Iraqi Journal for Computer Science and Mathematics

Journal Homepage: http://journal.esj.edu.iq/index.php/IJCM

ISSN: 2788-7421 p-ISSN: 2958-0544

Some Methods to Estimate the Parameters of Generalized Exponential Rayleigh Model by Simulation

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DOI: https://doi.org/10.52866/ijcsm.2023.02.02.010
Received November 2022 ; Accepted Februry 2023 ; Available online March 2023

ABSTRACT: The paper show how to estimate the three parameters of generalized exponential Rayleigh distribution by utilizing the three estimation methods which are The first one is the maximum likelihood estimator method the second one is moment employing estimation method (MEM), the third one is rank set sampling estimator method (RSSEM)The simulation technique is used for all these estimation methods to find the parameters for generalized exponential Rayleigh distribution. Finally using the mean squares error criterion to compare between these estimation methods to find which of these methods are best to the others. in order to extract the experimental results, one of object orinated programming language visual basic. net was used

Keywords: Maximum Likelihood Estimator Method, Moment estimator method, and Rank Set Sampling Estimator Method

1. INTRODUCTION

Application of statistics play an important role in our divergent phenomenon life, especially in engineering and medicine. The researchers depending on many procedures to find the new or mixture and compose distributions. In 2021 [7] introduced the mixed distribution where mixing between reliability function for exponential model and reliability function for Rayleigh model by using $X = \min(Y, Z)$ then the model called "Exponential – Rayleigh model"

In 2021 [1] applied maximum likehood method to estimate the values of survival function and hazard function based on real data for lung cancer and stomach cancer obtained from Iraqi [6]. In last research we introduced the (GER) distribution that strutting is to add the shape parameter to exponential Rayleigh distribution to get the (GER).

The aim of the research is to compare the (MLEM) method, the (MEM) method, and the (RSSEM) method by using the mean squares error criterion of the Monte Carlo simulation technique to find the best method for estimating the three parameters of the new distribution. The organization of this paper in the second section includes a generalized exponential Rayleigh distribution. In the third section, the estimation method. In the fourth section simulation technique. In the fifth section are the numerical results. In the sixth section, the conclusion.

2. STATISTICAL PROPERTIES OF GENERALIZED EXPONENTIAL RALEIGH DISTRIBUTION

In this section only, we mentioned the most important statistical properties that we got from the new distribution, and through these properties, we proved that the combination that we made between the two distributions, which are Rayleigh and exponential, is a new distribution, and it will depend on the student's discussion.

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2.1 THE PROBABILITY DENSITY FUNCTION FOR GENERALIZED EXPONENTIAL- RAYLEIGH DISTRIBUTION

$$f(x;\alpha,\beta,\lambda) = \left(\frac{\alpha}{\lambda}x^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}x^{\frac{2}{\lambda}-1}\right)e^{-\left(\alpha x^{\frac{1}{\lambda}} + \frac{\beta}{2}x^{\frac{2}{\lambda}}\right)} \qquad x \geq 0 \tag{1}$$

$\Omega = \{(\alpha,\beta,\lambda) : \alpha,\beta,\lambda > 0$ where α,β are scale parameter and λ is a shape parameter$\}$

2.2 THE CUMULATIVE DISTRIBUTION FUNCTION FOR GENERALIZED EXPONENTIAL- RAYLEIGH DISTRIBUTION IS

$$F(x;\alpha,\beta,\lambda) = 1 - e^{-\left(\alpha x^{\frac{1}{\lambda}} + \frac{\beta}{2}x^{\frac{2}{\lambda}}\right)} \tag{2}$$

$\Omega = \{(\alpha,\beta,\lambda) : \alpha,\beta,\lambda > 0$ where α,β are scale parameter and λ is a shape parameter$\}$

2.3 THE RELIABILITY FUNCTION OF THIS DISTRIBUTION IS

$$R(x;\alpha,\beta,\lambda) = e^{-\left(\alpha x^{\frac{1}{\lambda}} + \frac{\beta}{2}x^{\frac{2}{\lambda}}\right)} \qquad x \geq 0 \tag{3}$$

Then the hazard rate function for this distribution is:

$$h(x;\alpha,\beta,\lambda) = \frac{\alpha}{\lambda}x^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}x^{\frac{2}{\lambda}-1} \qquad x \geq 0 \tag{4}$$

$\Omega = \{(\alpha,\beta,\lambda) : \alpha,\beta,\lambda > 0$ where α, β are scale parameter and λ is a shape parameter$\}$

3. ESTIMATION METHODS

In this section, describing the estimation of the parameters of Generalized exponential Rayleigh distribution by employing three methods (maximum likelihood estimator method, moment estimation method and rank set sampling estimator method).

3.1 MAXIMUM LIKELIHOOD ESTIMATOR METHOD (MLEM)

The Maximum likelihood estimator method (MLEM) is one of the most popular and reliable methods that used to obtain appoint estimator for parameters for any distribution.

$$f(x,\alpha,\beta,\lambda) = \left(\frac{\alpha}{\lambda}X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}X_i^{\frac{2}{\lambda}-1}\right)e^{-\left(\alpha X_i^{\frac{1}{\lambda}} + \frac{\beta}{2}X_i^{\frac{2}{\lambda}}\right)}$$

$$L(\alpha,\beta,\lambda\,|\underline{X}) = \prod_{i=1}^{n} f(\underline{X}|\,\alpha,\beta,\lambda)$$

$$L(\alpha,\beta,\lambda\,|\underline{X}) = \prod_{i=1}^{n}\left[\left(\frac{\alpha}{\lambda}X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}X_i^{\frac{2}{\lambda}}\right)e^{-\left(\alpha X_i^{\frac{1}{\lambda}} + \frac{\beta}{2}X_i^{\frac{2}{\lambda}}\right)}\right]$$

$$L(\alpha,\beta,\lambda\,|\underline{X}) = \prod_{i=1}^{n}\left(\frac{\alpha}{\lambda}X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}X_i^{\frac{2}{\lambda}-1}\right)e^{-\sum_{i=1}^{n}\left(\alpha X_i^{\frac{1}{\lambda}} + \frac{\beta}{2}X_i^{\frac{2}{\lambda}}\right)}$$

$$\ln L(\alpha,\beta,\lambda\,|\underline{X}) = \sum_{i=1}^{n} ln\left(\frac{\alpha}{\lambda}X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}X_i^{\frac{2}{\lambda}-1}\right) - \sum_{i=1}^{n}\left(\alpha X_i^{\frac{1}{\lambda}} + \frac{\beta}{2}X_i^{\frac{2}{\lambda}}\right) \tag{5}$$

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{n}\frac{\frac{1}{\lambda}X_i^{\frac{1}{\lambda}-1}}{\left(\frac{\alpha}{\lambda}X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}X_i^{\frac{2}{\lambda}-1}\right)} - \sum_{i=1}^{n}X_i^{\frac{1}{\lambda}} \tag{6}$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^{n} \frac{\frac{1}{\lambda} X_i^{\frac{2}{\lambda}-1}}{\left(\frac{\alpha}{\lambda} X_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} X_i^{\frac{2}{\lambda}-1}\right)} - \frac{1}{2}\sum_{i=1}^{n} X_i^{\frac{2}{\lambda}} \tag{7}$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n} \frac{\left[-\frac{\alpha}{\lambda^3} X_i^{\frac{1}{\lambda}-1} \ln x_i - \frac{\alpha}{\lambda^2} x_i^{\frac{1}{\lambda}-1} - \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}-1} \ln x_i - \frac{\beta}{\lambda^2} x_i^{\frac{2}{\lambda}-1}\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)} + \sum_{i=1}^{n}\left[\frac{\alpha}{\lambda^2} x_i^{\frac{1}{\lambda}} \ln x_i + \frac{\beta}{\lambda^2} x_i^{\frac{2}{\lambda}} \ln x_i\right] \tag{8}$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = \sum_{i=1}^{n} \frac{-\frac{1}{\lambda} x_i^{\frac{1}{\lambda}-1} * \frac{1}{\lambda} x_i^{\frac{1}{\lambda}-1}}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\sum_{i=1}^{n} \frac{\frac{1}{\lambda^2} x_i^{\frac{4}{\lambda}-2}}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \sum_{i=1}^{n} \frac{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)\left[\frac{\alpha}{\lambda^5} x_i^{\frac{1}{\lambda}-1} (\ln x_i)^2 + \frac{3\alpha}{\lambda^4} x_i^{\frac{1}{\lambda}-1} \ln x_i + \frac{\alpha}{\lambda^4} x_i^{\frac{1}{\lambda}-1} \ln x_i\right.}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$+ \frac{\left.\frac{2\alpha}{\lambda^3} x_i^{\frac{1}{\lambda}-1} + \frac{4\beta}{\lambda^5} x_i^{\frac{2}{\lambda}-1} (\ln x_i)^2 + \frac{6\beta}{\lambda^4} x_i^{\frac{2}{\lambda}-1} \ln x_i + \frac{2\beta}{\lambda^4} x_i^{\frac{2}{\lambda}-1} \ln x_i + \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}-1}\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$- \frac{\left[\frac{\alpha}{\lambda^3} x_i^{\frac{1}{\lambda}-1} \ln x_i + \frac{\alpha}{\lambda^2} x_i^{\frac{1}{\lambda}-1} + \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}-1} \ln x_i + \frac{\beta}{\lambda^2} x_i^{\frac{2}{\lambda}-1}\right] * \left[\frac{\alpha}{\lambda^3} x_i^{\frac{1}{\lambda}-1} \ln x_i + \frac{\alpha}{\lambda^2} x_i^{\frac{1}{\lambda}-1} + \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}-1} \ln x_i + \frac{\beta}{\lambda^2} x_i^{\frac{2}{\lambda}-1}\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$-\sum_{i=1}^{n}\left[\frac{\alpha}{\lambda^4} x_i^{\frac{1}{\lambda}-1} (\ln x_i)^2 + \frac{2\alpha}{\lambda^3} x_i^{\frac{1}{\lambda}} \ln x_i + \frac{2\beta}{\lambda^4} x_i^{\frac{2}{\lambda}} (\ln x_i)^2 + \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}} \ln x_i\right]$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = \sum_{i=1}^{n} \frac{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right) * \left[\frac{1}{\lambda} x_i^{\frac{1}{\lambda}-1} \ln x_i \left(-\lambda^{-2}\right) + x_i^{\frac{1}{\lambda}-1} \cdot \left(-\lambda^{-2}\right)\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$- \frac{\frac{1}{\lambda} x_i^{\frac{1}{\lambda}-1} * \left[\left\{\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}} \ln x_i \left(-\lambda^{-2}\right) + x_i^{\frac{1}{\lambda}-1} \cdot \alpha\left(-\lambda^{-2}\right)\right\} + \left\{\frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1} \ln x_i \left(-2\lambda^{-2}\right) + x_i^{\frac{2}{\lambda}-1} \cdot \beta\left(-\lambda^{-2}\right)\right\}\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$-\sum_{i=1}^{n} x_i^{\frac{1}{\lambda}} \ln x_i \left(-\lambda^{-2}\right)$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = \sum_{i=1}^{n} \frac{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)\left[\left\{\frac{1}{\lambda} x_i^{\frac{2}{\lambda}-1} \ln x_i \left(-2\lambda^{-2}\right) + x_i^{\frac{2}{\lambda}-1} \left(-\lambda^{-2}\right)\right\}\right.}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$- \frac{\frac{1}{\lambda} x_i^{\frac{2}{\lambda}-1}\left[\left\{\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}} \ln x_i \left(-\lambda^{-2}\right) + x_i^{\frac{1}{\lambda}-1} \alpha\left(-\lambda^{-2}\right)\right\} + \left\{\frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1} \ln x_i \left(-2\lambda^{-2}\right) + x_i^{\frac{2}{\lambda}-1} \beta\left(-\lambda^{-2}\right)\right\}\right]}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$-\frac{1}{2}\sum_{i=1}^{n} x_i^{\frac{2}{\lambda}} \ln x_i \left(-2\lambda^{-2}\right)$$

$$\frac{\partial^2 \ln L}{\partial \lambda \partial \alpha} = \sum_{i=1}^{n} \frac{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right) \cdot \left(\frac{-1}{\lambda^3} x_i^{\frac{1}{\lambda}-1} \ln x_i - \frac{1}{\lambda^2} x_i^{\frac{1}{\lambda}-1}\right)}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$+ \frac{\left(\frac{\alpha}{\lambda^3} x_i^{\frac{1}{\lambda}-1} \ln x_i + \frac{\alpha}{\lambda^2} x_i^{\frac{1}{\lambda}-1} + \frac{2\beta}{\lambda^3} x_i^{\frac{2}{\lambda}-1} \ln x_i + \frac{\beta}{\lambda^2} x_i^{\frac{2}{\lambda}-1}\right)(\cdot \frac{1}{\lambda} x_i^{\frac{2}{\lambda}-1})}{\left(\frac{\alpha}{\lambda} x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda} x_i^{\frac{2}{\lambda}-1}\right)^2} + \sum_{i=1}^{n} \frac{1}{\lambda^2} x_i^{\frac{1}{\lambda}} \ln x_i$$

$$\frac{\partial^2 \ln L}{\partial\lambda\partial\beta} = \sum_{i=1}^{n} \frac{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)\left(-\frac{2}{\lambda^3}\, x_i^{\frac{2}{\lambda}-1} \ln x_i - \frac{1}{\lambda^2} x_i^{\frac{2}{\lambda}-1}\right)}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$+ \frac{\left(\frac{\alpha}{\lambda^3}\, x_i^{\frac{1}{\lambda}-1} \ln x_i + \frac{\alpha}{\lambda^2}\, x_i^{\frac{1}{\lambda}-1} - \frac{2\beta}{\lambda^3}\, x_i^{\frac{2}{\lambda}-1} \ln x_i - \frac{\beta}{\lambda^2}\, x_i^{\frac{2}{\lambda}-1}\right) \cdot \frac{1}{\lambda}\, x_i^{\frac{2}{\lambda}-1}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2} + \sum_{i=1}^{n} \frac{1}{\lambda^2} x_i^{\frac{2}{\lambda}} \ln x_i$$

$$\frac{\partial^2 \ln L}{\partial\alpha\partial\beta} = -\sum_{i=1}^{n} \frac{\left(\frac{1}{\lambda}\, x_i^{\frac{1}{\lambda}-1} \cdot \frac{1}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2} = -\sum_{i=1}^{n} \frac{\frac{1}{\lambda^2} x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2 \ln L}{\partial\beta\partial\alpha} = -\sum_{i=1}^{n} \frac{\left(\frac{1}{\lambda}\, x_i^{\frac{2}{\lambda}-1} \cdot \frac{1}{\lambda}\, x_i^{\frac{1}{\lambda}-1}\right)}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2} = -\sum_{i=1}^{n} \frac{\frac{1}{\lambda^2} x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1} + \frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

3.2 MOMENT ESTIMATOR METHOD(MEM)

The method of moment is one of the simplest techniques that is commonly used in the field of parameters estimation. The idea of this method are equate between sample moments and population moments

$$M_1 = E(X) = \int \mathrm{x}\ \mathrm{f}(x;\alpha,\beta,\lambda)\,\mathrm{dx}$$

$$M_1 = E(X) = e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}} \Gamma\frac{n+\lambda+1}{2} + \left(\frac{2}{\beta}\right)^{\frac{n+\lambda}{2}} \Gamma\frac{n+\lambda+2}{2}\right] \quad \text{First population moment}$$

$$M_1' = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \quad \text{Sample moment}$$

$$M_1 = M_1'$$

$$e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}} \Gamma\frac{n+\lambda+1}{2} + \left(\frac{2}{\beta}\right)^{\frac{n+\lambda}{2}} \Gamma\frac{n+\lambda+2}{2}\right] = \bar{x}$$

$$f(\alpha) = e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}} \Gamma\frac{n+\lambda+1}{2} + e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda}{2}} \Gamma\frac{n+\lambda+2}{2} - \bar{x} \tag{9}$$

$$M_2 = E\left(X^2\right) = \int x^2\ \mathrm{f}(x,\alpha,\beta,\lambda)\,\mathrm{dx}$$

$$M_2 = E\left(X^2\right) = e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}} \Gamma\frac{n+2\lambda+1}{2} + \left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}} \Gamma\frac{n+2\lambda+1}{2}\right] \quad \text{Second population moment}$$

$$M_2' = \frac{\sum_{i=1}^{n} x_i^2}{n} \quad \text{Second sample moment}$$

$$M_2 = M_2'$$

$$e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}} \Gamma\frac{n+2\lambda+1}{2} + \left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}} \Gamma\frac{n+2\lambda+1}{2}\right] = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

$$f(B) = e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}} \Gamma\frac{n+2\lambda+1}{2} + e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}} \Gamma\frac{n+2\lambda+1}{2} - \frac{\sum_{i=1}^{n} x_i^2}{n} \tag{10}$$

$$M_3 = E\left(x^3\right) = \int_{\text{all x}} x^3 f(\,x;\alpha,\beta,\lambda)\,\mathrm{dx}$$

$$M_3 = E\left(x^3\right) = e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda+1}{2}}\Gamma\frac{n+3\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda}{2}}\Gamma\frac{n+3\lambda+1}{2}\right]$$

$$M_3^{'} = \frac{\sum_{i=1}^{n} x_i^3}{n}$$

$$M_3 = M_3^{'}$$

$$e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda+1}{2}}\Gamma\frac{n+3\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda}{2}}\Gamma\frac{n+3\lambda+1}{2}\right] = \frac{\sum_{i=1}^{n} x_i^3}{n}$$

$$f(\lambda) = e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}}\Gamma\frac{n+3\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda}{2}}\Gamma\frac{n+3\lambda}{2}e^{-\frac{\alpha}{2}}-\frac{\sum_{i=1}^{n} x_i^3}{n} \tag{11}$$

$$\begin{bmatrix}\alpha_{k+1}\\ \beta_{k+1}\\ \lambda_{k+1}\end{bmatrix}-\begin{bmatrix}\alpha_k\\ \beta_k\\ \lambda_k\end{bmatrix}-J^{-1}\begin{bmatrix}f(\alpha)\\ f(\beta)\\ f(\lambda)\end{bmatrix},\qquad J=\begin{bmatrix}\frac{\partial f(\alpha)}{\partial\alpha} & \frac{\partial f(\alpha)}{\partial\beta} & \frac{\partial f(\alpha)}{\partial\lambda}\\ \frac{\partial f(\beta)}{\partial\alpha} & \frac{\partial f(\beta)}{\partial\beta} & \frac{\partial f(\beta)}{\partial\lambda}\\ \frac{\partial f(\lambda)}{\partial\alpha} & \frac{\partial f(\lambda)}{\partial\beta} & \frac{\partial f(\lambda)}{\partial\lambda}\end{bmatrix}$$

$$\frac{\partial f(\alpha)}{\partial\alpha}=\frac{1}{2}e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}}\Gamma\frac{n+\lambda+1}{2}-\frac{1}{2}e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}}\Gamma\frac{n+\lambda+1}{2}-\frac{1}{2}e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda}{2}}\Gamma\frac{n+\lambda+2}{2}$$

$$\frac{\partial f(\alpha)}{\partial\beta}=-e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\Gamma\frac{n+\lambda+1}{2}\cdot\frac{2^{\left(\frac{n+\lambda+1}{2}\right)}}{\beta^{\left(\frac{n+\lambda+3}{2}\right)}}\cdot\left(\frac{n+\lambda+1}{2}\right)+\Gamma\frac{n+\lambda+2}{2}\left(\frac{n+\lambda}{2}\right)\cdot\frac{2^{\left(\frac{n+\lambda}{2}\right)}}{\beta^{\left(\frac{n+\lambda+2}{2}\right)}}\right.$$

$$\frac{\partial f(\alpha)}{\partial\lambda}=e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+1}{2}}\left[\left(\frac{2}{\beta}\right)^{\frac{\lambda}{2}}\ln\left(\frac{2}{\beta}\right)\cdot\frac{1}{2}\cdot\Gamma\frac{n+\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{\lambda}{2}}\Gamma'\left(\frac{n+1}{2}+\frac{\lambda}{2}\right)\cdot\frac{1}{2}\right]+e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n}{2}}$$

$$\left[\left(\frac{2}{\beta}\right)^{\frac{\lambda}{2}}\ln\left(\frac{2}{\beta}\right)\cdot\frac{1}{2}\cdot\Gamma\frac{n+\lambda+2}{2}+\left(\frac{2}{\beta}\right)^{\frac{\lambda}{2}}\Gamma'\left(\frac{n+2}{2}+\frac{\lambda}{2}\right)\cdot\frac{1}{2}\right]$$

$$\frac{\partial f(\beta)}{\partial\alpha}=-\frac{1}{2}e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}}\Gamma\frac{n+2\lambda+1}{2}\left[\left(\frac{2}{\beta}\right)^{\frac{1}{2}}-\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{1}{2}}-1\right]$$

$$\frac{\partial f(\beta)}{\partial\beta}=-e^{-\frac{\alpha}{2}}\left[\frac{\alpha}{2}\Gamma\frac{n+2\lambda+1}{2}\cdot\left(\frac{n+2\lambda+1}{2}\right)\cdot\frac{2^{\left(\frac{n+2\lambda+1}{2}\right)}}{\beta^{\left(\frac{n+2\lambda+3}{2}\right)}}+\Gamma\frac{n+2\lambda+1}{2}\left(\frac{n+2\lambda}{2}\right)\cdot\frac{2^{\left(\frac{n+2\lambda}{2}\right)}}{\beta^{\left(\frac{n+2\lambda+2}{2}\right)}}\right]$$

$$\frac{\partial f(\beta)}{\partial\lambda}=e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\cdot\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}}\ln\left(\frac{2}{\beta}\right)\Gamma\frac{n+2\lambda+1}{2}+e^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\cdot\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda+1}{2}}\Gamma'\left(\frac{n+1}{2}+\lambda\right)$$

$$+e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}}\ln\left(\frac{2}{\beta}\right)\Gamma\frac{n+2\lambda+1}{2}+e^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+2\lambda}{2}}\Gamma'\left(\frac{n+2\lambda+1}{2}+\lambda\right)$$

$$\frac{f(\lambda)}{\partial\alpha}=\frac{1}{2}\mathrm{e}^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda+1}{2}}\left(1-\frac{\alpha}{2}\right)-\frac{1}{2}\mathrm{e}^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n+3\lambda}{2}}\Gamma\frac{n+3\lambda+1}{2}$$

$$\frac{f(\lambda)}{\partial\beta}=\mathrm{e}^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\Gamma\frac{n+3\lambda+1}{2}\cdot-\left(\frac{n+3\lambda+1}{2}\right)\cdot\frac{2^{\left(\frac{n+3\lambda+1}{2}\right)}}{\beta^{\left(\frac{n+3\lambda+3}{2}\right)}}+\mathrm{e}^{-\frac{\alpha}{2}}\Gamma\frac{n+3\lambda+1}{2}\cdot-\left(\frac{n+3\lambda}{2}\right)\cdot\frac{2^{\left(\frac{n+3\lambda}{2}\right)}}{\beta^{\left(\frac{n+3\lambda+2}{2}\right)}}$$

$$=-\mathrm{e}^{-\frac{\alpha}{2}}\Gamma\frac{n+3\lambda+1}{2}\left[\left(\frac{\alpha}{2}\right)\cdot\left(\frac{n+3\lambda+1}{2}\right)\frac{2^{\left(\frac{n+3\lambda+1}{2}\right)}}{\beta^{\left(\frac{n+3\lambda+3}{2}\right)}}+\left(\frac{n+3\lambda}{2}\right)\cdot\frac{2^{\left(\frac{n+3\lambda}{2}\right)}}{\beta^{\left(\frac{n+3\lambda+2}{2}\right)}}\right]$$

$$\frac{f(\lambda)}{\partial\lambda}=\mathrm{e}^{-\frac{\alpha}{2}}\left(\frac{\alpha}{2}\right)\left(\frac{2}{\beta}\right)^{\frac{n+1}{2}}\left[\left(\frac{2}{\beta}\right)^{\frac{3\lambda}{2}}\ln\left(\frac{2}{\beta}\right)\cdot\frac{3}{2}\cdot\Gamma\frac{n+3\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{3\lambda}{2}}\Gamma'\left(\frac{n+1}{2}+\frac{3\lambda}{2}\right)\cdot\frac{3}{2}\right]$$

$$+\mathrm{e}^{-\frac{\alpha}{2}}\left(\frac{2}{\beta}\right)^{\frac{n}{2}}\left[\left(\frac{2}{\beta}\right)^{\frac{3\lambda}{2}}\ln\left(\frac{2}{\beta}\right)\cdot\frac{3}{2}\cdot\Gamma\frac{n+3\lambda+1}{2}+\left(\frac{2}{\beta}\right)^{\frac{3\lambda}{2}}\cdot\Gamma'\left(\frac{n+1}{2}+\frac{3\lambda}{2}\right)*\frac{3}{2}\right.$$

3.3 RANK SET SAMPLING ESTIMATOR METHOD (RSSEM)

One of the Keys to statistical inference is to estimate the parameters of the distribution from the exits samples. The most common mechanisms for obtaining such data is that of a simple random sample.

$$g(y_i) = \frac{n\,!}{(i-1)!\,(n-i)!}\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{i-1}\left[e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{n-i}\cdot\left(\frac{\alpha}{\lambda}\,x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}; \qquad k=\frac{n\,!}{(i-1)!\,(n-i)!}$$

$$g(y_i) = k\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{i-1}\left[e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{n-i+1}\cdot\left(\frac{\alpha}{\lambda}\,x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)$$

$$L(g(y_i)) = k^n\prod_{i=1}^{n}\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{i-1}.\prod_{i=1}^{n}\left[e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^{n-i+1}\cdot\prod_{i=1}^{n}\left(\frac{\alpha}{\lambda}\,x_i^{\frac{1}{\lambda}-1}\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)$$

$$\ln L(g(y_i)) = n\ln k+\sum_{i=1}^{n}(i-1)\ln\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right)-\sum_{i=1}^{n}(n-i+1)\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)+\sum_{i=1}^{n}\ln\left(\frac{\alpha}{\lambda}\,x_i^{\frac{1}{\lambda}}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)$$

$$\frac{\partial\ln L}{\partial\alpha} = \sum_{i=1}^{n}(i-1)\frac{\left(x_i^{\frac{1}{\lambda}}\right)\cdot e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}}{\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right)}-\sum_{i=1}^{n}(n-i+1)\;x_i^{\frac{1}{\lambda}}+\sum_{i=1}^{n}\frac{\frac{1}{\lambda}\;x_i^{\frac{1}{\lambda}-1}}{\left(\frac{\alpha}{\lambda}\;x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)} \tag{12}$$

$$\frac{\partial\ln L}{\partial\beta} = \sum_{i=1}^{n}(i-1)\frac{\frac{x_i^{\frac{2}{\lambda}}}{2}\;e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}}{1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}}-\sum_{i=1}^{n}(n-i+1)\frac{x_i^{\frac{2}{\lambda}}}{2}+\sum_{i=1}^{n}\frac{\frac{1}{\lambda}\;x_i^{\frac{2}{\lambda}-1}}{\frac{\alpha}{\lambda}\;x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}} \tag{13}$$

$$\frac{\partial\ln L}{\partial\lambda} = \frac{-\sum_{i=1}^{n}(i-1)e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\frac{\ln x_i}{\lambda^2}\left[\alpha x_i^{\frac{1}{\lambda}}+\beta\,x_i^{\frac{2}{\lambda}}\right]}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]}+\sum_{i=1}^{n}(n-i+1)*\frac{1}{\lambda^2}\ln x_i\left[\alpha x_i^{\frac{1}{\lambda}}+\beta\,x_i^{\frac{2}{\lambda}}\right] \tag{14}$$

$$\frac{-\sum_{i=1}^{n}\left[\frac{\alpha}{\lambda^3}\,x_i^{\frac{1}{\lambda}-1}\ln x_i+\frac{\alpha}{\lambda^2}\,x_i^{\frac{1}{\lambda}-1}+\frac{2\beta}{\lambda^3}\,x_i^{\frac{2}{\lambda}-1}\ln x_i+\frac{\beta}{\lambda^2}\,x_i^{\frac{2}{\lambda}-1}\right.}{\left(\frac{\alpha}{\lambda}\,x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)}$$

$$\frac{\partial\ln L}{\partial\alpha\partial\beta} = -\frac{1}{2}\;\frac{\sum_{i=1}^{n}(i-1)\;x_i^{\frac{3}{\lambda}}\;e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}}{\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right)^2}-\sum_{i=1}^{n}\frac{\frac{1}{\lambda^2}x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\;x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2\ln L}{\partial\alpha^2} = \sum_{i=1}^{n}\frac{(i-1)\;x_i^{\frac{2}{\lambda}}\;e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}}{\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right)^2}-\sum_{i=1}^{n}\frac{\frac{1}{\lambda^2}\;x_i^{\frac{2}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\;x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\,x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2\ln L}{\partial\alpha\partial\lambda} = \frac{\sum_{i=1}^{n}(i-1)\left[x_i^{\frac{2}{\lambda}}\cdot e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\alpha}{\lambda^2}\cdot\ln x_i+x_i^{\frac{3}{\lambda}}\,e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\beta\ln x_i}{\lambda^2}-\frac{1}{\lambda^2}\,e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\cdot x_i^{\frac{1}{\lambda}}\ln x_i\right]\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right)}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\,x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$+\frac{(i-1)\, x_i^{\frac{2}{\lambda}}\, e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\alpha}{\lambda^2}\ \ln x_i+(i-1)\, x_i^{\frac{3}{\lambda}}\, e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\beta}{\lambda^2}\ \ln x_i}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$+\sum_{i=1}^{n}(n-i+1)\, x_i^{\frac{1}{\lambda}}\, \ln x_i\cdot\frac{1}{\lambda^2}-\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^4}\, x_i^{\frac{2}{\lambda}-2}\ln x_i}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}-\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^4}\, x_i^{\frac{3}{\lambda}-2}\ln x_i}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}-\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^3}\, x_i^{\frac{2}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}-\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^3}\, x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$+\frac{\frac{\alpha}{\lambda^4}\, x_i^{\frac{2}{\lambda}-2}\ \ln x_i+\frac{\alpha}{\lambda^3}\ x_i^{\frac{2}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}+\frac{\frac{2\beta}{\lambda^4}\, x_i^{\frac{3}{\lambda}-2}\ln x_i+\frac{\beta}{\lambda^3}x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2\ln L}{\partial\beta\partial\alpha}=\frac{\frac{-1}{2}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{3}{\lambda}}\, e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}-\sum_{i=1}^{n}\frac{\frac{1}{\lambda^2}\, x_i^{\frac{3}{\lambda}-2}}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right]^2}$$

$$\frac{\partial^2\ln L}{\partial\beta^2}=\frac{-\frac{1}{4}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{4}{\lambda}}\, e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}+\frac{1}{4}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{4}{\lambda}}\, e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}-\frac{\frac{1}{2}\sum_{i=1}^{n}(i-1)\, x_i^{\frac{4}{\lambda}}\, e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$-\sum_{i=1}^{n}\frac{\frac{1}{\lambda^2}\, x_i^{\frac{4}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\, x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial\ln L}{\partial\beta\,\partial\lambda}=\frac{\frac{1}{2}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{3}{\lambda}}\cdot\frac{\alpha}{\lambda^2}\, e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\ln x_i+\frac{1}{2}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{4}{\lambda}}\cdot\frac{\beta}{\lambda^2}\, e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot\ln x_i}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$-\frac{\sum_{i=1}^{n}\frac{1}{\lambda^2}\, e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot x_i^{\frac{2}{\lambda}}\cdot\ln x_i\left(1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right)}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$+\frac{\frac{1}{2}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{3}{\lambda}}e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\alpha}{\lambda^2}\ \ln x_i+\frac{1}{2}\ \sum_{i=1}^{n}(i-1)\, x_i^{\frac{4}{\lambda}}\cdot e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\beta}{\lambda^2}\ \ln x_i}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}}\right)}\right]^2}+\sum_{i=1}^{n}\frac{1}{\lambda^2}\ (n-i+1)\cdot x_i^{\frac{2}{\lambda}}\ln x_i$$

$$-\frac{\sum_{i=1}^{n}\frac{2\alpha}{\lambda^4}\, x_i^{\frac{3}{\lambda}-2}\ln x_i-\sum_{i=1}^{n}\frac{2\beta}{\lambda^4}\, x_i^{\frac{4}{\lambda}-2}\ln x_i}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}-\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^3}x_i^{\frac{3}{\lambda}-2}-\sum_{i=1}^{n}\frac{\beta}{\lambda^3}x_i^{\frac{4}{\lambda}-2}}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}+\frac{\sum_{i=1}^{n}\frac{2\beta}{\lambda^4}\, x_i^{\frac{4}{\lambda}-2}\ln x_i}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}+\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^3}\, x_i^{\frac{4}{\lambda}-2}}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}$$

$$+\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^4}x_i^{\frac{3}{\lambda}-2}\ln x_i}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}+\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^3}x_i^{\frac{3}{\lambda}-2}}{\left[\frac{\alpha}{\lambda}\, x_i^{\frac{1}{\lambda}-2}+\frac{\beta}{2}\, x_i^{\frac{2}{\lambda}-1}\right]^2}$$

$$\frac{\partial \ln L}{\partial\lambda\partial\beta}=\frac{\left[\frac{1}{2}\sum_{i=1}^{n}(i-1)\ \frac{\alpha}{\lambda^2}\ x_i^{\frac{3}{\lambda}}\ \ln x_i\ e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}\right)}-\sum_{i=1}^{n}(i-1)\ \frac{x_i^{\frac{2}{\lambda}}}{\lambda^2}\ \ln x_i\ e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}\right)}]+\frac{1}{2}\sum_{i=1}^{n}(i-1)\ \frac{\beta}{\lambda^2}\ \ln x_i\ x_i^{\frac{3}{\lambda}}\ e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}\right)}\right]}{\left[\mathbf{1}-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}x_i^{\frac{2}{\lambda}}\right)}\right]^{\mathbf{2}}}$$

$$\cdot\left(1-e^{-\alpha\ x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}}\right)+\frac{1}{2}\frac{\sum_{i=1}^{n}(i-1)\ e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\alpha}{\lambda^2}\ x_i^{\frac{3}{\lambda}}\ \ln x_i+}{\left[\mathbf{1}-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}x_i^{\frac{2}{\lambda}}\right)}\right]^{\mathbf{2}}}\frac{1}{2}\frac{\sum_{i=1}^{n}(i-1)\ e^{-2\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}\ x_i^{\frac{2}{\lambda}}\right)}\cdot\frac{\beta}{\lambda^2}\ \ln x_i\ x_i^{\frac{4}{\lambda}}}{\left[\mathbf{1}-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}x_i^{\frac{2}{\lambda}}\right)}\right]^{\mathbf{2}}}$$

$$+\sum_{i=1}^{n}(n-i+1)\cdot\frac{x_i^{\frac{2}{\lambda}}}{\lambda^2}\ \ln x_i-\frac{\sum_{i=1}^{n}\frac{2\alpha}{\lambda^4}\ x_i^{\frac{3}{\lambda}-2}\ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}-\frac{\sum_{i=1}^{n}\frac{2\beta}{\lambda^4}x_i^{\frac{4}{\lambda}-2}\ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}-\frac{\frac{\alpha}{\lambda^3}\ x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$-\frac{\frac{\beta}{\lambda^3}\ x_i^{\frac{4}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}+\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^4}\ x_i^{\frac{3}{\lambda}-2}\ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}+\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^3}x_i^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}+\frac{\sum_{i=1}^{n}\frac{2\beta}{\lambda^4}\ x_i^{\frac{4}{\lambda}-2}\ \ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}+\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^3}x_i^{\frac{4}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i^{\frac{2}{\lambda}-1}\right)^2}$$

$$\frac{\partial^2\ln}{\partial\lambda\partial\alpha}=\frac{\left[-\sum_{i=1}^{n}(i-1)x_i{}^{\frac{1}{\lambda}}\ \mathrm{e}^{-\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}\cdot\frac{\ln x_i}{\lambda^2}+\sum_{i=1}^{n}(i-1)x_i{}^{\frac{2}{\lambda}}\ \frac{\alpha}{\lambda^2}\ \mathrm{e}^{-\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}\ln x_i+\ \sum_{i=1}^{n}(i-1)x_i{}^{\frac{3}{\lambda}}\ \frac{\beta}{\lambda^2}\ \mathrm{e}^{-\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}\ln x_i\right]}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$\cdot\left[1-e^{-\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}\right]+\frac{\sum_{i=1}^{n}(i-1)x_i{}^{\frac{2}{\lambda}}\ \frac{\alpha}{\lambda^2}\ \ln x_i\ \mathrm{e}^{-2\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}+\sum_{i=1}^{n}(i-1)\ \mathrm{e}^{-2\left(\alpha x_i{}^{\frac{1}{\lambda}}+\frac{\beta}{2}\right)}\ \frac{\ln x_i}{\lambda^2}\beta x_i{}^{\frac{3}{\lambda}}}{\left[1-e^{-\left(\alpha x_i^{\frac{1}{\lambda}}+\frac{\beta}{2}x_i^{\frac{2}{\lambda}}\right)}\right]^2}$$

$$+\sum_{i=1}^{n}(n-i+1)\ \frac{1}{\lambda^2}x_i{}^{\frac{1}{\lambda}}\ln x_i-\frac{\sum_{i=1}^{n}\frac{\alpha}{\lambda^4}\ x_i{}^{\frac{2}{\lambda}-2}\ \ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i{}^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i{}^{\frac{2}{\lambda}-1}\right)^2}-\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^4}\ x_i{}^{\frac{3}{\lambda}-2}\ \ln x_i-\sum_{i=1}^{n}\frac{\alpha}{\lambda^3}\ x_i{}^{\frac{2}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i{}^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i{}^{\frac{2}{\lambda}-1}\right)^2}$$

$$-\frac{\sum_{i=1}^{n}\frac{\beta}{\lambda^3}\ x_i{}^{\frac{3}{\lambda}-2}+\sum_{i=1}^{n}\frac{\alpha}{\lambda^4}\ x_i{}^{\frac{2}{\lambda}-2}\ \ln x_i}{\left(\frac{\alpha}{\lambda}\ x_i{}^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i{}^{\frac{2}{\lambda}-1}\right)^2}+\frac{\frac{\alpha}{\lambda^3}\ x_i{}^{\frac{2}{\lambda}-2}\ +2\frac{\beta}{\lambda^4}\ x_i{}^{\frac{3}{\lambda}-2}\ln x_i+\frac{\beta}{\lambda^3}x_i{}^{\frac{3}{\lambda}-2}}{\left(\frac{\alpha}{\lambda}\ x_i{}^{\frac{1}{\lambda}-1}+\frac{\beta}{\lambda}\ x_i{}^{\frac{2}{\lambda}-1}\right)^2}$$

4. SIMULATION RESULTS TECHNIQUE:

It has been highlighted that the Monte Carlo approach is the most general and widely used method in simulation techniques for generating data (samples) for any distribution. This approach (the simulation process) is adaptable, allowing for several tests and experiments.

$$F(x;\alpha,\beta,\lambda)=1-e^{-\left(\alpha x^{\frac{1}{\lambda}}+\frac{\beta}{2}x^{\frac{2}{\lambda}}\right)}\qquad Then\quad u=1-e^{-\left(\alpha x^{\frac{1}{\lambda}}+\frac{\beta}{2}x^{\frac{2}{\lambda}}\right)},\quad x=\left|\frac{-\alpha-\sqrt{\alpha^2-2\ln\beta+2\ln u-\beta}}{\beta}\right|^{\lambda}$$

Because (x) is positive, the negative values resulting from this generation are ignored. According to above equation, generating different sizes of samples $n = 15, 25, 50, 75, 100$. And estimation the unknown parameters by using (MLEM, MEM, RSSEM) methods. Therefore, calculating the value of mean square error to compare between the methods. $MSE = \sum_{i=1}^{L}\frac{(\hat{\theta}-\theta)^2}{L}$, where n is the size of sample and L which is the number of Repeating the experiments ($L = 1000$), $n = 15, 25, 50, 75, 100$.

5. NUMERICAL RESULTS

In this section using Monte-Carlo approach is simulation technique to find the mean squares error for all methods be using in this research.

The algorithm of the numerical method is:

1. start
2. define the initial parameters $(n, \alpha, \beta, \gamma)$
3. start iteration with $(I = 1)$
4. Generat random variable (x_i) such that (x_i Generalized Exponential Rayleigh distribution) with (α, β, γ) and sample size (n)
5. estimate (α, β, γ) by (Maximum Likelihood Estimator Method, Moment estimator method, and Rank Set Sampling Estimator Method)
6. repate steps (4-5) till iteration (I=1000)
7. the estimators for (α, β, γ) parameters will be the average for the (1000) estimators for each parameter
8. caclate the mean square error for each parameter
9. end

Table 1. The value of estimator and mean squares error for all estimation method with $(\alpha = 0.25)$ for first parameter (α)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
0.5	1	15	0.302478	3.22E-03	0.258324	2.90E-04	0.280876	1.73E-03	2
		25	0.401794	4.80E-02	0.251484	9.37E-06	0.406229	4.88E-02	2
		50	0.249573	1.88E-07	0.24988	5.29E-08	0.249698	9.38E-08	2
		75	0.249963	1.47E-09	0.250013	1.70E-10	0.249978	6.08E-10	2
		100	0.249651	1.94E-07	0.250002	4.08E-11	0.249652	1.92E-07	2
	2	15	0.293436	1.60E-02	0.232904	1.90E-03	0.331035	2.29E-02	2
		25	0.24734	2.72E-05	0.248796	2.53E-06	0.246983	7.15E-05	2
		50	0.249864	6.46E-08	0.249968	1.75E-07	0.249816	6.12E-08	3
		75	0.249604	1.63E-07	0.249944	3.36E-09	0.249643	1.56E-07	2
		100	0.276147	1.37E-03	0.249998	3.97E-12	0.276139	1.37E-03	2
1	1	15	0.466303	9.72E-02	0.276422	7.03E-04	0.461278	9.44E-02	2
		25	0.407085	4.66E-02	0.248133	3.98E-05	0.407972	4.75E-02	2
		50	0.35159	2.10E-02	0.249227	6.04E-07	0.35242	2.12E-02	2
		75	0.251388	3.18E-05	0.249995	1.30E-10	0.251415	3.13E-05	2
		100	0.257995	8.37E-05	0.250001	4.09E-11	0.257994	8.36E-05	2
	2	15	0.230313	7.57E-04	0.24759	7.13E-04	0.255602	6.83E-04	3
		25	0.23079	7.74E-04	0.248801	6.47E-06	0.227827	8.50E-04	2
		50	0.248281	3.25E-06	0.249781	7.54E-08	0.248244	3.24E-06	2
		75	0.263906	4.78E-04	0.250022	9.44E-10	0.263884	4.77E-04	2
		100	0.249734	9.78E-08	0.249999	2.16E-11	0.249734	1.00E-07	2
1.5	1	15	0.340615	0.011095	0.261887	2.86E-03	0.29159	1.19E-02	2
		25	0.27463	6.35E-04	0.249314	1.83E-06	0.26918	3.72E-04	2
		50	0.254865	1.10E-04	0.249688	1.21E-07	0.254949	1.16E-04	2
		75	0.270335	6.71E-04	0.25001	1.14E-09	0.270232	6.69E-04	2
		100	0.266482	2.75E-04	0.249998	5.18E-12	0.266475	2.75E-04	2
	2	15	0.322093	5.42E-03	0.300486	3.85E-03	0.326645	9.12E-03	2
		25	0.293182	2.80E-03	0.251763	4.91E-06	0.289892	2.29E-03	2
		50	0.259833	2.13E-04	0.250015	1.12E-08	0.260129	2.19E-04	2
		75	0.261225	2.84E-04	0.249996	1.44E-09	0.261264	2.85E-04	2
		100	0.25482	7.35E-05	0.250001	2.58E-11	0.254818	7.36E-05	2

Noting that the mean square error for moment estimator method are the best estimator from the other method indicating that $\frac{28}{30} * 100 = 93.3$

Table 2. The value of estimator and mean squares error for all estimation method with ($\alpha = 0.5$) for first parameter (α)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
0.5	1	15	1.716738	2.55236	0.471463	3.51E-03	1.681025	2.522908	2
		25	0.502041	5.28E-05	0.49674	1.22E-05	0.493574	4.17E-05	2
		50	0.518687	6.77E-04	0.500053	6.16E-07	0.51795	6.46E-04	2
		75	0.532624	2.13E-03	0.499971	8.62E-10	0.532664	2.13E-03	2
		100	0.499996	1.78E-11	0.500003	1.75E-11	0.499992	7.00E-11	2
	2	15	0.597037	4.32E-02	0.541988	2.15E-03	0.637807	5.22E-02	2
		25	0.753757	0.131067	0.494823	2.68E-05	0.754876	0.134905	2
		50	0.524495	1.22E-03	0.499973	8.44E-08	0.52446	1.20E-03	2
		75	0.499979	7.90E-10	0.499993	3.79E-09	0.500011	2.61E-10	3
		100	0.513913	3.86E-04	0.499997	1.88E-11	0.51391	3.86E-04	2
1	1	15	0.818048	0.106772	0.522962	9.82E-04	0.807814	9.68E-02	2
		25	0.524206	1.28E-03	0.502799	2.19E-05	0.519906	9.85E-04	2
		50	0.577006	1.19E-02	0.500024	2.61E-07	0.577578	1.20E-02	2
		75	0.561119	6.85E-03	0.499943	3.39E-09	0.56117	6.87E-03	2
		100	0.506755	9.19E-05	0.500001	3.34E-11	0.506753	9.19E-05	2
	2	15	0.584929	0.031675	0.521279	5.61E-04	0.652518	4.39E-02	2
		25	0.524825	9.25E-03	0.500872	2.20E-06	0.528473	9.65E-03	2
		50	0.534066	1.27E-03	0.499857	7.08E-07	0.533299	1.24E-03	2
		75	0.503971	3.46E-05	0.499975	1.03E-09	0.503994	3.53E-05	2
		100	0.583437	1.32E-02	0.499996	1.60E-11	0.583435	1.32E-02	2
1.5	1	15	0.95365	0.208278	0.502806	1.23E-04	0.954799	0.210204	2
		25	0.513556	8.93E-04	0.500774	4.14E-06	0.514627	1.37E-03	2
		50	0.545004	4.78E-03	0.499691	1.26E-07	0.545042	4.77E-03	2
		75	0.516812	6.17E-04	0.499985	4.49E-09	0.516841	6.18E-04	2
		100	0.498253	9.60E-04	0.50001	1.03E-11	0.498251	9.60E-04	2
	2	15	0.506553	9.94E-04	0.475719	8.10E-04	0.532363	1.65E-03	2
		25	0.553689	4.35E-03	0.50049	2.68E-07	0.552109	4.19E-03	2
		50	0.583316	7.04E-03	0.500079	1.91E-08	0.583509	7.07E-03	2
		75	0.505491	7.32E-04	0.499952	4.50E-09	0.505429	7.32E-04	2
		100	0.512007	5.52E-04	0.50001	7.95E-13	0.512012	5.53E-04	2

Noting that the mean square error for moment estimator method are the best estimator from the other method indicating that $\frac{29}{30} * 100 = 96.666$

Table 3. The value of estimator and mean squares error for all estimation method with ($\alpha = 0.25$) for second parameter (β)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
0.5	1	15	0.276888	5.90E-02	0.464726	2.13E-03	0.308156	4.40E-02	2
		25	0.470936	1.50E-03	0.49602	1.71E-05	0.473143	1.16E-03	2
		50	0.406185	8.82E-03	0.499896	1.32E-07	0.406871	8.69E-03	2
		75	0.528901	1.70E-03	0.500021	5.23E-10	0.528861	1.69E-03	2
		100	0.519604	9.39E-04	0.499999	1.60E-12	0.519607	9.39E-04	2
	2	15	0.477249	1.26E-02	0.503915	0.002633	0.410077	2.34E-02	2
		25	0.453927	6.50E-03	0.497068	1.61E-05	0.460512	5.77E-03	2
		50	0.537702	7.66E-03	0.499636	4.78E-07	0.537768	7.83E-03	2
		75	0.521553	4.68E-04	0.499982	1.10E-09	0.521582	4.69E-04	2
		100	0.495295	5.18E-03	0.499996	2.07E-11	0.4953	5.18E-03	2
1	1	15	0.936256	4.12E-03	1.000258	5.45E-05	0.966719	1.12E-03	2
		25	0.849232	5.49E-02	1.00157	1.63E-05	0.851385	5.54E-02	2
		50	0.831599	3.09E-02	0.999919	7.23E-09	0.831131	3.11E-02	2
		75	1.101376	1.62E-02	1.000016	2.28E-09	1.101373	1.62E-02	2
		100	0.964636	2.15E-03	1.000003	9.15E-12	0.964635	2.15E-03	2
	2	15	0.780797	7.80E-02	1.02124	7.32E-04	0.890328	2.94E-02	2
		25	1.296925	0.165816	0.997192	1.76E-05	1.29881	0.167411	2
		50	0.942116	3.65E-03	1.000228	6.03E-08	0.941599	3.70E-03	2
		75	1.106308	1.15E-02	1.000026	1.23E-09	1.106368	1.15E-02	2
		100	1.031289	1.43E-03	1.000002	4.93E-12	1.031283	1.43E-03	2
1.5	1	15	1.627147	0.264655	1.535848	2.07E-03	1.699108	0.269348	2
		25	1.418428	2.33E-02	1.501646	3.21E-06	1.417923	2.48E-02	2
		50	1.643418	2.06E-02	1.49972	1.82E-07	1.643555	2.06E-02	2
		75	1.527767	8.34E-03	1.499987	1.04E-09	1.527761	8.33E-03	2
		100	1.392974	1.57E-02	1.500001	3.60E-12	1.392979	1.57E-02	2
	2	15	1.292102	0.125215	1.513715	3.52E-04	1.371252	9.70E-02	2
		25	1.600839	1.10E-02	1.502052	9.46E-06	1.60414	1.15E-02	2
		50	1.728666	5.74E-02	1.50017	1.68E-07	1.728829	5.74E-02	2
		75	1.504669	8.70E-03	1.500005	1.68E-09	1.50469	8.72E-03	2
		100	1.446701	0.011732	1.500002	5.42E-12	1.4467	1.17E-02	2

Noting that the mean square error for moment estimator method are the best estimator from the other method indicating that $\frac{30}{30} * 100 = 100$

Table 4. The value of estimator and mean squares error for all estimation method with $(\alpha = 0.5)$ **for second parameter** (β)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
		15	0.590458	8.58E-03	0.494696	4.84E-04	0.594724	8.98E-03	2
		25	0.562376	1.40E-02	0.502311	2.13E-05	0.564095	1.48E-02	2
	1	50	0.546967	3.84E-03	0.500028	5.21E-08	0.547276	3.93E-03	2
		75	0.560523	5.70E-03	0.500003	2.73E-11	0.560521	5.70E-03	2
0.5		100	0.490985	1.10E-03	0.499995	2.63E-11	0.490978	1.10E-03	2
		15	0.472583	8.09E-03	0.466132	2.05E-03	0.452697	0.010676	2
		25	0.542519	1.83E-03	0.495919	1.96E-05	0.539887	1.60E-03	2
	2	50	0.500048	4.50E-03	0.500254	2.29E-07	0.500172	4.45E-03	2
		75	0.470661	1.62E-03	0.49995	2.55E-09	0.470626	1.62E-03	2
		100	0.467586	1.13E-03	0.499998	1.18E-11	0.467584	1.13E-03	2
		15	1.026282	2.14E-03	0.912091	7.75E-03	1.066111	0.013416	1
		25	1.118413	6.70E-02	0.994416	4.72E-05	1.121384	7.06E-02	2
	1	50	0.860787	0.037953	0.999855	2.11E-08	0.860745	3.80E-02	2
		75	0.914917	1.18E-02	0.999983	1.05E-09	0.914983	1.18E-02	2
1		100	1.09097	1.45E-02	0.999994	4.35E-11	1.090967	1.45E-02	2
		15	1.086974	4.96E-02	1.027409	7.82E-04	1.083458	3.39E-02	2
		25	1.117319	3.25E-02	0.999809	8.26E-06	1.111195	3.26E-02	2
	2	50	0.833278	0.059014	0.999732	7.19E-08	0.832933	5.90E-02	2
		75	0.858398	2.01E-02	0.999991	2.99E-09	0.85843	2.00E-02	2
		100	0.931366	0.011594	1.000111	6.79E-12	0.931375	1.16E-02	2
		15	1.434303	8.54E-03	1.534589	1.72E-03	1.439078	6.58E-03	2
		25	1.742678	6.06E-02	1.49877	1.31E-05	1.747734	6.36E-02	2
		50	1.428397	2.66E-02	1.500254	1.76E-07	1.429109	2.64E-02	2
		75	1.476955	3.17E-02	1.499987	1.93E-09	1.476942	3.17E-02	2
1.5	1	100	1.478857	1.45E-03	1.499996	1.72E-11	1.478852	1.45E-03	2
		15	1.549421	1.01E-02	1.546281	2.55E-03	1.557398	3.99E-03	2
		25	1.443696	4.54E-02	1.49895	1.40E-05	1.451145	4.71E-02	2
	2	50	1.640089	2.25E-02	1.500443	3.85E-07	1.640077	2.26E-02	2
		75	1.503069	3.70E-03	1.499934	4.86E-09	1.503031	3.71E-03	2
		100	1.657451	2.58E-02	1.50001	7.55E-14	1.657447	2.58E-02	2

Noting that the mean square error for moment estimator method are the best estimator from the other method indicating that $\frac{29}{30} * 100 = 96.666$

Table 5. The value of estimator and mean squares error for all estimation method with $(\alpha = 0.25)$ **for third parameter** (λ)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
		15	0.995161	6.07E-04	1.022009	1.63E-03	1.008858	2.98E-04	3
		25	1.001659	1.21E-05	1.004028	4.96E-05	1.004139	4.36E-05	1
	1	50	1.001347	2.55E-06	1.003065	1.14E-05	1.008468	7.17E-05	1
		75	1.003925	1.76E-05	1.002866	8.28E-06	1.006733	4.60E-05	2
0.5		100	1.006994	4.90E-05	1.006753	4.57E-05	1.004828	2.36E-05	3
		15	2.039973	2.15E-03	2.034129	1.37E-03	2.039918	4.61E-03	2
		25	2.003892	1.89E-05	2.001371	1.98E-06	1.997908	5.91E-06	2
	2	50	2.004196	2.62E-05	2.004431	3.54E-05	2.003077	1.04E-05	3
		75	2.006954	4.99E-05	2.000866	7.56E-07	2.001749	4.07E-06	2
		100	2.007275	5.42E-05	2.003829	1.49E-05	2.00206	6.07E-06	3
		15	1.003737	6.81E-04	1.045415	2.20E-03	1.01181	1.08E-03	1
		25	1.001864	1.20E-05	1.006134	3.85E-05	1.005273	4.85E-05	1
	1	50	1.001192	1.43E-06	1.007375	5.72E-05	1.007133	5.09E-05	1
		75	1.002972	1.44E-05	1.003575	1.76E-05	1.003591	1.37E-05	3
1		100	1.006452	4.19E-05	1.006029	3.64E-05	1.006902	4.78E-05	2
		15	2.057489	3.39E-03	1.959454	1.64E-03	1.981152	5.75E-04	3
		25	2.004578	6.14E-05	2.001257	4.05E-05	2.001369	2.38E-05	3
	2	50	2.00858	7.36E-05	2.005149	2.93E-05	2.007006	4.95E-05	2
		75	2.006574	4.72E-05	2.004803	2.92E-05	2.002943	1.23E-05	3
		100	2.002858	1.56E-05	2.001061	1.81E-06	2.004611	3.54E-05	2
		15	1.018912	4.91E-03	1.012872	6.58E-04	1.039115	1.61E-03	2
		25	1.005595	4.70E-05	1.004237	1.80E-05	1.004803	3.28E-05	2
	1	50	1.006588	5.31E-05	1.004006	1.64E-05	1.002064	4.41E-06	3
		75	1.000734	7.13E-07	1.002076	5.61E-06	1.004686	2.64E-05	1
1.5		100	1.00602	4.38E-05	1.005588	4.29E-05	1.004797	2.48E-05	3
		15	2.000105	8.90E-05	1.991298	1.65E-04	2.053148	3.92E-03	1
		25	2.005276	2.79E-05	2.007421	5.60E-05	2.008959	8.51E-05	1
	2	50	2.000877	1.31E-06	2.005379	3.34E-05	2.003791	2.07E-05	1
		75	2.002138	4.92E-06	2.006528	4.47E-05	2.006623	4.67E-05	1
		100	2.003904	2.06E-05	2.0023	6.78E-06	2.006569	4.88E-05	3

Noting that the mean square error for rank set sample estimator method are the best estimator from the other method indicating that $\frac{11}{30} * 100 = 36.666$

Table 6. The value of estimator and mean squares error for all estimation method with ($\alpha = 0.5$) for third parameter (λ)

β	λ	n	M_1		M_2		M_3		best
			$\hat{\lambda}$	MLEM	$\hat{\lambda}$	MEM	$\hat{\lambda}$	RSSEM	
0.5	1	15	1.006135	4.31E-05	0.99244	4.25E-03	0.973986	2.37E-03	1
		25	0.997612	1.10E-05	1.004876	3.20E-05	0.999798	1.57E-06	3
		50	1.004555	2.76E-05	1.005147	2.69E-05	1.003559	1.74E-05	3
		75	1.001843	5.74E-06	1.001836	3.41E-06	1.002769	8.35E-06	2
		100	1.003306	1.75E-05	1.004728	2.90E-05	1.00659	4.72E-05	1
	2	15	2.003136	1.27E-03	2.017247	2.27E-03	2.04349	2.50E-03	1
		25	2.009261	8.95E-05	2.000963	3.15E-05	2.006528	5.04E-05	2
		50	2.001517	2.32E-06	2.003722	1.50E-05	2.006926	4.80E-05	1
		75	2.003993	1.63E-05	2.005732	3.49E-05	2.003882	1.52E-05	2
		100	2.002624	9.36E-06	2.003336	1.65E-05	2.008138	6.69E-05	1
1	1	15	1.012362	4.15E-04	0.990828	8.83E-05	1.00798	1.96E-03	2
		25	1.004621	2.51E-05	1.007518	6.11E-05	1.00114	1.51E-06	3
		50	1.004631	2.33E-05	1.006165	3.84E-05	1.003824	1.70E-05	3
		75	1.004621	3.90E-05	1.002112	4.65E-06	1.004708	2.55E-05	2
		100	1.004278	2.47E-05	1.005808	3.44E-05	1.003804	1.49E-05	3
	2	15	2.016716	1.32E-03	2.044973	2.06E-03	1.981064	1.71E-03	1
		25	2.005741	3.35E-05	2.005273	2.94E-05	2.006356	6.41E-05	2
		50	2.003225	1.09E-05	2.005824	3.44E-05	2.001037	1.15E-06	3
		75	2.003697	2.28E-05	2.000635	4.57E-07	2.001569	4.07E-06	2
		100	2.004825	2.38E-05	2.005714	3.37E-05	2.005374	4.25E-05	1
1.5	1	15	0.982403	4.54E-04	1.023702	1.00E-03	1.027934	1.72E-03	1
		25	1.003497	1.54E-05	1.006682	4.47E-05	1.00322	2.04E-05	1
		50	1.006458	4.18E-05	1.004768	2.32E-05	1.004526	2.17E-05	3
		75	1.003338	1.12E-05	1.004042	2.99E-05	1.002453	7.59E-06	3
		100	1.005287	2.80E-05	1.002447	1.12E-05	1.006721	4.60E-05	2
	2	15	1.962465	1.79E-03	2.0764	6.10E-03	1.967793	2.61E-03	1
		25	2.009167	1.02E-04	1.998982	3.05E-06	2.004548	2.55E-05	1
		50	2.003629	1.39E-05	2.00293	1.56E-05	2.004788	4.02E-05	1
		75	2.002002	4.46E-06	2.004249	2.93E-05	2.00352	1.24E-05	1
		100	2.006082	3.70E-05	2.007059	5.37E-05	2.00506	2.87E-05	3

Noting that the mean square error for maximum likelihood estimator method are the best estimator from the other method indicating that $\frac{13}{30} * 100 = 43.33$

6. DISCUSSION AND CONCLUSION

In this paper, we suggest a novel distribution. It studies characteristics. Classical estimating methods were investigated in order to estimate the three unknown parameters of the new distribution. A simulation study was also performed to produce alternative sample sizes. We compared the various estimation techniques using the mean squared error. It should be noted that the moment estimation approach is the best since it has the lowest error for all sample sizes.

7. FUNDING

None

8. ACKNOLEDGEMENT

The author we would like to thank the reviewers for their valuable contribution in the publication of this paper.

9. CONFLICTS OF INTEREST

The author declares no conflict of interest.

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