R_0$* , then  $M$  is so.

**Proof.** Assume that  $\tilde{m} \in \tilde{E}(M_{\tilde{b}}, E)$ , where  $\tilde{b} \in \tilde{E}(B, D)$  and  $(F, E)$  is a  $\tilde{\tau}_i$ -soft open set of  $\tilde{m}$  in  $M$ . Now  $(F, E) = \phi^{-1}(F^*, E^*)$ , where  $(F^*, E^*)$  is a  $\tilde{\tau}_i$ -soft open set of  $\tilde{m}^* = \phi(\tilde{m})$  in  $M^*$ . Since  $M^*$  is *fw-ps  $R_0$*  there exists a soft nbd  $(H, D)$  of  $\tilde{b}$  such that  $M_{(H, D)}^* \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}^*\} \subset \tilde{E}(F^*, E^*)$ . Then  $M_{(H, D)} \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}\} \subset \phi^{-1}(M_{(H, D)}^* \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}^*\}) = \phi^{-1}(F^*, E^*) = (F, E)$  and so  $M$  is *fw-ps  $R_0$* , anywhere  $i, j=1, 2, i \neq j$ .

The class of *fw-ps  $R_0$*  spaces is finitely multiplicative, in the following sense.

**Proposition 2.7.** Assume that  $\{M_r, \tilde{\tau}_{r1}, \tilde{\tau}_{r2}, E_r\}$  is a finite family of *fw-ps  $R_0$*  spaces over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Then the *fw* soft bitopological product  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E) = \prod_B (M_r, \tilde{\tau}_{r1}, \tilde{\tau}_{r2}, E_r)$  is *fw-ps  $R_0$* .

**Proof.** Assume that  $\tilde{m} \in \tilde{E}(M_{\tilde{b}}, E)$ , where  $\tilde{b} \in \tilde{E}(B, D)$ . Study a  $\tilde{\tau}_i$ -soft open set  $(F, E) = \prod_B (F_r, E_r)$  of  $\tilde{m}$  in  $M$ , anywhere  $(F_r, E_r)$  is a  $\tilde{\tau}_i$ -soft open set of  $\prod_r(\tilde{m}) = \tilde{m}_r$  in  $M_r$  for each index  $r$ . Since  $M_r$  is *fw-ps  $R_0$*  there is a soft nbd  $(H_r, D_r)$  from  $\tilde{m}$  in  $(B, D)$  so as  $((M_r, E_r) - (H_r, D_r)) \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}_r\} \subset \tilde{E}(F_r, E_r)$ . Then the intersection  $(H, D)$  of the  $(H_r, D_r)$  is a soft nbd of  $\tilde{b}$  such that  $((M, E) - (H, D)) \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}\} \subset \tilde{E}(F, E)$  and so  $M = \prod_B M_r$  is *fw-ps  $R_0$* , anywhere  $i, j = 1, 2, i \neq j$ .

The similar supposition holds for infinite fibrewise soft products provided all of the elements are fibrewise soft nonempty.

**Proposition 2.8.** Suppose that  $\phi : M \rightarrow N$  is a s-closed, s-continuous fibrewise surjection function, anywhere  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ ,  $(N, \tilde{\sigma}_1, \tilde{\sigma}_2, K)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $M$  is *fw-ps  $R_0$* , then  $N$  is so.

**Proof.** Assume that  $\tilde{n} \in \tilde{E}(N_{\tilde{b}}, K)$ , where  $\tilde{b} \in \tilde{E}(B, D)$ , and  $(O, K)$  is a  $\tilde{\tau}_i$ -soft open set of  $\tilde{n}$  in  $N$ . Pick  $\tilde{m} \in \phi^{-1}(\tilde{n})$ . Then  $(F, E) = \phi^{-1}(O, K)$  is a  $\tilde{\tau}_i$ -soft open set of  $\tilde{m}$ . Since  $M$  is *fw-ps  $R_0$*  there exists a soft nbd  $(H, D)$  of  $\tilde{b}$  such that  $M_{(H, D)} \tilde{\cap} \tilde{\tau}_j\text{-Cl}\{\tilde{m}\} \subset \tilde{E}(F, E)$ . Then  $N_{(H, D)} \tilde{\cap} \phi(\tilde{\tau}_j\text{-Cl}\{\tilde{m}\}) \subset \phi(F, E) = (O, K)$ . Since  $\phi$

is s-closed, then  $\phi(\tilde{\tau}_j - Cl\{\tilde{m}\}) = \tilde{\sigma}_j - Cl(\phi\{\tilde{m}\})$ . Therefore  $N_{(H, D)} \tilde{\cap} \tilde{\sigma}_j - Cl(\phi\{\tilde{m}\}) \subset \tilde{C}(O, K)$  and so  $K$  is  $fw$ -ps  $R_0$ , where  $i, j = 1, 2, i \neq j$ .

At this time we introduce the type of fibrewise pairwise soft Hausdorff spaces in the following.

**Definition 2.9.** The  $fw$ -sfts  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is named fibrewise pairwise soft Hausdorff (briefly  $fw$ -ps Hausdorff) where  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , and  $\tilde{b} \in \tilde{B}(B, D)$  and  $\tilde{m}_1 \neq \tilde{m}_2$ , there exists disjoint pair  $\tilde{\tau}_i$ - soft open set  $(F_i, E)$  of  $\tilde{m}_1$  and  $\tilde{\tau}_j$ - soft open  $(F_j, E)$  of  $\tilde{m}_2$  respectively in  $M$ , anywhere  $i, j = 1, 2, i \neq j$ .

**Example 2.10.** Let  $M = \{m_1, m_2, m_3\}$ ,  $B = \{a, b\}$ ,  $E = \{e_1, e_2\}$ ,  $D = \{d_1, d_2\}$  and let  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  be a  $fw$ -sfts over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Define  $f : M \rightarrow B$  and  $u : E \rightarrow D$ , such that  $f(m_1) = f(m_3) = \{a\}$ ,  $f(m_2) = \{b\}$ ,  $u(e_1) = \{d_1\}$ ,  $u(e_2) = \{d_2\}$ . Then  $\tilde{\tau}_1 = \{\tilde{\Phi}, \tilde{M}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ ,  $\tilde{\tau}_2 = \{\tilde{\Phi}, \tilde{M}, (F_6, E), (F_7, E), (F_8, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E)$  are soft sets over  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{m_1\})\}, & (F_6, E) &= \{(e_2, \{m_2\})\}, \\ (F_2, E) &= \{(e_2, \{m_3\})\}, & (F_7, E) &= \{(e_1, \{m_2\}), (e_2, \{m_1\})\}, \\ (F_3, E) &= \{(e_1, \{m_1\}), (e_2, \{m_3\})\}, & (F_8, E) &= \{(e_1, \{m_2\}), (e_2, \{m_1, m_2\})\}, \\ (F_4, E) &= \{(e_1, \{m_1, m_3\})\} \\ (F_5, E) &= \{(e_1, \{m_1, m_3\}), (e_2, \{m_3\})\} \end{aligned}$$

$\tilde{\gamma}_1 = \{\tilde{\Phi}, \tilde{B}, (L, D)\}$ ,  $\tilde{\gamma}_2 = \{\tilde{\Phi}, \tilde{B}, (O, D)\}$ ,  $(L, D) = \{(d_1, \{a\})\}$ ,  $(O, D) = \{(d_2, \{b\})\}$  and then the projection function  $P_{fu} : M \rightarrow B$  be a s-continuous, then there exists disjoint  $\tilde{\tau}_i$ - soft open  $\{(e_1, \{m_1\})\}$  of  $\tilde{m}_1$  and  $\tilde{\tau}_j$ - soft open  $\{(e_2, \{m_2\})\}$  of  $\tilde{m}_2$  such that  $\tilde{m}_1 \in \tilde{\tau}_i$  and  $\tilde{m}_2 \in \tilde{\tau}_j$ . Also,  $M$  is  $fw$ -ps Hausdorff space, where  $i, j = 1, 2, i \neq j$ .

**Remark 2.11.** If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps Hausdorff space over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  then  $(M_{B^*}, \tilde{\tau}_{1B^*}, \tilde{\tau}_{2B^*}, E_{B^*})$  is  $fw$ -ps Hausdorff over  $B^*$  for all soft subspace  $(B^*, \tilde{\gamma}_{1^*}, \tilde{\gamma}_{2^*}, D^*)$  of  $B$ . Especially the fibres pairwise soft from  $M$  is Hausdorff spaces.

On the other hand a  $fw$ -sfts with pairwise soft Hausdorff fibres is not essentially pairwise soft Hausdorff :

**Example 2.12.** Assume that  $M = \{m_1, m_2, m_3\}$ ,  $B = \{a, b\}$ ,  $E = \{e_1, e_2\}$ ,  $D = \{d_1, d_2\}$  and let  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  be a  $fw$ -sfts over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Define  $f : M \rightarrow B$  and  $u : E \rightarrow D$ , such that  $f(m_1) = \{a\}$ ,  $f(m_2) = f(m_3) = \{b\}$ ,  $u(e_1) = \{d_1\}$ ,  $u(e_2) = \{d_2\}$ . Then  $\tilde{\tau}_1 = \{\tilde{\Phi}, \tilde{M}, (F_1, E), (F_2, E)\}$ ,  $\tilde{\tau}_2 = \{\tilde{\Phi}, \tilde{M}, (F_3, E), (F_4, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$  are soft sets over  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{m_2\}), (e_2, \{m_1, m_2\})\}, & (F_3, E) &= \{(e_1, \{m_1\}), (e_2, \{m_1, m_3\})\}, & (F_2, E) &= \\ & \{(e_1, \{m_1\}), (e_2, \{m_1\})\} & (F_4, E) &= \{(e_1, \{m_1\}), (e_2, M)\}, \end{aligned}$$

$\tilde{\gamma}_1 = \{\tilde{\Phi}, \tilde{B}, (L, D)\}$ ,  $\tilde{\gamma}_2 = \{\tilde{\Phi}, \tilde{B}, (O, D)\}$ ,  $(L, D) = \{(d_1, \{a\}), (d_2, \{a\})\}$  and  $(O, D) = \{(d_1, \{a\}), (d_2, M)\}$  then the projection function  $P_{fu} : M \rightarrow B$  is a s-continuous. Since we have :  $M_{\tilde{b}} = \{\tilde{m}_2, \tilde{m}_3\}$ ,  $\tilde{\tau}_{1M_{\tilde{b}}} = \{\tilde{\Phi}, \tilde{M}_{\tilde{b}}, \{(e_2, \{m_2\})\}\}$ ,  $\tilde{\tau}_{2M_{\tilde{b}}} = \{\tilde{\Phi}, \tilde{M}_{\tilde{b}}, \{(e_2, \{m_3\})\}, \{(e_2, \{m_2, m_3\})\}\}$ . Then there is  $\tilde{\tau}_{1M_{\tilde{b}}}$ -soft open set  $\{(e_2, \{m_2\})\}$  where  $\tilde{m}_2 \in \tilde{\tau}_{1M_{\tilde{b}}}$ ,  $\tilde{m}_3 \notin \tilde{\tau}_{1M_{\tilde{b}}}$  and there exists  $\tilde{\tau}_{2M_{\tilde{b}}}$ -soft open set  $\{(e_2, \{m_3\})\}$  where  $\tilde{m}_3 \in \tilde{\tau}_{2M_{\tilde{b}}}$ ,  $\tilde{m}_2 \notin \tilde{\tau}_{2M_{\tilde{b}}}$ . Since we have:  $M_{\tilde{b}} = \{\tilde{m}_2, \tilde{m}_3\}$  is  $fw$ -ps Hausdorff. But  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is not  $fw$ -ps Hausdorff since  $\tilde{m}_2$  and  $\tilde{m}_3 \in \tilde{M}_{\tilde{b}}, \tilde{m}_2 \neq \tilde{m}_3$  and there exists no disjoint  $\tilde{\tau}_i$ - soft open sets of  $\tilde{m}_2$  and  $\tilde{m}_3$ .



**Proposition 2.13.** The  $fw$ - $sfts$   $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is  $fw$ - $ps$  Hausdorff iff the soft diagonal embedding  $\Delta : M \rightarrow M \times_B M$  is  $\tilde{\tau}_i \times_B \tilde{\tau}_i$   $s$ -closed .

**Proof.** ( $\Rightarrow$ ) Assume that  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ . Since  $\Delta(M)$  is  $\tilde{\tau}_i \times_B \tilde{\tau}_i$   $s$ -closed in  $M \times_B M$ , then  $(\tilde{m}_1, \tilde{m}_2)$ , existence a point of the soft complement, confesses a fibrewise product  $\tilde{\tau}_i \times_B \tilde{\tau}_j$  soft open set  $(F_1, E) \times_B (F_2, E)$  which do not give  $\Delta(M)$ , also then  $(F_1, E)$  is  $\tilde{\tau}_i$ -soft open of  $\tilde{m}_1$  and  $(F_2, E)$  is  $\tilde{\tau}_j$ -soft open set of  $\tilde{m}_2$ , where  $i, j = 1, 2, i \neq j$ .

( $\Leftarrow$ ) The reverse way is like.

The soft subspaces of  $fw$ - $ps$  Hausdorff spaces are  $fw$ - $ps$  Hausdorff spaces, In fact we have.

**Proposition 2.14** Assume that  $\phi: M \rightarrow M^*$  is a  $s$ -continuous embedding fibrewise function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are  $fw$ - $sfts$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $M^*$  is  $fw$ - $ps$  Hausdorff, then is so  $M$ .

**Proof.** Assume that  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ . Then  $\phi(\tilde{m}_1), \phi(\tilde{m}_2) \in \tilde{M}_{\tilde{b}^*}, E^*$  are distinct, since  $M^*$  is  $fw$ - $ps$  Hausdorff, then we have a  $\tilde{\tau}_i^*$ -soft open sets  $(F_1^*, E^*)$  of  $\phi(\tilde{m}_1)$  and  $\tilde{\tau}_j^*$ -soft open  $(F_2^*, E^*)$  of  $\phi(\tilde{m}_2)$  in  $M^*$  which are disjoint. Because  $\phi$  is  $s$ -continuous, the inverse images  $\phi^{-1}(F_1^*, E^*) = (F_1, E), \phi^{-1}(F_2^*, E^*) = (F_2, E)$  such that  $(F_1, E)$  is  $\tilde{\tau}_i$ -soft open set of  $\tilde{m}_1$  and  $(F_2, E)$  is  $\tilde{\tau}_j$ -soft open set of  $\tilde{m}_2$  in  $M$  which are disjoint therefore  $M$  is  $fw$ - $ps$  Hausdorff, where  $i, j = 1, 2, i \neq j$ .

**Proposition 2.15.** Assume that  $\phi: M \rightarrow N$  is a  $s$ -continuous fibrewise function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(N, \tilde{\sigma}_1, \tilde{\sigma}_2, K)$  are  $fw$ - $sfts$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $N$  is  $fw$ - $ps$  Hausdorff, now the fibrewise soft graph  $G : M \rightarrow M \times_B N$  of  $\phi$  is a  $\tilde{\tau}_i \times_B \tilde{\sigma}_j$  - soft closed embedding.

**Proof.** The fibrewise soft graph is clear in the similar method as the usual soft graph, on the other hand with standards in the fibrewise soft product, so as to the figure presented below is commutative.

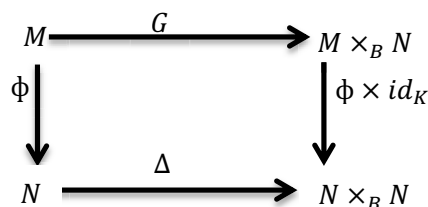


Fig. (1): Diagram of Proposition (2.15).

Since  $\Delta(N)$  is  $\sigma_i \times_B \sigma_i$ -soft closed in  $N \times_B N$  by Proposition (2. 13), so  $G(M) = (\phi \times id_K)^{-1}(\Delta(N))$  is  $\tau_i \times_B \sigma_j$  - soft closed in  $M \times_B N$ , where  $i, j = 1, 2, i \neq j$ .

The class from  $fw$ - $ps$  Hausdorff spaces is multiplicative, in the following sense.

**Proposition 2.16.** Assume that  $\{M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r\}$  is a family of  $fw$ - $ps$  Hausdorff spaces over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Then the fibrewise soft bitopological product  $M = \prod_B M_r$  is  $fw$ - $ps$  Hausdorff .

**Proof.** Assume that  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ . Then  $\pi_r(\tilde{m}_1) = \tilde{m}_{1r} \neq \pi_r(\tilde{m}_2) = \tilde{m}_{2r}$  for particular directory  $r$ . Since  $M_r$  is  $fw$ - $ps$  Hausdorff, then there is a  $\tau_{ir}$  - soft open set  $(F_{1r}, E_r)$ , of  $\tilde{m}_{1r}$  and  $\tau_{jr}$  -soft open  $(F_{2r}, E_r)$  of  $\tilde{m}_{2r}$  in  $M_r$  which are soft disjoint. Since  $\pi_r$  is  $s$ -continuous, then the inverse

images  $\pi_r^{-1}(F_{1r}, E_r), \pi_r^{-1}(F_{2r}, E_r)$  are disjoint  $\tilde{\tau}_i$ -soft open and  $\tilde{\tau}_i$ -soft open sets of  $\tilde{m}_1, \tilde{m}_2$  in  $M$  where  $i, j=1, 2, i \neq j$ .

The pairwise soft functionally type of the fibrewise pairwise soft Hausdorff axiom is sturdier than the non pairwise soft functional type on the other hand its features are fairly like. Here and away we usage the closed unit interval  $[0, 1]$  in the real line.

**Definition 2.17.** The *fw-sbts*  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is called fibrewise pairwise soft functionally (briefly, *fw-ps functionally*) Hausdorff if whenever  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$ , where  $\tilde{b} \in \tilde{B}, D$  and  $\tilde{m}_1 \neq \tilde{m}_2$ , there exists a soft nbd  $(H, D)$  of  $\tilde{b}$  and disjoint  $\tilde{\tau}_i$ -soft open and  $\tilde{\tau}_j$ -soft open sets  $(F, E), (V, E)$  of  $\tilde{m}_1, \tilde{m}_2$  in  $M$  and a continuous function  $\lambda : M_{(H,D)} \rightarrow [0,1]$  such that  $M_{\tilde{b}} \tilde{\cap} (F,E) \subseteq \lambda^{-1}(0), M_{\tilde{b}} \tilde{\cap} (V, E) \subseteq \lambda^{-1}(1)$ , where  $i, j=1, 2, i \neq j$ .

**Example 2.18.** Assume that  $M = \{m_1, m_2, m_3\}, B = \{a, b\}, E = \{e\}, G = \{d\}$  and let  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  be a *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Define  $f : M \rightarrow B$  and  $u : E \rightarrow D$ , such that  $f(m_1)=\{a\}, f(m_2) = f(m_3)=\{b\}, u(e)=\{d\}$ . Then  $\tilde{\tau}_1 = \{\tilde{\Phi}, \tilde{M}, (F_1, E), (F_2, E), (F_3, E)\}, \tilde{\tau}_2 = \{\tilde{\Phi}, \tilde{M}, (F_4, E), (F_5, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$  are soft sets over  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e, \{h_2\})\}, & (F_4, E) &= \{(e, \{h_2, h_3\})\}, \\ (F_2, E) &= \{(e, \{h_1, h_2\})\}, & (F_5, E) &= \{(e, \{h_3\})\}, \\ (F_3, E) &= \{(e, \{h_1\})\} \end{aligned}$$

$\tilde{\gamma}_1 = \{\tilde{\Phi}, \tilde{B}, (L, D)\}, \tilde{\gamma}_2 = \{\tilde{\Phi}, \tilde{B}, (O, D)\}, (L, D) = \{(d, \{a\})\}$  and  $(O, D) = \{(d, \{b\})\}$ . It clear that  $M_{(H,D)} = \{(e, \{h_2, h_3\})\}$ , then the projection function  $P_{ju} : M \rightarrow B$  is a s-continuous. Let  $\lambda : M_{(H,D)} \rightarrow [0,1]$  where  $\lambda(\tilde{m}_2) = 0, \lambda(\tilde{m}_3) = 1, \tilde{\tau}_{1M_{(H,D)}} = \{\tilde{\Phi}, \tilde{M}_{(H,D)}, \{(e, \{m_2\})\}\}, \tilde{\tau}_{2M_{(H,D)}} = \{\tilde{\Phi}, \tilde{M}_{(H,D)}, \{(e, \{m_3\})\}\}$ , then  $\lambda$  is continuous function, and there exist disjoint  $\tilde{\tau}_i$ -soft open  $\{(e, \{m_1\})\}$  of  $\tilde{m}_1$  and  $\tilde{\tau}_j$ -soft open  $\{(e, \{m_2\})\}$  of  $\tilde{m}_2$  such that  $M_{\tilde{b}} \tilde{\cap} \{(e, \{m_2\})\} \subseteq \lambda^{-1}(0), M_{\tilde{b}} \tilde{\cap} \{(e, \{m_3\})\} \subseteq \lambda^{-1}(1)$ . Also,  $M$  is *fw-ps functionally* Hausdorff space, where  $i, j = 1, 2, i \neq j$ .

**Remark 2.19.** If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps functionally* Hausdorff space over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ , then  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*) = (M_{B^*}, \tilde{\tau}_{1B^*}, \tilde{\tau}_{2B^*}, E_{B^*})$  is *fw-ps functionally* Hausdorff over  $B^*$  for all soft subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $B$ . Especially the fibres soft of  $M$  are pairwise soft functionally Hausdorff spaces.

The soft subspaces of *fw-ps functionally* Hausdorff spaces are *fw-ps functionally* Hausdorff spaces. We have actually.

**Proposition 2.20.** suppose that  $\phi : M \rightarrow M^*$  is a s-continuous embedding fibrewise function, anywhere  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $M^*$  is *fw-ps functionally* Hausdorff, then is so  $M$

**Proof.** Let  $\tilde{m}_1, \tilde{m}_2 \in \tilde{M}_{\tilde{b}}, E$  and  $\tilde{m}_1 \neq \tilde{m}_2; \tilde{b} \in \tilde{B}, D$ . Then  $\phi(\tilde{m}_1) = \tilde{m}_1^*, \phi(\tilde{m}_2) = \tilde{m}_2^* \in (M_{\tilde{b}}^*, E^*), \tilde{m}_1^* \neq \tilde{m}_2^*$ . Since  $M^*$  is *fw-ps functionally* Hausdorff, then we have a nbd  $(H, D)$  of  $\tilde{b}$  in  $B$  and disjoint pair of  $\tau_i^*$ -soft open set  $(F_1^*, E^*)$  of  $\tilde{m}_1^*$  and  $\tau_j^*$ -soft open set  $(F_2^*, E^*)$  of  $\tilde{m}_2^*$  and a continuous function  $\lambda^* : M_{(M,D)}^* \rightarrow [0, 1]$  where  $M_{\tilde{b}}^* \tilde{\cap} (F_1^*, E^*) \subseteq (\lambda^*)^{-1}(0)$  and  $M_{\tilde{b}}^* \tilde{\cap} (F_2^*, E^*) \subseteq (\lambda^*)^{-1}(1)$ . Now, since  $\phi$  is s-continuous,  $\phi^{-1}(F_1^*, E^*) = (F_1, E)$  and  $\phi^{-1}(F_2^*, E^*) = (F_2, E)$  are disjoint pair of  $\tau_i$ -soft open set of  $\tilde{m}_1$  and  $\tau_j$ -soft open set of  $\tilde{m}_2$  respectively and the continuous function  $\lambda$  where  $\lambda = \lambda^* \circ \phi : M_{(N,D)} \rightarrow [0,1]$  such that  $M_{\tilde{b}} \tilde{\cap} (F_1, E) \subseteq \lambda^{-1}(0)$  and  $M_{\tilde{b}} \tilde{\cap} (F_2, E) \subseteq \lambda^{-1}(1)$ , where  $i, j = 1, 2, i \neq j$ .

Furthermore the class of  $fw$ -ps functionally Hausdorff spaces is multiplicative, as the following.

**Proposition 2.21.** Suppose that  $\{M_r, \tilde{\tau}_{r1}, \tilde{\tau}_{r2}, E_r\}$  is a set of  $fw$ -ps functionally Hausdorff spaces over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Now the fibrewise soft bitopological product  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E) = \prod_B(M_r, \tilde{\tau}_{r1}, \tilde{\tau}_{r2}, E)$  with the family of fibrewise soft projection  $\pi_r: M = \prod_B M_r \rightarrow M_r$  is  $fw$ -ps functionally Hausdorff.

**Proof.** Let  $\tilde{m}_1, \tilde{m}_2 \in (M_{\tilde{b}}, E); \tilde{b} \in (B, D)$ , and  $\tilde{m}_1 \neq \tilde{m}_2$ . Then  $\pi_r(\tilde{m}_1) = \tilde{m}_{r1}, \pi_r(\tilde{m}_2) = \tilde{m}_{r2} \in (M_r)_{\tilde{b}}, E_r$  for some index  $r$  where  $\tilde{m}_{r1} \neq \tilde{m}_{r2}$ . Since  $M_r$  is  $fw$ -ps functionally Hausdorff, then we have a soft nbd  $(H_r, D_r)$  of  $\tilde{b}$  in  $B$  and disjoint pair of  $\tau_{ij}$ - soft open set  $(F_{r1}, E_r)$  of  $\tilde{m}_{r1}$ , and  $\tau_{jr}$ - soft open set  $(F_{r2}, E_r)$  of  $\tilde{m}_{r2}$  and a continuous function  $\lambda : M_{r(H_r, D_r)} \rightarrow [0,1]$  such that  $(M_r)_{\tilde{b}} \tilde{\cap}(F_{r1}, E_r) \subset \tilde{\lambda}^{-1}(0)$  and  $(M_r)_{\tilde{b}} \tilde{\cap}(F_{r2}, E_r) \subset \tilde{\lambda}^{-1}(1)$ . Now the intersection of  $(H_r, D_r)$  is a soft nbd  $(H, D)$  of  $\tilde{b}$  in  $B$ , and since  $\pi_r$  is s-continuous, then  $\pi_r^{-1}(F_{r1}, E_r) = (F_1, E)$  and  $\pi_r^{-1}(F_{r2}, E_r) = (F_2, E)$  are disjoint pair of  $\tau_i$ - soft open set of  $\tilde{m}_1$  and  $\tau_j$ - soft open set of  $\tilde{m}_2$  respectively and the continuous function  $\Omega$  where  $\Omega = \lambda \circ \pi_r : M_{(H, D)} \rightarrow [0,1]$  where  $M_{\tilde{b}} \tilde{\cap}(F_1, E) \subset \tilde{\Omega}^{-1}(0)$  and  $M_{\tilde{b}} \tilde{\cap}(F_2, E) \subset \tilde{\Omega}^{-1}(1)$  where  $i, j=1, 2, i \neq j$ .

### 3. Fibrewise pairwise Soft bitopological Regular and pairwise Soft Normal Spaces

Now we proceed to consider the fibrewise soft types of the higher soft near separation axioms, beginning with fibrewise soft near regularity and fibrewise soft near completely regularity.

**Definition 3.1.** The  $fw$ -sfts  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is named fibrewise pairwise soft regular (briefly  $fw$ -ps regular) space if for all point  $\tilde{m} \in (M_{\tilde{b}}, E); \tilde{b} \in (B, D)$ , and for all  $\tilde{\tau}_i$ - soft open set  $(F, E)$  of  $\tilde{m}$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , we have a  $\gamma_i$ - soft nbd  $(N, D)$  of  $b$  in  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  and a  $\tilde{\tau}_i$ - soft open set  $(V, E)$  of  $\tilde{m}$  in  $(M_{(N, D)}, \tau_{1(N, D)}, \tau_{2(N, D)}, E_{(N, D)})$  such that the  $\tilde{\tau}_i$ - closure soft of  $(F, E)$  in  $M_{(N, D)}$  is contained in  $(V, E)$  (i.e.,  $M_{(N, D)} \tilde{\cap} \tau_i - Cl(F, E) \subset (V, E)$ ), .

Such as, the trivial fibrewise soft bitopological spaces with soft regular fibre are fibrewise pairwise soft bitopological regular space.

#### Remark 3.2.

- (a) The nbds of  $m$  are given by a fibrewise soft basis it is enough if the condition in Definition (3.1) is satisfied for each fibrewise soft basic nbds.
- (b) If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps regular space over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, G)$  then  $(M_{B^*}, \tau_{1B^*}, \tau_{2B^*}, E_{B^*})$  is  $fw$ -ps regular space over  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  for all subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, G)$ .

Soft subspaces of  $fw$ -ps regular space spaces are fibrewise pairwise soft bitopological regular space, where  $i=1, 2$ . Actually we have.

**Proposition 3.3.** Suppose  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is a *fw-ps* irresolute embedding function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are *fw-ps* bitopological spaces on  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$ . If  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is *fw-ps* regular then  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  so is.

**Proof.** Let  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ , and  $(F, E)$  be a  $\tilde{\tau}_i$ -soft open set of  $\tilde{m}$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Then  $(F, E) = \phi^{-1}(F^*, E^*)$ , where  $(F^*, E^*)$  is a  $\tilde{\tau}_i^*$ -soft open set of  $\tilde{m}^* = \phi(\tilde{m})$  in  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$ . Since  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is *fw-ps* regular, there is a  $\tilde{\gamma}_i$ -soft nbd  $(N, D)$  of  $b$  in  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  and a  $\tilde{\tau}_i^*$ -soft open set  $(V^*, E^*)$  of  $\tilde{m}^*$  in  $M_{(N,D)}^*$  such that  $M_{(N,D)}^* \tilde{\cap} \tilde{\tau}_i - Cl(V^*, E^*) \subseteq (F^*, E^*)$ . Then  $(V, E) = \phi^{-1}(V^*, E^*)$  is a  $\tilde{\tau}_i$ -soft opens set of  $\tilde{m}$  in  $M_{(N,D)}$  such that  $M_{(N,D)} \tilde{\cap} \tilde{\tau}_i - Cl(V, E) \subseteq (F, E)$ , and so  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* regular, where  $i = 1, 2$ , as required.

Class of *fw-ps* regular spaces is fibrewise soft multiplicative;  $i = 1, 2$ , in the next.

**Proposition 3.4.** Suppose  $\{(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)\}$  be a finite family of *fw-ps* regular spaces over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . So, the *fw-ps* regular product  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E) = \prod_B(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)$  is *fw-ps* regular.

**Proof.** Let  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ . Consider a  $\tilde{\tau}_i$ -soft open set  $(F, E) = \prod_B(F_r, E_r)$  of  $m$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , where  $(F, E)$  is a  $\tilde{\tau}_i$ -soft open set of  $\pi_r(m) = m_r$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  for each index  $r$ . Since  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* regular there is a  $\tilde{\gamma}_i$ -soft nbd  $(N_r, D_r)$  of  $b$  in  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  and a  $\tilde{\tau}_i$ -soft open set  $(V_r, E_r)$  of  $m_r$  in  $(M_r, E_r) - (N_r, D_r)$  where, the closure soft  $((M_r, E_r) - (N_r, D_r)) \tilde{\cap} \tilde{\tau}_i - Cl(V_r, E_r)$  of  $(V_r, E_r)$  in  $((M_r, E_r) - (N_r, D_r))$  is contained in  $(F_r, E_r)$ . Therefore the intersection  $(N, D)$  of the  $(N_r, D_r)$  is a  $\tilde{\tau}_i$ -soft nbd of  $b$  and  $(V, E) = \prod_B(V_r, E_r)$  is a  $\tilde{\tau}_i$ -soft open set of  $m$  in  $M_{(N,D)}$  where the  $\tilde{\tau}_i$ -closure soft  $M_{(N,D)} \tilde{\cap} \tilde{\tau}_i - Cl(V, D)$  of  $(V, E)$  in  $(M_{(N,D)}, \tau_{(N,D)}, E_{(N,D)})$  is contained in  $(F, E)$ , and so  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E) = \prod_B(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)$  is *fw-ps* regular.

The similar conclusion embraces for infinite fibrewise soft products provided all of the factors is fibrewise soft non-empty.

**Proposition 3.5.** Suppose  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  be a soft open, soft closed and soft irresolute fibrewise surjection function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Then  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* regular iff  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  so is.

**Proof.**  $(\Rightarrow)$  Let  $\tilde{k} \in (K_{\tilde{b}}, L)$ ;  $\tilde{b} \in (B, D)$ , and let  $(U, L)$  be a  $\sigma_i$ -soft open set of  $k$  in  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$ . Pick  $m \in \phi^{-1}(k)$ . Then  $(F, E) = \phi^{-1}(U, L)$  is a  $\tilde{\tau}_i$ -soft open set of  $m$ . Since  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* regular there is a  $\tilde{\gamma}_i$ -soft nbd  $(N, D)$  of  $b$  and a  $\tilde{\tau}_i$ -soft open set  $(F^*, E^*)$  of  $m$  where  $M_{(N,D)} \tilde{\cap} \tilde{\tau}_i - Cl(F^*, E^*) \subseteq (F, E)$ . Then  $K_{(N,D)} \tilde{\cap} \phi(\tilde{\tau}_i - Cl(F^*, E^*)) \subseteq \phi(F, E) = (U, L)$ . Since  $\phi$  is soft closed, then  $\phi(\tilde{\tau}_i - Cl(F^*, E^*)) = \sigma_i Cl(\phi(F^*, E^*))$  and since  $\phi$  is soft open, then  $\phi(F^*, E^*)$  is a soft open set of  $k$ . Thus  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  is *fw-ps* regular as asserted.

( $\Leftarrow$ ) Let  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ , and let  $(F, E)$  be a  $\tilde{\tau}_i$ -soft open set of  $m$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Pick  $k \in \phi(m)$ . Then  $\phi(F, E) = (U, L)$  is a  $\tilde{\tau}_i$ -soft open set of  $k$ . Since  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  is  $fw$ -ps regular there is a  $\tilde{\tau}_i$ -soft nbd  $(N, D)$  of  $b$  and a  $\tilde{\tau}_i$ -soft open set  $(U^*, L^*)$  of  $k$  where  $K_{(N,D)} \cap \tilde{\tau}_i - Cl(U^*, L^*) \subset (U, L)$ . Then  $M_{(N,D)} \cap \tilde{\tau}_i - \phi^{-1}(Cl(U^*, L^*)) \subset \phi^{-1}(U, L) = (F, E)$ . Since  $\phi$  is closed, then  $\phi^{-1}(Cl(U^*, L^*)) = \tilde{\sigma}_i - Cl(\phi^{-1}(U^*, L^*))$  and since  $\phi$  is soft irresolute, then  $\phi^{-1}(U^*, L^*)$  is a soft open set of  $h$ . Thus  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps regular.

The soft functionally type of the fibrewise soft regularity axiom is stronger than the non soft functionally type but its properties are equally like. In the ordinary theory the term completely regular is always used instead of functionally regular and we spread this usage to the fibrewise soft theory.

**Definition 3.6.** The  $fw$ -sfts  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is named fibrewise pairwise soft completely regular (briefly,  $fw$ -ps completely regular) if for all point  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ , and for all  $\tilde{\tau}_i$ -soft open set  $(F, E)$  of  $m$  there exists a  $\tilde{\gamma}_i$ -soft nbd  $(N, D)$  of  $b$  and a  $\tilde{\tau}_j$ -soft open set  $(V, E)$  of  $m$  in  $M_{(N,D)}$  and a continuous function  $\lambda : M_{(N,D)} \rightarrow I$  where that  $M_b \cap (V, E) \subset \lambda^{-1}(0)$  and  $M_{(N,D)} \cap (M_{(N,D)}, E_{(N,D)}) - (F, E) \subset \lambda^{-1}(1)$ , where  $i, j = 1, 2, i \neq j$ .

For example,  $B \times T$  is  $fw$ -ps completely regular space for all  $fw$ -ps completely regular spaces  $(T, \tilde{\tau}_1, \tilde{\tau}_2, E)$ .

**Remark 3.7.**

- (a) The soft nbds of  $m$  are given by a fibrewise soft basis it is sufficient if the condition in Definition (3.6) is satisfied for every fibrewise soft basic nbds.
- (b) If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps completely regular space on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  then  $(M_{B^*}, \tau_{1B^*}, \tau_{2B^*}, E_{B^*})$  is  $fw$ -ps completely regular space on  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  for all subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ .

Soft subspaces of  $fw$ -ps completely regular spaces are  $fw$ -ps completely regular spaces. Actually we have.

**Proposition 3.8.** Suppose  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is a  $fw$ -sfts irresolute embedding ;  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are  $fw$ -sfts over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is  $fw$ -ps completely regular then so is  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ .

**Proof.** The proof is like to the proof of Proposition (3.3).

The class of  $fw$ -ps completely regular spaces is finitely multiplicative, as the next.

**Proposition 3.9.** Suppose  $\{(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)\}$  is a finite family of *fw-ps* completely regular spaces on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . The *fw-sbts* product  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E) = \prod_B(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)$  is *fw-ps* completely regular.

**Proof.** Let  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ . Consider a *fw-ps*  $\tilde{\tau}_i$ -open set  $\prod_B(F_r, E_r)$  of  $m$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ , where  $(F_r, E_r)$  is a  $\tilde{\tau}_{ir}$ -soft open set of  $\pi_r(m) = m_r$  in  $\{(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)\}$  for each index  $r$ . Since  $\{(M_r, \tilde{\tau}_{1r}, \tilde{\tau}_{2r}, E_r)\}$  is *fw-ps* completely regular we have a  $\tilde{\gamma}_i$ -soft nbd  $(N_r, D_r)$  of  $b$  and a  $\tilde{\tau}_j$ -soft open set  $(V, E)$  of  $m_r$  in  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and a continuous function  $\lambda_r : M_{(N_r, D_r)} \rightarrow I$ ;  $(M_r)_b \tilde{\cap} (V, E) \tilde{\subset} \lambda_r^{-1}(0)$  and  $M_{(N_r, D_r)} \tilde{\cap} (M_{(N_r, D_r)} - (F_r, E_r)) \tilde{\subset} \lambda_r^{-1}(1)$ . So the intersection  $(N, D)$  of the  $(N_r, D_r)$  is a  $\gamma_i$ -soft nbd of  $b$  and  $\lambda : M_{(N, D)} \rightarrow I$  is a continuous function where:

$$\lambda(\xi) = \inf_{r=1, 2, \dots, n} \{\lambda_r(\xi_r)\} \text{ for } \xi = (\xi_r) \in M_{(N, D)}.$$

Since  $M_b \tilde{\cap} \pi_r^{-1}(V, E) \tilde{\subset} \pi_r^{-1}[(M_r)_b \tilde{\cap} (V, E)] \tilde{\subset} \pi_r^{-1}(\lambda_r^{-1}(0)) = (\lambda_r \circ \pi_r)^{-1}$  and  $M_{(N, D)}$

$$\pi_r^{-1}(M_{(N_r, D_r)}, E_{(N_r, D_r)}) - (F_r, E_r) \tilde{\subset} \pi_r^{-1}[M_{(N_r, D_r)} \tilde{\cap} (M_{(N_r, D_r)}, E_{(N_r, D_r)}) - (F_r, E_r)] \tilde{\subset} \pi_r^{-1}(\lambda_r^{-1}(1)) = (\pi_r \circ \lambda_r)^{-1}(1), \text{ where } i, j = 1, 2, i \neq j \text{ this proves the result.}$$

**Lemma 3.10.** Let  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  be a soft open and soft closed fibrewise surjection function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . Let  $C : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (\mathbb{R}, \tilde{\vartheta}_1, \tilde{\vartheta}_2, O)$  be a soft continuous real-valued function which is fibrewise soft bound above, in the sense that  $C$  is bounded above on each fiber soft of  $M$ . Then  $\varphi : (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L) \rightarrow (\mathbb{R}, \tilde{\vartheta}_1, \tilde{\vartheta}_2, O)$  is soft continuous, where

$$\varphi(\mu) = \text{Sup}_{\zeta \in \phi^{-1}(\mu)} C(\zeta).$$

The similar conclusion holds for infinite fibrewise soft products provided that all of the factors is fibrewise soft non-empty.

**Proposition 3.11.** Suppose  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  is a soft open, soft closed and soft irresolute fibrewise surjection function;  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  are *fw-sbts* on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* completely regular then  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  is so.

**Proof.** Let  $\tilde{k} \in (K_{\tilde{b}}, L)$ ;  $\tilde{b} \in (B, D)$ , and let  $(U_k, L_k)$  be a  $\tilde{\sigma}_i$ -soft open set of  $k$ . Pick  $\tilde{m} \in (M_{\tilde{b}}, E)$ ;  $\tilde{b} \in (B, D)$ , so that  $(F_m, E_m) = \phi^{-1}(U_k, L_k)$  is a  $\tilde{\tau}_i$ -soft open set of  $m$ . Since  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* completely regular we have a  $\tilde{\gamma}_i$ -soft nbd  $(N, D)$  of  $b$  and a  $\tilde{\tau}_i$ -soft open set  $(V_m, E_m)$  of  $m$  in  $M_{(N, D)}$  and a continuous function  $\lambda : M_{(N, D)} \rightarrow I$  where  $M_b \tilde{\cap} (V_m, E_m) \tilde{\subset} \lambda^{-1}(0)$  and  $M_{(N, D)} \tilde{\cap} (M_{(N, D)} - (F_m, E_m)) \tilde{\subset} \lambda^{-1}(1)$ . Using Lemma (3.10) we get a continuous function  $\omega : K_{(N, D)} \rightarrow I$  where  $K_b \tilde{\cap} (M_k, L_k) \tilde{\subset} \omega^{-1}(0)$  and  $K_{(N, D)} \tilde{\cap} (K_{(N, D)}, L_{(N, D)}) - (U_k, L_k) \tilde{\subset} \omega^{-1}(1)$ , where  $i, j = 1, 2, i \neq j$ .

At this time we present the type of fibrewise soft near normal space as next.

**Definition 3.12.** The  $fw$ -sfts  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is named fibrewise pairwise soft normal (briefly,  $fw$ -ps normal) if for all point  $\tilde{b} \in (B, D)$  and each disjoint pair  $\tilde{\tau}_i - (C, E)$ , and  $\tilde{\tau}_j - (S, E)$  of soft closed sets of  $M$ , there exists a  $\tilde{\gamma}_i$  soft nbd  $(N, D)$  of  $b$  and a disjoint pair of  $\tilde{\tau}_i$ - soft open sets  $(F, E)$ , and  $\tilde{\tau}_j$ -soft open set  $(V, E)$  of  $M_{(N,D)} \tilde{\cap} (C, E), M_{(N,D)} \tilde{\cap} (S, E)$  in  $M_{(N,D)}$ , where  $i, j = 1, 2, i \neq j$ .

**Remark 3.13.** If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps normal space on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  then  $(M_{B^*}, \tau_{1B^*}, \tau_{2B^*}, E_{B^*})$  is  $fw$ -ps normal space on  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  for all soft subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$

Soft closed subspaces of  $fw$ -ps normal spaces are  $fw - ps$ normal. Actually we have.

**Proposition 3.14.** Suppose  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is a soft closed  $fw$ -ps irresolute embedding ;  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are  $fw$ -sfts over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is  $fw$ -ps normal then so is  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ .

**Proof.** Let  $\tilde{b} \in (B, D)$  and let  $\tilde{\tau}_i - (C, E)$  and  $\tilde{\tau}_j - (S, E)$  be disjoint pair of soft closed sets of  $M$ . Then  $\phi(C, E), \phi(S, E)$  are disjoint pair of  $\tilde{\tau}_i^*$ - and  $\tilde{\tau}_j^*$ - soft closed sets of  $M^*$ . Because of  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is  $fw$ -ps normal we have a  $\tilde{\gamma}_i$ - soft nbd  $(N, D)$  of  $b$  and a disjoint pair of  $\tilde{\tau}_i^*$ - and  $\tilde{\tau}_j^*$ - soft open sets  $(V, E), (U, E)$  of  $M_{(N,D)}^* \tilde{\cap} \phi(C, E), M_{(N,D)}^* \tilde{\cap} \phi(S, E)$  in  $M_{(N,D)}^*$ . Then  $\phi^{-1}(V, E)$  and  $\phi^{-1}(U, E)$  are disjoint pair of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ - soft open sets of  $M_{(N,D)} \tilde{\cap} (C, E), M_{(N,D)} \tilde{\cap} (S, E)$  in  $M_{(N,D)}$ .

**Definition 3.15.** A soft mapping  $P_{fu} : (M, E) \rightarrow (B, D)$  is called soft- biclosed (briefly, S-biclosed) map if the image of every S-closed set in  $M$  is S-closed set in  $K$ .

**Proposition 3.16.** Let  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  be a soft- biclosed continuous fibrewise surjection function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  are  $fw$ -sfts over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps normal then so is  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$ .

**Proof.** Let  $\tilde{b} \in (B, D)$  and let  $(C, L), (S, L)$  be disjoint pair of  $\tilde{\sigma}_i$ - and  $\tilde{\sigma}_j$ - soft closed sets of  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  . Then  $\phi^{-1}(C, L), \phi^{-1}(S, L)$  are disjoint pair of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ - soft closed sets of  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Since  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is  $fw$ -ps normal there exists a  $\tilde{\gamma}_i$ - soft nbd  $(N, D)$  of  $b$  and a pair of disjoint of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ - soft open sets  $(F, E), (U, E)$  of  $M_{(N,D)} \tilde{\cap} \phi^{-1}(C, L)$  and  $M_{(N,D)} \tilde{\cap} \phi^{-1}(S, L)$ . Since  $\phi$  is soft biclosed the sets  $K_{(N,D)} - \phi(M_{(N,D)}, E_{(N,D)}) - (F, E), (K_{(N,D)}, L_{(N,D)}) - \phi(M_{(N,D)}, E_{(N,D)}) - (U, E)$  are  $\tilde{\sigma}_i$ - soft open in

$(K_{(N,D)}, \sigma_{1(N,D)}, \sigma_{2(N,D)}, L_{(N,D)})$ , and form a disjoint pair of a  $\tilde{\sigma}_i$ - and  $\tilde{\sigma}_i$ - soft open sets of  $K_{(N,G)} \tilde{\cap} (C, E)$ ,  $K_{(N,G)} \tilde{\cap} (S, E)$  in  $(K_{(N,D)}, \sigma_{1(N,D)}, \sigma_{2(N,D)}, L_{(N,D)})$ , as required.

Lastly, we present the type of fibrewise soft near functionally normal space in the next.

**Definition 3.17.** The *fw-sbts*  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  on  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  is named fibrewise pairwise functionally (briefly, *fw-ps* functionally) normal if for all point  $\tilde{b} \tilde{\in} (B, D)$  and all pair of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ -  $(C, E), (S, E)$  of disjoint soft closed sets of  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  there exists a  $\tilde{\gamma}_i$  soft nbd  $(N, D)$  of  $b$  and a pair of disjoint  $\tilde{\tau}_j$ - and  $\tilde{\tau}_i$ - soft open sets  $(F, E), (U, E)$  and a continuous function  $\lambda : M_{(N,D)} \rightarrow I$  where  $M_{(N,D)} \tilde{\cap} (C, E) \tilde{\cap} (F, E) \tilde{\subset} \lambda^{-1}(0)$  and  $M_{(N,D)} \tilde{\cap} (S, E) \tilde{\cap} (U, E) \tilde{\subset} \lambda^{-1}(1)$  in  $M_{(N,D)}$ .

For example,  $B \times T$  is *fw-ps* functionally normal space whenever  $(T, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* functionally normal space.

**Remark 3.18.** If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* functionally normal space over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$  then  $(M_{B^*}, \tau_{1B^*}, \tau_{2B^*}, E_{B^*})$  is *fw-ps* functionally normal space on  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  for each subspace  $(B^*, \tilde{\gamma}_1^*, \tilde{\gamma}_2^*, D^*)$  of  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ .

Soft closed subspaces of *fw-ps* functionally normal spaces are *fw-ps* functionally normal. In fact we have.

**Proposition 3.19.** Let  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  be a soft closed fibrewise irresolute embedding, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is *fw-ps* functionally normal then  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is so.

**Proof.** Let  $\tilde{b} \tilde{\in} (B, D)$  and let  $(C, E), (S, E)$  be disjoint pair of  $\tilde{\tau}_j$ - and  $\tilde{\tau}_i$ - soft closed sets of  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Then  $\phi(C, E), \phi(S, E)$  are disjoint pair of  $\tilde{\tau}_j$ - and  $\tilde{\tau}_i$ -soft closed sets of  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$ . Because of  $(M^*, \tilde{\tau}_1^*, \tilde{\tau}_2^*, E^*)$  is *fw-ps* functionally normal there exists a  $\tilde{\gamma}_i$ - soft nbd  $(N, D)$  of  $b$  and a disjoint pair of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ - soft open sets  $(F, E), (U, E)$  and a continuous function  $\lambda : M_{(N,D)}^* \rightarrow I$  where  $M_{(N,D)}^* \tilde{\cap} \phi(C, E) \tilde{\cap} (F, E) \tilde{\subset} \lambda^{-1}(0)$  and  $M_{(N,D)}^* \tilde{\cap} \phi(S, E) \tilde{\cap} (U, E) \tilde{\subset} \lambda^{-1}(1)$  in  $(M_{(N,D)}^*, \tau_{(N,D)}^*, E_{(N,D)}^*)$ . Then  $\omega = \lambda \circ \phi : M_{(N,D)} \rightarrow I$  is a continuous function such that  $M_{(N,D)} \tilde{\cap} (C, E) \tilde{\cap} \phi^{-1}(F, E) \tilde{\subset} \omega^{-1}(0)$  and  $M_{(N,D)} \tilde{\cap} (S, E) \tilde{\cap} \phi^{-1}(U, E) \tilde{\subset} \omega^{-1}(1)$  in  $(M_{(N,D)}, \tau_{(N,D)}, E_{(N,D)})$ , as required.

**Definition 3.20.** A mapping  $P_{fu} : (M, E) \rightarrow (B, D)$  is called soft biopen (briefly, S-biopen) map if the image of every  $\tilde{\tau}_i$ - soft open set in  $M$  is  $\tilde{\sigma}_i$ - open set in  $K$ .



**Proposition 3.21.** Let  $\phi : (M, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  be a soft biopen, soft closed and continuous fibrewise pairwise surjection function, where  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  and  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$  are *fw-sbts* over  $(B, \tilde{\gamma}_1, \tilde{\gamma}_2, D)$ . If  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* functionally normal then so is  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$ .

**Proof.** Let  $\tilde{b} \tilde{\in} (B, D)$  and let  $(C, E), (S, E)$  be disjoint pair of  $\tilde{\sigma}_j$ - and  $\tilde{\sigma}_i$ -soft closed sets of  $(K, \tilde{\sigma}_1, \tilde{\sigma}_2, L)$ . Then  $\phi^{-1}(C, E), \phi^{-1}(S, E)$  are disjoint pair of  $\tilde{\tau}_j$ - and  $\tilde{\tau}_i$ - soft closed sets of  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Since  $(M, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is *fw-ps* functionally normal there exists a  $\tilde{\gamma}_i$ - soft nbd  $(N, D)$  of  $b$  and a disjoint pair of  $\tilde{\tau}_i$ - and  $\tilde{\tau}_j$ - soft open sets  $(F, E), (U, E)$  and a continuous function  $\lambda : M_{(N,D)} \rightarrow I$  where  $M_{(N,D)} \tilde{\cap} \phi^{-1}(C, E) \tilde{\cap} (F, E) \tilde{\subset} \lambda^{-1}(0)$  and  $M_{(N,D)} \tilde{\cap} \phi^{-1}(S, E) \tilde{\cap} (U, E) \tilde{\subset} \lambda^{-1}(1)$  in  $M_{(N,D)}$ . Now a function  $\omega : K_{(N,D)} \rightarrow I$  is given by

$$\omega(k) = \sup_{h \in \phi^{-1}(k)} \lambda(m); k \in K_{(N,D)}.$$

Since  $\phi$  is soft -biopen and soft closed, in addition to a continuous, it follows that  $\omega$  is continuous. Because of  $K_{(N,D)} \tilde{\cap} (C, E) \tilde{\cap} \phi(F, E) \tilde{\subset} \omega^{-1}(0)$  and  $K_{(N,D)} \tilde{\cap} (S, E) \tilde{\cap} \phi(U, E) \tilde{\subset} \omega^{-1}(1)$  in  $M_{(N,D)}$ . This proves the proposition.

## References

- [1] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, , 2009 "*On some new operations in soft set theory*", Computer and Mathematics with Applications, 57, 1547-1553.
- [2] Aygunoglu and H. Aygun, 2012 "*Some notes on soft topological spaces*", Neural Comput and Applic., Neural Comput. and Applic, 21(1), 113-119.
- [3] K. V. Babitha, J. J. Sunil , 2010 "*Soft set relations and functions*". Comput Math Appl; 60: 1840-1849.
- [4] G. Birkhoff, 1940 "*Lattic theory*", American Math. Soc., New York, NY, USA, 1st edition.
- [5] N. Çağman, S. Karataş Enginoğlu, 2011 "*Soft topology*", Computers and Mathematics with Applicatons,. 62:351-358.
- [6] D. Chen, 2005 "*The parametrization reduction of soft sets and its applications*", Comput. Math. Appl. 49 757-763.
- [7] R. Engking, 1989 "*Outline of general topology*", Amsterdam.
- [8] B. M. Ittanagi, December 2014, 1-4 "*Soft Bitopological Spaces*", International Journal of Computer Applications, Volume 107 - No. 7.
- [9] I. M. James, 1989 "*Fibrewise Topology*", Cambridge University Press, London.
- [10] J. C. Kelly, 1963 "*Bitopological spaces*", Proc. London Math. Soc., Vol.13,pp.71-89.
- [11] P. K. Maji, R. Biswas and A. R. Roy, 2003 "*Soft set theory*", Computer and Math. with Appl., 555-562.
- [12] D. Molodtsov, 1999 "*Soft set theory-First results*", Comput. Math. Appl. 37(4-5), 19-31.
- [13] S. Mondal, M. Chiney, S. K. Samanta, July 2011 "*Urysohn's lemma and Tietze's extension theorem in soft topology*", Annals of Fuzzy Mathematics and Informatics, Volume 2, No. 1, 1-130.
- [14] S. K. Nazmul and S. K. Samanta, 2013 "*Neighbourhood properties of soft topological spaces*", Annals of Fuzzy Mathematics and Informatics, Volume 6, No. 1, 1-15, July.
- [15] T. Y. Ozturk, 2015 "*A new approach to soft uniform spaces*", Turk J Math; DOI: 10.3906/mat-1506-98.
- [16] M. Shabir and M. Naz, , 2011 "*On soft topological spaces*", Comput. Math. Appl. 61, 1786-1799.
- [17] Y. Y. Yousif, 2008 "*Some result on fibrewise Topological space*", Ibn Al-haitham Journal for pure and Applied science. University of Baghdad – Collage of Education - Ibn Al-haitham Vol. 21, No. 2, 118-132.
- [18] Y. Y. Yousif, L. A. Hussain, , February 2017 "*Fibrewise bitoplogical Spaces*", International Journal of Science and Research (IJSR) Volume 6, Issue 2, 978-982.
- [19] Y. Y. Yousif, L. A. Hussain, M. A. Hussain 2018, preprint "*Fibrewise Soft Bitopological Spaces*" Baghdad Science Journal
- [20] Y. Y. Yousif, M. A. Hussain , February 2017 "*Fibrewise Soft Near Topolgical Spaces*", International Journal of Science and Research (IJSR), Volume 6 Issue 2, 1010-1019.
- [21] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, 2012 "*Remarks on soft topological spaces*", Annals of Fuzzy Mathematics and Informatics, 3(2), 171-185.