



UNIVERSITY OF BAGHDAD
COLLEGE OF EDUCATION IBN AL-HAITHAM

IBN AL-HAITHAM JOURNAL

For Pure & Applied Science



ISSN 1609 - 4042

VOLUME (24)

2011

NUMBER (1)

On Modified Pre-Test Double Stage Shrunken Estimators for Estimate the Parameters of Simple Linear Regression Model

A. N. Salman and M. A. Mohammed

Department of Mathematics, College of Education -Ibn Al-Haitham, Baghdad University

Received in Feb,7, 2010

Accepted in April,25, 2010

Abstract

In this paper, we proposed modified preliminary test double stage Shrunken estimator to estimate the parameters of simple linear regression model, using shrinkage weight function $\psi(\cdot)$, in first stage and proposed modify technique in second stage.

The expression for bias, mean squared error, relative efficiency, expected sample size, probability for avoiding the second sample are derived for considered estimators. Numerical results and conclusions concern bias ratio $[B(\cdot)]$ and relative efficiency $[R.Eff(\cdot)]$ are reported for different constants involved in the expressions mentioned above.

Key words: Estimation, Parameters, Shrinkage, Prior information, Double stage estimator, Linear Regression

Introduction

Assume the following simple linear regression model;

$$y_i = A + Bx_i + e_i, \quad i = 1, 2, \dots, n \quad \dots(1)$$

Where A and B are parameters of the above model, x_i is an independent variable, y_i is a response variable and e_i is a random error that distribute normally with zero mean and $cov(e_i, e_j) = 0, i \neq j$, that is; $e_i \sim N(0, \sigma^2)$.

To estimate the parameter θ (θ may be refer to A or to B) when a prior estimate θ_0 about θ available using double stage-Shrinkage technique, we observe n_1 sample and calculate the classical estimator $\hat{\theta}_1$ based on the n_1 observations. If $\hat{\theta}_1$ implies that our prior estimate was reasonable ($\hat{\theta}_1 \in R$), we stop sampling and shrink $\hat{\theta}_1$ toward θ_0 . Otherwise, we observe second sample of size n_2 ($n = n_1 + n_2$) and calculate the classical estimator $\hat{\theta}_2$. Furthermore, calculate polling estimator $\hat{\theta}_p$ based on all n observation.

Hence, we introduce a new double stage Shrinkage estimators (modified double stage shrinkage estimator with the following form;-

$$\tilde{\theta} = \begin{cases} \psi(\hat{\theta})(\hat{\theta}_1 - \theta_0) + \theta_0 & , \text{ if } \hat{\theta}_1 \in R \\ (\hat{\theta}_p + \theta_0)/2 & , \text{ if } \hat{\theta}_1 \notin R \end{cases} \quad \dots(2)$$

where $\hat{\theta}_p = \frac{n\hat{\theta}_1 + n_2\hat{\theta}_2}{n}$, and R is a pre-test region of acceptance of sizes α for testing the hypothesis $H_0 : \theta = \theta_0$ against the hypothesis $H_1 : \theta \neq \theta_0$.

The aim of this paper is to use a preliminary test double stage shrunken estimator define in (2) to estimate the parameters A and B for simple linear regression model (1).

The expressions for bias, mean square error and relative efficiency, expected sample size and probability of avoiding the second sample of considered estimator are given.

Numerical results are presented for above expressions. These results are compared with the last studies in the sense of MSE and relative efficiency.

Noted that, the general double stage shrinkage estimator (DSSE) has the following form:

$$\tilde{\theta} = \begin{cases} \psi_1(\hat{\theta}_1)(\hat{\theta}_1 - \theta_0) + \theta_0 & , \text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_p & , \text{if } \hat{\theta}_1 \notin R \end{cases} \dots(3)$$

Many authors have studied the double stage shrunken estimator (DSSE) in (3); for example see Katti [1962], Al-Bayyati and Arnold [1972], Waikar Schuurmann and Raghunathan [1984], Kambo, Handa, and Al-Hemyari [1988], Al-Hemyari [1990], Al-Nazzal [1996], Al-Kanane [1997], and Kalaf [2007].

Modified Preliminary Test Double Stage Shrunken Estimators \tilde{A} and \tilde{B}

In this section, we have to estimate the parameters A and B using modified method (2).

The preliminary double stage shrunken estimators \tilde{A} and \tilde{B} (for estimate A and B respectively), have the forms below:

$$\tilde{A} = \begin{cases} k_1(\hat{A}_1 - A_0) + A_0 & , \text{if } \hat{A}_1 \in R_1 \\ \frac{1}{2}(\hat{A}_p + A_0) & , \text{if } \hat{A}_1 \notin R_1 \end{cases} \dots(4)$$

and

$$\tilde{B} = \begin{cases} k_2(\hat{B}_1 - B_0) + B_0 & , \text{if } \hat{B}_1 \in R_2 \\ \frac{1}{2}(\hat{B}_p + B_0) & , \text{if } \hat{B}_1 \notin R_2 \end{cases} \dots(5)$$

Where k_i ($i = 1,2$) is shrinkage weight factor, such that $0 \leq k_i \leq 1$, which is found by minimizing the MSE of \tilde{A} and \tilde{B} respectively.

$$\hat{A}_p = \frac{n_1 \hat{A}_1 + n_2 \hat{A}_2}{n_1 + n_2}, \hat{B}_p = \frac{n_1 \hat{B}_1 + n_2 \hat{B}_2}{n_1 + n_2}; A_0 \text{ and } B_0 \text{ are prior information about A and B}$$

respectively, R_1 and R_2 are pre-test regions of acceptance of size (α) for testing the hypothesis $H_{01} : A = A_0$ against the hypothesis $H_{11} : A \neq A_0$ and $H_{02} : B = B_0$ against the hypothesis $H_{12} : B \neq B_0$ respectively,

in that,

$$R_1 = \left[A_0 - t_{1-\frac{\alpha}{2}, n_1-2} \sqrt{\frac{\sigma^2 \sum x_{li}^2}{n_1 SS_{x_1}}}, A_0 + t_{1-\frac{\alpha}{2}, n_1-2} \sqrt{\frac{\sigma^2 \sum x_{li}^2}{n_1 SS_{x_1}}} \right] \dots(6)$$

$$R_2 = \left[B_0 - t_{1-\frac{\alpha}{2}, n-2} \sqrt{\frac{\sigma^2}{SS_{x_1}}}, B_0 + t_{1-\frac{\alpha}{2}, n-2} \sqrt{\frac{\sigma^2}{SS_{x_1}}} \right] \dots(7)$$

It is known that:

$$\hat{A}_1 = \bar{y} - \hat{B}_1 \bar{x} \text{ and } \hat{B}_1 = \frac{\sum_{j=1}^{n_1} (x_{1j} - \bar{x})(y_j - \bar{y})}{\sum_{j=1}^{n_1} (x_{1j} - \bar{x})^2} \text{ are unbiased estimators of A and B}$$

respectively see [1], [3].

Also,

$$\hat{A}_1 \sim N\left(A, \frac{\sigma^2 \sum_{i=1}^{n_1} x_{1i}^2}{n_1 SS_x}\right), \hat{B}_1 \sim N\left(B, \frac{\sigma^2}{SS_x}\right)$$

where $SS_x = \sum_{i=1}^n (x_i - \bar{x})^2$, $SS_{x_1} = \sum_{j=1}^{n_1} (x_{1j} - \bar{x})^2$ and $SS_{x_2} = \sum_{j=1}^{n_2} (x_{2j} - \bar{x})^2$ see [1], [3].

The expressions for bias and MSE of estimators \tilde{A} and \tilde{B} respectively given by:

$$\text{Bias}(\tilde{A}|A, R_1) = E(\tilde{A} - A)$$

$$\begin{aligned} &= \int_{\hat{A}_2=-\infty}^{\infty} \int_{\hat{A}_1 \in R} [k_1(\hat{A}_1 - A_0) + A_0 - A] f_1(\hat{A}_1) f_2(\hat{A}_2) d\hat{A}_1 d\hat{A}_2 + \\ &\int_{\hat{A}_2=-\infty}^{\infty} \int_{\hat{A}_1 \in R} \left[\frac{1}{2}(\hat{A}_p + A_0) - A \right] f_1(\hat{A}_1) f_2(\hat{A}_2) d\hat{A}_1 d\hat{A}_2 \\ &= \sqrt{\text{var}(\hat{A}_1)} \left[\lambda_1(1 - k_1) + \frac{1}{2}\lambda_1 - \frac{n_1}{2n} J_1(a, b) - \frac{1}{2}\lambda_1 J_0(a, b) \right], 0 \leq k_1 \leq 1 \dots (8) \end{aligned}$$

$$\text{where } f(\hat{A}_1) = \frac{1}{\sqrt{2\pi \left(\frac{\sigma^2 \sum_{i=1}^{n_1} x_i^2}{n_1 SS_x} \right)}} \text{Exp} \left[- \left(\frac{n_1 SS_x (\hat{A}_1 - A)^2}{2\sigma^2 \sum_{i=1}^{n_1} x_i^2} \right) \right], \text{ for } -\infty < \hat{A} < \infty$$

and

$$\text{Bias}(\tilde{B}|B, R_2) = E(\tilde{B} - B)$$

$$= \sqrt{\text{var}(\hat{B}_1)} \left[\lambda_2(1 - k_2) + \frac{1}{2}\lambda_2 - \frac{n_1}{2n} J_1^*(c, d) - \frac{1}{2}\lambda_2 J_0^*(c, d) \right], 0 \leq k_2 \leq 1 \dots (9)$$

where

$$J_i(a, b) = \int_a^b \frac{1}{\sqrt{2\pi}} (Z)^i e^{-\frac{z^2}{2}} dZ, i = 0, 1, 2 \dots (10)$$

$$J_i^*(c, d) = \int_c^d \frac{1}{\sqrt{2\pi}} (Z^*)^i e^{-\frac{z^{*2}}{2}} dZ^*, i = 0, 1, 2 \dots (11)$$

$$Z = (\hat{A}_1 - A) / \sqrt{\text{var}(\hat{A}_1)}, Z^* = (\hat{B}_1 - B) / \sqrt{\text{var}(\hat{B}_1)} \text{ and}$$

$$a = -\lambda_1 - t_{1-\frac{\alpha}{2}, n-2},$$

$$b = -\lambda_1 - t_{1-\frac{\alpha}{2}, n-2},$$

$$c = -\lambda_2 - t_{1-\frac{\alpha}{2}, n-2},$$

$$d = -\lambda_2 - t_{1-\frac{\alpha}{2}, n-2}.$$

and $R_1^* = [a, b]$, and $R_2^* = [c, d]$.

Bias Ratio (B.R.) of \tilde{A} :

$$B.R(\tilde{A}|A, R_1) = \frac{\text{Bias}(\tilde{A}|A, R_1)}{\sqrt{\text{var}(\hat{A}_1)}}$$

and

Bias Ratio (B.R.) of \tilde{B} :

$$B.R(\tilde{B}|B, R_2) = \frac{\text{Bias}(\tilde{B}|B, R_2)}{\sqrt{\text{var}(\hat{B}_1)}}$$

The expressions for mean squared error of \tilde{A} and \tilde{B} are respectively given as below:
 $MSE(\tilde{A}|A, R_1) = E[(\tilde{A} - A)^2]$

$$= \text{var}(\hat{A}_1) \left\{ \left[k_1^2 J_2(a, b) + \left\{ \left[k_1^2 \lambda_1 J_0(a, b) - 2k_1^2 \lambda_1 J_1(a, b) + \lambda_1^2 J_0(a, b) + 2k_1 \lambda_1 J_1(a, b) - \right. \right. \right. \right. \\ \left. \left. \left. 2k_1 \lambda_1^2 J_0(a, b) \right] + \frac{1}{4} \left(\frac{n_1}{n} \right)^2 + \frac{1}{4} \left(\frac{n_2}{n} \right)^2 w + \frac{1}{4} \lambda_1^2 - \left[\left(\frac{n_1}{n} \right)^2 J_2(a, b) + \right. \right. \right. \right. \\ \left. \left. \left. \left(\frac{n_2}{n} \right)^2 J_0(a, b) w + \lambda_1^2 J_0(a, b) + 2 \left(\frac{n_1}{n} \right) \lambda_1 J_1(a, b) \right] \right\} \quad \dots(12)$$

here

$$w = \frac{\text{var}(\hat{A}_2)}{\text{var}(\hat{A}_1)} = \frac{\sigma^2 \sum_{j=1}^{n_2} x_{2j}^2 / n_2 SS_{x_2}}{\sigma^2 \sum_{j=1}^{n_1} x_{1j}^2 / n_1 SS_{x_1}} = \frac{\sum_{j=1}^{n_2} x_{2j}^2}{\sum_{j=1}^{n_1} x_{1j}^2} \cdot \frac{SS_{x_2}}{SS_{x_1}} \cdot \frac{n_1}{n_2} = \frac{d_2}{d_1} \cdot \frac{h_1}{h_2} \cdot \frac{1}{u}, \quad d_p = \sum_{i=1}^{n_p} x_{pi}^2, \quad h_p = SS_{x_p},$$

$$SS_{x_p} = \sum_{i=1}^{n_p} (x_{pi} - \bar{x})^2, \quad p = 1, 2. \quad \text{Also, } \frac{n_2}{n_1} = u, \quad \frac{n_1}{n} = \frac{1}{1+u}, \quad \frac{n_2}{n} = \frac{u}{1+u}.$$

Therefore;

$$MSE(\tilde{A}|A, R_1) = \text{var}(\hat{A}_1) \left\{ k_1^2 J_2(a, b) + k_1^2 \lambda_1 J_0(a, b) - 2k_1^2 \lambda_1 J_1(a, b) + \lambda_1^2 J_0(a, b) + 2k_1 \lambda_1 J_1(a, b) - \right. \\ \left. 2k_1 \lambda_1^2 J_0(a, b) + \frac{1}{4} \left(\frac{1}{1+u} \right)^2 + \frac{1}{4} \left(\frac{u}{1+u} \right)^2 w + \frac{1}{4} \lambda_1^2 - \left[\left(\frac{1}{1+u} \right)^2 J_2(a, b) + \right. \right. \\ \left. \left. \left(\frac{u}{1+u} \right)^2 J_0(a, b) w + \lambda_1^2 J_0(a, b) + 2 \left(\frac{1}{1+u} \right) \lambda_1 J_1(a, b) \right] \right\} \quad \dots(13)$$

Similarly,

$$MSE(\tilde{B}|B, R_2) = E[(\tilde{B} - B)^2] \\ = \text{var}(\hat{B}_1) \left\{ k_2^2 J_2'(c, d) + k_2^2 \lambda_2 J_0'(c, d) - 2k_2^2 \lambda_2 J_1'(c, d) + \lambda_2^2 J_0'(c, d) + 2k_2 \lambda_2 J_1'(c, d) - \right. \\ \left. 2k_2 \lambda_2^2 J_0'(c, d) + \frac{1}{4} \left(\frac{1}{1+u} \right)^2 + \frac{1}{4} \left(\frac{u}{1+u} \right)^2 w' + \frac{1}{4} \lambda_2^2 - \left[\left(\frac{1}{1+u} \right)^2 J_2'(c, d) + \right. \right. \\ \left. \left. \left(\frac{u}{1+u} \right)^2 J_0'(c, d) w' + \lambda_2^2 J_0'(c, d) + 2 \left(\frac{1}{1+u} \right) \lambda_2 J_1'(c, d) \right] \right\} \quad \dots(14)$$

where $w^* = \frac{\text{var}(\hat{B}_2)}{\text{var}(\hat{B}_1)} = \frac{\sigma^2}{SS_{x_2}} / \frac{\sigma^2}{SS_{x_1}} = \frac{SS_{x_1}}{SS_{x_2}} = \frac{h_1}{h_2}$.

Also, the value of k_i ($i=1,2$) can be found by minimizing the mean squared of \tilde{A} and \tilde{B} respectively,

$$\text{i.e. } \frac{d}{dK_1} \text{MSE}(\tilde{A}|A, R_1) = 0 \text{ and } \frac{d}{dK_2} \text{MSE}(\tilde{B}|B, R_2) = 0$$

and by simple calculation we have:

$$K_1 = \frac{2\lambda_1^2 J_0(a, b) - 2\lambda_1 J_1(a, b)}{2J_2(a, b) + 2\lambda_1 J_0(a, b) - 4\lambda_1 J_1(a, b)}; \text{ w.r.t. } \tilde{A}$$

and,

$$K_2 = \frac{2\lambda_2^2 J_0^*(c, d) - 2\lambda_2 J_1^*(c, d)}{2J_2^*(c, d) + 2\lambda_2 J_0^*(c, d) - 4\lambda_2 J_1^*(c, d)}; \text{ w.r.t. } \tilde{B}.$$

Remark: To be ensure that:

(i) K_i ($i=1,2$) $\in [0,1]$, we suggest the value of K_i ($i=1,2$) as follows:

$$K = \begin{cases} 0 & , \text{ if } K_i < 0 \\ K_i & , \text{ if } 0 \leq K_i \leq 1 \\ 1 & , \text{ if } K_i > 1 \end{cases}$$

(ii) K_i is minimize $\text{MSE}(\tilde{\theta})$, we test the second derivatives for the $\text{MSE}(\tilde{\theta})$ with respect to K_i ,

$$\text{i.e. } \frac{d^2}{dK_i^2} \text{MSE}(\tilde{\theta}|\theta, R) > 0$$

where θ may be denoted to A or B and the region R may be denoted to R_1 or R_2 .

The relative efficiency of both \tilde{A} and \tilde{B} are respectively given as:

$$\text{R.Eff}(\tilde{A}|A, R_1) = \frac{\text{MSE}(\hat{A}|A)}{\text{MSE}(\tilde{A}|A, R_1)} \cdot \left(\frac{n}{E(n|A, R)} \right) = \frac{\text{var}(\hat{A}_1)}{\text{MSE}(\tilde{A}|A, R_1) \cdot \{E\}}$$

$$\text{where } E = \frac{E(n|A, R_1)}{n};$$

$$E(n|A, R_1) = n \left(1 - \frac{u}{1+u} J_0(a, b) \right) \text{ denote to expected sample size w.r.t.A.}$$

Therefore;

$$\begin{aligned} \text{R.Eff}(\tilde{A}|A, R_1) = & \left[\left\{ k_1^2 J_2(a, b) + k_1^2 \lambda_1 J_0(a, b) - 2k_1^2 \lambda_1 J_1(a, b) + \lambda_1^2 J_0(a, b) + 2k_1 \lambda_1 J_1(a, b) - \right. \right. \\ & 2k_1 \lambda_1^2 J_0(a, b) + \frac{1}{4} \left(\frac{1}{1+u} \right)^2 + \frac{1}{4} \left(\frac{u}{1+u} \right)^2 w + \frac{1}{4} \lambda_1^2 - \left[\left(\frac{1}{1+u} \right)^2 J_2(a, b) + \right. \\ & \left. \left. \left(\frac{u}{1+u} \right)^2 J_0(a, b) w + \lambda_1^2 J_0(a, b) + 2 \left(\frac{1}{1+u} \right) \lambda_1 J_1(a, b) \right] \cdot \left(1 - \frac{u}{1+u} J_0(a, b) \right) \right]^{-1} \dots (15) \end{aligned}$$

and

$$\text{R.Eff}(\tilde{B}|B, R_2) = \text{MSE}(\hat{B}|B) / \text{MSE}(\tilde{B}|B, R_2) \cdot E(n|B, R_2) = \frac{\text{var}(\hat{B})}{\text{MSE}(\tilde{B}|B, R_2) \cdot \{E^*\}}$$

$$\text{where } E^* = \frac{E^*(n|B, R_2)}{n};$$

$E^*(n|B, R_2) = n(1 - \frac{u}{1+u} J_0^*(c, d))$ refer to expected sample size w.r.t.B.

Therefore;

$$R.Eff(\tilde{B}|B, R_2) = \left[\left\{ k_2^2 J_2^*(c, d) + k_2^2 \lambda_2 J_0^*(c, d) - 2k_2^2 \lambda_2 J_1^*(c, d) + \lambda_2^2 J_0^*(c, d) + 2k_2 \lambda_2 J_1^*(c, d) - 2k_2 \lambda_2^2 J_0^*(c, d) + \frac{1}{4} \left(\frac{1}{1+u} \right)^2 + \frac{1}{4} \left(\frac{u}{1+u} \right)^2 w^* + \frac{1}{4} \lambda_2^2 - \left[\left(\frac{1}{1+u} \right)^2 J_2^*(c, d) + \left(\frac{u}{1+u} \right)^2 J_0^*(c, d) w^* + \lambda_2^2 J_0^*(c, d) + 2 \left(\frac{1}{1+u} \right) \lambda_2 J_1^*(c, d) \right] \right\} \left(1 - \frac{u}{1+u} J_0^*(c, d) \right) \right]^{-1} \dots (16)$$

where $\lambda_1 = (A - A_0) / \sqrt{\text{var}(\hat{A}_1)}$, $\lambda_2 = (B - B_0) / \sqrt{\text{var}(\hat{B}_1)}$, and

Numerical Results and Conclusions

1- \tilde{A} and \tilde{B} are consistence estimators.

i.e. $\lim_{n \rightarrow \infty} \text{MSE}(\tilde{A}|A, R_1) = 0$, and $\lim_{n \rightarrow \infty} \text{MSE}(\tilde{B}|B, R_2) = 0$.

2- \tilde{A} and \tilde{B} are dominates to \hat{A} and \hat{B} respectively with the large sample size (n).

i.e. $\lim_{n \rightarrow \infty} [\text{MSE}(\tilde{A}|A, R_1) - \text{MSE}(\hat{A}|A)] \leq 0$, and $\lim_{n \rightarrow \infty} [\text{MSE}(\tilde{B}|B, R_2) - \text{MSE}(\hat{B}|B)] \leq 0$.

3- The estimators \tilde{A} and \tilde{B} are biased when $A = A_0$ and $B = B_0$ respectively.

4- The R.Eff. of \tilde{A} and \tilde{B} are even function with λ_1 and λ_2 respectively.

5- The computation of relative efficiency R.Eff(.) and bias ratio B(.) were used for the estimators \tilde{A} and \tilde{B} , these computation were performed for $\alpha=0.01, 0.05, 0.1$ and $\lambda_i=0.0(0.1)1, i=1,2$ and $n=4,8,12,20$. Some of these computations are given in the table leads to the following conclusions:

i. The relative efficiency of \tilde{A} and that of \tilde{B} are adversely proportional with small values of α and those of n.

ii. The relative efficiency of \tilde{A} and that of \tilde{B} are maximum when $A \approx A_0$ and $B \approx B_0$ respectively (i.e. $\lambda_1 \approx 0$ and $\lambda_2 \approx 0$).

iii. The bias ratio $[B(\cdot) = \text{Bias}(\tilde{\theta}|\theta, R) / \sqrt{\text{var}(\hat{\theta})}]$ of \hat{A} and \hat{B} are reasonably small when $A \approx A_0$ and $B \approx B_0$ respectively, and maximum when λ_1 and λ_2 are maximum.

iv. The bias ratio B(.) of \tilde{A} and \tilde{B} are reasonably small with small sample size (n).

6- The considered estimators \tilde{A} and \tilde{B} are better than the classical estimators \hat{A} and \hat{B} , also than the estimators of Al-Bayyati and Arnold [1972] and Al-Kanane [1997].

7- The relative efficiency [R.Eff(.)] and bias ratio [B(.)] are decreasing function w.r.t.(n₁), and the bias ratio [B(.)] of considered estimators are decreasing functions w.r.t. α .

8- The considered estimators are increasing function w.r.t. (u) especially when $(\lambda_1 \approx 0)$ and $(\lambda_2 \approx 0)$.

9- The expected sample size closed to n₁ especially when $(\lambda_1 \approx 0)$ and $(\lambda_2 \approx 0)$.

10- The probability for avoiding the second stage is very heigh when $A \approx A_0$ and $B \approx B_0$.

11- The effective intervals [The value of λ_i which make the R.Eff. greater than 1] of \tilde{A} is [-1,1] and [-1,1] for \tilde{B} for all k_i, u, α and n.

References

1. Al-Bayyati, H.A and Arnold, J.C. (1972), "On Double Stage Estimation in Sample Linear Regression Using Prior Knowledge", *Technometrics*, 14:405-414.
2. Al-Hemyari, Z.A. (1990), "A Two-stage Estimator of the Mean of Normal Distribution", *Proc. Int. Conf. statistic, comp. Cairo*, 3, 2, 139-153.
3. Al-Kanane, I.H. (1997), "Single and Double Stage Shrunken Estimators for the Linear Regression Models", Ph.D. Thesis College of Administration and Economic, Al-Mustansiriyah University.
4. Al-Nazzal, R.I. (1996), "Double Stage Shrunken Bayesian Estimators", Ph.D. Thesis, College of Management and Economics, University of Baghdad.
5. Kalaf, B.A. (2007), "An Efficient Shrinkage Estimator for the Mean of Normal Distribution" M.Sc.Thesis College of Education Ibn-Al-Haitham, Baghdad University.
6. Kambo, N.S.; Handa, B.R. and Al-Hemyari, Z.A. (1988), "On Double Stage Shrunken Estimator for the Mean of Exponential Distribution", *Iapqr Transactions*, 13:19-33.
7. Katti, S.K. (1962) "Use of Some a Prior Knowledge in the Estimation of Means from Double Samples", *Biometrics*, 18:139-147.
8. Waikar, V.B.; Schuurmann, F.J. and Raghunathar, T.E. (1984) "On a Two-Stage Shrinkage Testimator of the Mean of a Normal Distribution", *Commum. Statist-Theor. Meth.*, A13(15):1901-1913.

Table (1) Showed Bias Ratio [B(.)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_1 when $k_1 = 0.01$ and $u = 0.5$

α	n_1	λ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.980	
		R.Eff _A	15000.000	151.500	38.164	16.986	9.559	6.119	4.250	3.123	2.391	1.889	1.530	0.383	
		R.Eff _B	15000.000	151.500	38.164	16.986	9.559	6.119	4.250	3.123	2.391	1.889	1.530	0.383	
	8	Bias	0.000	0.099	0.198	0.297	0.396	0.496	0.595	0.694	0.793	0.793	0.892	0.992	1.967
		R.Eff _A	3205.963	145.652	37.695	16.861	9.505	6.088	4.229	3.107	2.378	1.879	1.521	0.379	
		R.Eff _B	3219.326	145.681	37.697	16.862	9.505	6.088	4.229	3.107	2.378	1.879	1.521	0.379	
	12	Bias	0.000	0.099	0.199	0.298	0.398	0.498	0.590	0.690	0.700	0.799	0.899	0.997	1.882
		R.Eff _A	697.272	124.196	35.830	16.388	9.310	5.984	4.164	3.062	2.345	1.853	1.500	0.379	
		R.Eff _B	701.897	124.350	35.845	16.392	9.312	5.985	4.165	3.063	2.345	1.853	1.501	0.379	
	20	Bias	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	1.997	1.882
		R.Eff _A	320.453	101.909	33.451	15.777	9.066	5.859	4.089	3.013	2.310	1.827	1.481	0.382	
		R.Eff _B	323.015	102.179	33.484	15.786	9.070	5.861	4.090	3.013	2.311	1.828	1.481	0.382	
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.979	
		R.Eff _A	10828.228	150.851	38.110	16.971	9.552	6.115	4.247	3.120	2.389	1.887	1.529	0.381	
		R.Eff _B	10840.437	150.853	38.111	16.971	9.552	6.115	4.247	3.120	2.389	1.887	1.529	0.381	
	8	Bias	0.000	0.101	0.203	0.304	0.405	0.506	0.606	0.706	0.805	0.902	0.992	1.776	
		R.Eff _A	116.143	64.599	27.707	14.195	8.436	5.544	3.908	2.898	2.233	1.774	1.443	0.396	
		R.Eff _B	117.364	64.988	27.786	14.219	8.446	5.549	3.911	2.900	2.235	1.775	1.444	0.397	
	12	Bias	0.000	0.103	0.205	0.307	0.409	0.510	0.610	0.709	0.807	0.902	0.992	1.704	
		R.Eff _A	74.244	48.891	24.154	13.107	7.994	5.326	3.786	2.824	2.186	1.743	1.424	0.411	
		R.Eff _B	75.147	49.293	24.260	13.142	8.009	5.335	3.791	2.828	2.189	1.745	1.425	0.411	
	20	Bias	0.000	0.103	0.206	0.309	0.411	0.512	0.612	0.710	0.809	0.902	0.992	1.658	
		R.Eff _A	58.504	41.416	22.079	12.423	7.711	5.188	3.710	2.824	2.159	1.726	1.414	0.422	
		R.Eff _B	59.280	41.814	22.201	12.466	7.730	5.198	3.716	2.828	2.162	1.729	1.416	0.423	
0.01	4	Bias	0.000	0.100	0.200	0.300	0.400	0.500	0.599	0.699	0.799	0.898	0.997	1.889	
		R.Eff _A	357.006	105.483	33.872	15.887	9.110	5.881	4.102	3.021	2.316	1.831	1.484	0.381	
		R.Eff _B	359.786	105.736	33.901	15.895	9.113	5.883	4.103	3.022	2.317	1.832	1.485	0.381	
	8	Bias	0.000	0.104	0.208	0.311	0.413	0.514	0.613	0.711	0.805	0.897	0.986	1.596	
		R.Eff _A	44.439	33.74	19.604	11.552	7.343	5.008	3.613	2.724	2.126	1.708	1.404	0.439	
		R.Eff _B	45.099	34.132	19.743	11.606	7.368	5.021	3.621	2.729	2.130	1.711	1.407	0.440	
	12	Bias	0.000	0.105	0.209	0.312	0.414	0.515	0.613	0.709	0.802	0.892	0.977	1.544	
		R.Eff _A	36.010	28.606	17.702	10.838	7.036	4.858	3.534	2.680	2.102	1.696	1.400	0.456	
		R.Eff _B	36.600	28.985	17.856	10.900	7.065	4.875	3.544	2.686	2.107	1.699	1.403	0.457	
	20	Bias	0.000	0.105	0.209	0.313	0.415	0.515	0.613	0.708	0.808	0.897	0.987	1.512	
		R.Eff _A	32.010	26.005	16.651	10.424	6.855	4.771	3.488	2.656	2.090	1.690	1.399	0.468	
		R.Eff _B	32.566	26.378	16.812	10.492	6.888	4.789	3.499	2.663	2.095	1.694	1.402	0.469	

Table (2) Showed Bias Ratio [B(·)] and relative efficiency [R.Eff.] of \tilde{A} and \tilde{B} w.r.t. α , n_1 and λ_j when $k_1 = 0.01$ and $u = 1$

α	n_1	λ_j	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.980	
		R.Eff _A	20000.000	202.000	50885	22.648	12.746	8.159	5.667	4.164	3.188	2.519	2.040	2.040	0.510
		R.Eff _B	20000.000	202.000	50885	22.648	12.746	8.159	5.667	4.164	3.188	2.519	2.040	2.040	0.510
	8	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.694	0.793	0.892	0.991	0.991	1.959
		R.Eff _A	6472.285	197.445	50514	22.545	12.699	8.131	5.647	4.148	3.175	2.507	2.030	2.030	0.498
		R.Eff _B	6472.285	197.445	50514	22.545	12.699	8.131	5.647	4.148	3.175	2.507	2.030	2.030	0.498
	12	Bias	0.000	0.099	0.199	0.298	0.396	0.497	0.596	0.695	0.794	0.893	0.992	0.992	1.910
		R.Eff _A	1574.835	179.247	48.983	22.144	12.526	8.034	5.582	4.100	3.137	2.475	2.002	2.002	0.484
		R.Eff _B	1574.835	179.247	48.983	22.144	12.526	8.034	5.582	4.100	3.137	2.475	2.002	2.002	0.484
	20	Bias	0.000	0.100	0.199	0.299	0.399	0.498	0.598	0.697	0.796	0.894	0.992	0.992	1.859
		R.Eff _A	734.686	157.542	46.917	21.605	12.300	7.910	5.503	4.044	3.093	2.440	1.972	1.972	0.477
		R.Eff _B	734.686	157.542	46.917	21.605	12.300	7.910	5.503	4.044	3.093	2.440	1.972	1.972	0.477
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	0.990	1.976
		R.Eff _A	16415.887	201.503	50.843	22.635	12.740	8.155	5.664	4.161	3.186	2.517	2.039	2.039	0.506
		R.Eff _B	16415.887	201.503	50.843	22.635	12.740	8.155	5.664	4.161	3.186	2.517	2.039	2.039	0.506
	8	Bias	0.000	0.101	0.201	0.302	0.402	0.501	0.601	0.699	0.796	0.893	0.987	0.987	1.746
		R.Eff _A	266.371	112.919	41.365	20.099	11.674	7.576	5.295	3.900	2.987	2.357	1.906	1.906	0.476
		R.Eff _B	266.371	112.919	41.365	20.099	11.674	7.576	5.295	3.900	2.987	2.357	1.906	1.906	0.476
	12	Bias	0.000	0.101	0.202	0.303	0.404	0.503	0.602	0.699	0.795	0.889	0.889	0.980	1.672
		R.Eff _A	169.157	90.077	37.470	18.968	11.201	7.327	5.143	3.798	2.914	2.303	1.864	1.864	0.482
		R.Eff _B	169.157	90.077	37.470	18.968	11.201	7.327	5.143	3.798	2.914	2.303	1.864	1.864	0.482
	20	Bias	0.000	0.102	0.203	0.304	0.404	0.504	0.602	0.698	0.793	0.885	0.975	0.975	1.625
		R.Eff _A	132.499	78.140	35.006	18.212	10.881	7.160	5.043	3.732	2.868	2.269	1.839	1.839	0.489
		R.Eff _B	132.499	78.140	35.006	18.212	10.881	7.160	5.043	3.732	2.868	2.269	1.839	1.839	0.489
0.01	4	Bias	0.000	0.100	0.199	0.299	0.398	0.498	0.597	0.696	0.795	0.894	0.992	0.992	1.867
		R.Eff _A	817.517	161.239	47.291	21.703	12.341	7.932	5.517	4.054	3.101	2.446	1.977	1.977	0.478
		R.Eff _B	817.517	161.239	47.291	21.703	12.341	7.932	5.517	4.054	3.101	2.446	1.977	1.977	0.478
	8	Bias	0.000	0.102	0.204	0.305	0.405	0.504	0.601	0.696	0.789	0.878	0.965	0.965	1.563
		R.Eff _A	99.646	65.064	31.858	17.197	10.446	6.933	4.908	3.645	2.808	2.228	1.810	1.810	0.501
		R.Eff _B	99.646	65.064	31.858	17.197	10.446	6.933	4.908	3.645	2.808	2.228	1.810	1.810	0.501
	12	Bias	0.000	0.102	0.204	0.305	0.405	0.503	0.599	0.693	0.783	0.871	0.954	0.954	1.511
		R.Eff _A	79.893	55.788	29.268	16.315	10.063	6.735	4.792	3.571	2.759	2.195	1.788	1.788	0.515
		R.Eff _B	79.893	55.788	29.268	16.315	10.063	6.735	4.792	3.571	2.759	2.195	1.788	1.788	0.515
	20	Bias	0.000	0.102	0.204	0.305	0.404	0.502	0.597	0.690	0.779	0.865	0.946	0.946	1.479
		R.Eff _A	70.491	50.917	27.765	15.782	9.829	6.614	4.722	3.528	2.732	2.177	1.776	1.776	0.525
		R.Eff _B	70.491	50.917	27.765	15.782	9.829	6.614	4.722	3.528	2.732	2.177	1.776	1.776	0.525

Table (3) Showed Bias Ratio [B(.)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_1 when $k_1 = 0.01$ and $n = 9$

α	n_1	λ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.980
		R.Eff _A	1000000.000	1009.999	254.427	113.239	63.728	40.796	28.334	20.818	15.940	12.595	10.202	2.551
		R.Eff _B	1000000.000	1009.999	254.427	113.239	63.728	40.796	28.334	20.818	15.940	12.595	10.202	2.551
	8	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.890	0.989	1.941
		R.Eff _A	91201.063	1007.011	253.753	112.873	63.455	40.557	28.105	20.587	15.698	12.336	9.920	1.887
		R.Eff _B	81796.012	1005.640	253.647	112.845	63.443	40.550	28.101	20.584	15.695	12.334	9.918	1.886
	12	Bias	0.000	0.099	0.198	0.297	0.396	0.494	0.592	0.691	0.789	0.887	0.984	1.869
		R.Eff _A	63569.847	990.467	250.018	110.956	62.122	39.474	27.145	19.689	14.831	11.483	9.072	1.336
		R.Eff _B	39944.157	980.925	249.307	110.781	62.051	39.436	27.121	19.672	14.818	11.472	9.064	1.333
	20	Bias	0.000	0.099	0.198	0.297	0.395	0.494	0.592	0.689	0.786	0.883	0.978	1.805
		R.Eff _A	42475.070	962.453	243.691	107.830	60.053	37.885	25.816	18.522	13.776	10.516	8.181	1.090
		R.Eff _B	20652.437	938.905	241.940	107.412	59.889	37.800	25.765	18.487	13.751	10.496	8.166	1.087
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.971
		R.Eff _A	99041.434	1009.729	254.366	113.204	63.701	40.770	28.308	20.790	15.909	12.559	10.161	2.345
		R.Eff _B	98059.401	1009.616	254.357	113.201	63.700	40.769	28.307	20.790	15.908	12.559	10.161	2.345
	8	Bias	0.000	0.099	0.197	0.297	0.393	0.490	0.587	0.682	0.776	0.869	0.959	1.674
		R.Eff _A	18545.521	863.855	221.440	97.338	53.532	33.217	22.199	15.587	11.333	8.455	6.436	0.847
		R.Eff _B	6598.465	794.310	216.044	96.097	53.070	32.993	22.071	15.505	11.277	8.415	6.406	0.842
	12	Bias	0.000	0.098	0.196	0.294	0.391	0.487	0.582	0.675	0.767	0.856	0.943	1.594
		R.Eff _A	11378.902	777.520	201.978	88.536	48.362	29.750	19.693	13.696	9.870	7.309	5.532	0.775
		R.Eff _B	3579.939	673.778	193.451	86.592	47.655	29.416	19.508	13.581	9.794	7.255	5.492	0.768
	20	Bias	0.000	0.098	0.196	0.297	0.389	0.484	0.578	0.670	0.760	0.847	0.932	1.545
		R.Eff _A	8455.064	716.736	188.273	82.481	44.917	27.522	18.144	12.571	9.033	6.676	5.049	0.747
		R.Eff _B	2495.550	592.841	177.585	80.046	44.041	27.114	17.902	12.435	8.943	6.613	5.003	0.739
0.01	4	Bias	0.000	0.099	0.198	0.297	0.395	0.494	0.592	0.690	0.787	0.883	0.979	1.815
		R.Eff _A	45436.276	967.931	244.928	108.433	60.446	38.181	26.059	18.731	13.960	10.680	8.329	1.122
		R.Eff _B	22897.775	947.107	243.381	108.063	60.300	38.105	26.013	18.699	13.937	10.662	8.315	1.118
	8	Bias	0.000	0.097	0.194	0.291	0.386	0.480	0.572	0.663	0.749	0.834	0.915	1.484
		R.Eff _A	5780.190	633.452	169.464	74.324	40.390	24.675	16.220	11.215	8.050	5.951	4.510	0.723
		R.Eff _B	1584.262	487.668	155.884	71.214	39.282	24.167	15.946	11.049	7.942	5.877	4.456	0.714
	12	Bias	0.000	0.097	0.193	0.288	0.382	0.475	0.565	0.653	0.738	0.820	0.898	1.433
		R.Eff _A	4188.529	561.49	153.150	67.366	36.614	22.359	14.696	10.167	7.309	5.418	4.121	0.713
		R.Eff _B	1083.235	402.478	137.143	63.660	35.300	21.761	14.377	9.976	7.185	5.333	4.060	0.703
	20	Bias	0.000	0.096	0.192	0.286	0.380	0.471	0.561	0.647	0.731	0.811	0.887	1.403
		R.Eff _A	3451.185	518.769	143.444	63.270	34.422	21.036	13.840	9.588	6.906	5.132	3.916	0.711
		R.Eff _B	863.158	354.666	126.037	59.201	32.981	20.383	13.493	9.381	6.773	5.041	3.580	0.699

Table (4) Showed Bias Ratio [B(-)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_1 when $k_i = 0.01$ and $n = 12$

α	n_1	λ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.980
		R.Eff _A	130000.000	1312.999	330.755	147.210	82.847	53.034	36.834	27.064	20.722	16.373	13.263	3.316
		R.Eff _B	130000.000	1312.999	330.755	147.210	82.847	53.034	36.834	27.064	20.722	16.373	13.263	3.316
	8	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.980
		R.Eff _A	122607.823	1308.793	329.660	146.593	82.379	52.622	36.437	26.662	20.300	15.922	12.772	2.243
		R.Eff _B	108591.698	1306.859	329.511	146.554	82.362	52.612	36.431	26.657	20.297	15.919	12.769	2.241
	12	Bias	0.000	0.099	0.198	0.297	0.396	0.494	0.593	0.691	0.789	0.887	0.984	1.977
		R.Eff _A	95149.082	1284.561	323.506	143.343	80.095	50.761	34.787	25.123	18.822	14.477	11.350	1.481
		R.Eff _B	54617.727	1271.115	322.506	143.098	79.996	50.708	34.754	25.099	18.804	14.463	11.338	1.478
	20	Bias	0.000	0.099	0.198	0.296	0.395	0.493	0.591	0.689	0.786	0.882	0.977	1.970
		R.Eff _A	69081.477	1242.858	313.096	138.091	76.604	48.084	32.563	23.187	17.094	12.918	9.938	1.174
		R.Eff _B	28188.424	1209.767	310.653	137.511	76.377	47.968	32.493	23.140	17.060	12.892	9.918	1.169
0.01	4	Bias	0.000	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990	1.970
		R.Eff _A	129231	1312.634	330.658	147.152	82.800	52.990	36.789	27.015	20.667	16.311	13.191	2.963
		R.Eff _B	127937.234	1312.475	330.645	147.149	82.798	52.989	36.788	27.014	20.667	16.311	13.191	2.963
	8	Bias	0.000	0.099	0.197	0.295	0.393	0.490	0.586	0.681	0.775	0.867	0.958	1.669
		R.Eff _A	23599.990	1096.708	277.419	121.060	65.048	40.601	26.874	18.626	13.372	9.847	7.398	0.885
		R.Eff _B	8675.580	1000.272	270.067	119.382	64.429	40.303	26.675	18.520	13.301	9.796	7.359	0.879
	12	Bias	0.000	0.098	0.196	0.294	0.390	0.486	0.581	0.674	0.765	0.855	0.941	1.589
		R.Eff _A	20089.419	972.042	247.585	107.497	58.153	35.389	23.151	15.901	11.314	8.273	6.186	0.801
		R.Eff _B	4553.739	830.112	236.202	104.929	57.229	34.957	22.914	15.756	11.219	8.206	6.137	0.794
	20	Bias	0.000	0.098	0.195	0.292	0.388	0.483	0.577	0.669	0.758	0.845	0.929	1.541
		R.Eff _A	14819.435	886.571	227.343	98.538	53.113	32.185	20.968	14.352	10.187	7.441	5.565	0.768
		R.Eff _B	3103.200	718.732	213.281	95.372	51.987	31.667	20.688	14.183	10.077	7.364	5.509	0.760
0.01	4	Bias	0.000	0.099	0.198	0.297	0.395	0.494	0.592	0.689	0.786	0.883	0.978	1.812
		R.Eff _A	73054.133	1251.033	315.125	139.099	77.262	48.578	32.964	23.528	17.390	13.177	10.166	1.212
		R.Eff _B	31298.809	1221.751	312.963	138.584	77.059	48.473	32.900	23.485	17.359	13.154	10.147	1.208
	8	Bias	0.000	0.097	0.194	0.290	0.385	0.478	0.570	0.660	0.747	0.831	0.912	1.479
		R.Eff _A	9963.384	772.674	200.564	86.920	46.741	28.246	18.360	12.551	8.910	6.519	4.891	0.741
		R.Eff _B	1910.708	577.864	183.041	82.858	45.346	27.613	18.023	12.350	8.781	6.430	4.827	0.731
	12	Bias	0.000	0.096	0.192	0.287	0.381	0.473	0.563	0.651	0.736	0.817	0.895	1.428
		R.Eff _A	7088.348	677.189	178.234	77.400	41.633	25.164	16.369	11.208	7.978	5.860	4.419	0.728
		R.Eff _B	1272.635	467.226	157.919	72.761	40.006	24.432	15.982	10.980	7.832	5.760	4.347	0.716
	20	Bias	0.000	0.096	0.191	0.285	0.378	0.470	0.559	0.645	0.728	0.808	0.883	1.398
		R.Eff _A	5769.799	621.869	165.331	71.956	38.750	23.450	15.278	10.483	7.482	5.513	4.173	0.725
		R.Eff _B	998.441	406.591	143.449	66.915	36.983	22.657	14.860	10.237	7.325	5.406	4.097	0.712

Table (5) Showed Bias Ratio [B(·)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_i when $k_i = 0.1$ and $u = 0.5$

α	n_1	λ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.800
		R.Eff _A	150.000	82.873	35.377	18.094	10.745	7.059	4.973	3.686	2.839	2.252	1.829	0.462
		R.Eff _B	150.000	82.873	35.377	18.094	10.745	7.059	4.973	3.686	2.839	2.252	1.829	0.462
	8	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.787
		R.Eff _A	145.117	81.259	35.021	17.973	10.688	7.025	4.951	3.670	2.825	2.241	1.820	0.459
		R.Eff _B	145.145	81.268	35.023	17.974	10.688	7.026	4.951	3.670	2.826	2.241	1.820	0.459
	12	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.750
		R.Eff _A	126.279	74.731	33.590	17.514	10.486	6.915	4.882	3.623	2.791	2.215	1.799	0.459
		R.Eff _B	126.430	74.787	33.603	17.519	10.488	6.916	4.883	3.623	2.791	2.215	1.799	0.460
	20	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.798
		R.Eff _A	105.643	66.756	31.721	16.918	10.231	6.783	4.803	3.571	2.755	2.189	1.780	0.464
		R.Eff _B	105.920	66.872	31.751	16.928	10.236	6.785	4.805	3.572	2.756	2.189	1.780	0.464
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.798
		R.Eff _A	149.474	82.697	35.337	18.079	10.738	7.054	4.970	3.684	2.837	2.250	1.827	0.460
		R.Eff _B	149.476	82.698	35.337	18.079	10.738	7.054	4.970	3.684	2.837	2.250	1.827	0.460
	8	Bias	0.000	0.092	0.183	0.275	0.366	0.457	0.548	0.638	0.727	0.816	0.903	1.622
		R.Eff _A	68.910	49.655	27.013	15.350	9.568	6.448	4.612	3.452	2.677	2.136	1.744	0.480
		R.Eff _B	69.338	49.884	27.087	15.378	9.581	6.455	4.616	3.455	2.680	2.138	1.745	0.481
	12	Bias	0.000	0.092	0.185	0.277	0.368	0.460	0.550	0.639	0.728	0.814	0.899	1.563
		R.Eff _A	52.664	40.518	23.953	14.250	9.098	6.215	4.483	3.376	2.630	2.106	1.725	0.496
		R.Eff _B	53.117	40.794	24.057	14.292	9.118	6.226	4.490	3.380	2.633	2.109	1.728	0.497
	20	Bias	0.000	0.093	0.186	0.278	0.370	0.461	0.551	0.640	0.727	0.812	0.895	1.525
		R.Eff _A	44.778	35.640	22.111	13.550	8.793	6.064	4.401	3.329	2.602	2.090	1.717	0.508
		R.Eff _B	45.232	35.934	22.233	13.601	8.818	6.078	4.410	3.335	2.606	2.093	1.719	0.509
0.01	4	Bias	0.000	0.091	0.181	0.272	0.363	0.453	0.544	0.634	0.724	0.814	0.904	1.718
		R.Eff _A	109.020	68.130	32.055	17.025	10.277	6.806	4.817	3.580	2.761	2.193	1.783	0.463
		R.Eff _B	109.278	68.235	32.082	17.034	10.281	6.808	4.818	3.581	2.762	2.194	1.784	0.463
	8	Bias	0.000	0.093	0.186	0.279	0.371	0.462	0.551	0.639	0.725	0.808	0.889	1.475
		R.Eff _A	36.586	30.218	19.858	12.646	8.393	5.867	4.296	3.27-	2.569	2.072	1.708	0.526
		R.Eff _B	37.033	30.529	20.001	12.710	8.425	5.885	4.308	3.278	2.575	2.076	1.712	0.527
	12	Bias	0.000	0.094	0.187	0.280	0.371	0.462	0.551	0.637	0.721	0.803	0.881	1.432
		R.Eff _A	31.046	26.324	18.084	11.895	8.053	5.700	4.209	3.223	2.544	2.061	1.705	0.543
		R.Eff _B	31.483	26.644	18.244	11.971	8.092	5.723	4.223	3.233	2.551	2.066	1.710	0.545
	20	Bias	0.000	0.094	0.187	0.280	0.371	0.462	0.550	0.635	0.716	0.799	0.875	1.406
		R.Eff _A	28.225	24.266	17.086	11.455	7.851	5.601	4.159	3.197	2.532	2.056	1.705	0.555
		R.Eff _B	28.657	24.590	17.255	11.538	7.895	5.627	4.174	3.208	2.539	2.062	1.710	0.557

Table (6) Showed Bias Ratio [B(·)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_1 when $k_i = 0.1$ and $u = 1$

α	n_1	λ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.800
		R.Eff _A	200.000	110.497	47.170	24.125	14.327	9.412	6.631	4.915	3.785	3.003	2.439	0.615
		R.Eff _B	200.000	110.497	47.170	24.125	14.327	9.412	6.631	4.915	3.785	3.003	2.439	0.615
	8	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.779
		R.Eff _A	196.490	109.341	46.914	24.038	14.284	9.386	6.613	4.901	3.773	2.992	2.429	0.603
		R.Eff _B	196.490	109.341	46.914	24.038	14.284	9.386	6.613	4.901	3.773	2.992	2.429	0.603
	12	Bias	0.000	0.090	0.180	0.271	0.361	0.451	0.541	0.631	0.721	0.810	0.900	1.733
		R.Eff _A	181.742	104.393	45.845	23.690	14.126	9.296	6.554	4.857	3.739	2.963	2.404	0.589
		R.Eff _B	181.742	104.393	45.845	23.690	14.126	9.296	6.554	4.857	3.739	2.963	2.404	0.589
	20	Bias	0.000	0.090	0.181	0.271	0.361	0.452	0.542	0.631	0.721	0.810	0.898	1.689
		R.Eff _A	163.143	97.742	44.366	23.218	13.918	9.182	6.481	4.806	3.699	2.931	2.377	0.582
		R.Eff _B	163.143	97.742	44.366	23.218	13.918	9.182	6.481	4.806	3.699	2.931	2.377	0.582
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.796
		R.Eff _A	199.629	110.373	47.141	24.115	14.321	9.408	6.629	4.913	3.783	3.001	2.437	0.612
		R.Eff _B	199.629	110.373	47.141	24.115	14.321	9.408	6.629	4.913	3.783	3.001	2.437	0.612
	8	Bias	0.000	0.091	0.182	0.272	0.363	0.453	0.543	0.631	0.719	0.806	0.891	1.592
		R.Eff _A	121.772	80.802	40.198	21.862	13.330	8.868	6.286	4.672	3.600	2.854	2.314	0.580
		R.Eff _B	121.772	80.802	40.198	21.862	13.330	8.868	6.286	4.672	3.600	2.854	2.314	0.580
	12	Bias	0.000	0.091	0.182	0.273	0.363	0.453	0.542	0.630	0.716	0.801	0.884	1.530
		R.Eff _A	98.972	69.865	37.102	20.808	12.873	8.627	6.141	4.575	3.531	2.802	2.275	0.586
		R.Eff _B	98.972	69.865	37.102	20.808	12.873	8.627	6.141	4.575	3.531	2.802	2.275	0.586
	20	Bias	0.000	0.091	0.182	0.273	0.363	0.453	0.541	0.628	0.713	0.797	0.878	1.492
		R.Eff _A	86.614	63.351	35.066	20.088	12.557	8.462	6.043	4.512	3.487	2.770	2.251	0.593
		R.Eff _B	86.614	63.351	35.066	20.088	12.557	8.462	6.043	4.512	3.487	2.770	2.251	0.593
0.01	4	Bias	0.000	0.090	0.181	0.271	0.361	0.451	0.541	0.631	0.721	0.810	0.898	1.696
		R.Eff _A	166.383	98.938	44.637	23.304	13.956	9.203	6.494	4.815	3.706	2.936	2.381	0.583
		R.Eff _B	166.383	98.938	44.637	23.304	13.956	9.203	6.494	4.815	3.706	2.936	2.381	0.583
	8	Bias	0.000	0.091	0.183	0.273	0.363	0.452	0.539	0.625	0.708	0.789	0.868	1.442
		R.Eff _A	72.729	55.462	32.376	19.098	12.118	8.234	5.909	4.426	3.429	2.730	2.223	0.605
		R.Eff _B	72.729	55.462	32.376	19.098	12.118	8.234	5.909	4.426	3.429	2.730	2.223	0.605
	12	Bias	0.000	0.091	0.182	0.273	0.362	0.450	0.536	0.621	0.703	0.782	0.858	1.399
		R.Eff _A	62.658	49.318	30.083	18.216	11.722	8.029	5.791	4.352	3.381	2.698	2.201	0.618
		R.Eff _B	62.658	49.318	30.083	18.216	11.722	8.029	5.791	4.352	3.381	2.698	2.201	0.618
	20	Bias	0.000	0.091	0.182	0.272	0.361	0.449	0.534	0.618	0.698	0.776	0.850	1.374
		R.Eff _A	57.297	45.890	28.718	17.673	11.476	7.902	5.718	4.308	3.353	2.680	2.190	0.627
		R.Eff _B	57.297	45.890	28.718	17.673	11.476	7.902	5.718	4.308	3.353	2.680	2.190	0.627

Table (7) Showed Bias Ratio [B(.)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_i when $k_i = 0.1$ and $u = 9$

α	n_1	λ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.800
		R.Eff _A	1000.000	552.486	235.849	120.627	71.633	47.059	33.156	24.576	18.925	15.013	12.195	3.077
		R.Eff _B	1000.000	552.476	235.849	120.627	71.633	47.059	33.156	24.576	18.925	15.013	12.195	3.077
	8	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.629	0.717	0.809	0.898	1.76-
		R.Eff _A	1000.419	552.271	235.555	120.376	71.405	46.838	32.931	24.338	18.668	14.731	11.883	2.293
		R.Eff _B	999.158	551.859	235.464	120.345	71.390	46.828	32.924	24.333	18.664	14.728	11.880	2.291
	12	Bias	0.000	0.090	0.180	0.269	0.359	0.449	0.538	0.627	0.716	0.804	0.892	1.693
		R.Eff _A	998.915	549.449	233.461	118.874	70.198	45.775	31.940	23.379	17.719	13.781	10.926	1.636
		R.Eff _B	989.717	546.500	232.840	118.674	70.107	45.724	31.906	23.355	17.701	13.766	10.914	1.631
	20	Bias	0.000	0.090	0.179	0.269	0.358	0.447	0.536	0.624	0.712	0.799	0.884	1.634
		R.Eff _A	992.409	543.002	229.467	116.257	68.241	44.167	30.537	22.111	16.549	12.691	9.909	1.341
		R.Eff _B	968.499	535.426	227.914	115.772	68.029	44.053	30.466	22.061	16.513	12.663	9.887	1.335
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.790
		R.Eff _A	1000.094	552.491	235.830	120.607	71.612	47.036	33.132	24.548	18.893	14.975	12.151	2.836
		R.Eff _B	999.993	552.458	235.822	120.607	71.611	47.035	33.131	24.548	18.892	14.975	12.150	2.835
	8	Bias	0.000	0.089	0.178	0.266	0.354	0.442	0.529	0.614	0.702	0.782	0.863	1.520
		R.Eff _A	955.305	514.331	213.935	106.929	61.819	39.308	26.637	18.868	13.802	10.344	7.900	1.046
		R.Eff _B	873.803	488.848	208.895	105.433	61.203	38.994	26.452	18.749	13.719	10.283	7.854	1.038
	12	Bias	0.000	0.088	0.176	0.264	0.351	0.437	0.522	0.606	0.696	0.788	0.877	1.453
		R.Eff _A	912.792	485.030	199.368	98.754	56.568	35.614	23.887	16.751	12.139	9.025	6.851	0.956
		R.Eff _B	777.005	442.526	191.055	98.342	55.604	35.137	23.615	16.579	12.023	8.943	6.790	0.946
	20	Bias	0.000	0.088	0.175	0.262	0.348	0.433	0.517	0.600	0.680	0.769	0.855	1.412
		R.Eff _A	878.676	462.676	188.730	93.000	53.011	33.212	22.170	15.481	11.181	8.293	6.288	0.920
		R.Eff _B	703.969	407.677	177.991	89.915	51.796	32.620	21.838	15.275	11.043	8.196	6.217	0.908
0.01	4	Bias	0.000	0.090	0.179	0.269	0.358	0.447	0.536	0.625	0.712	0.800	0.886	1.644
		R.Eff _A	993.907	544.358	230.272	116.771	68.617	44.470	30.795	22.339	16.754	12.877	10.078	1.379
		R.Eff _B	972.958	537.709	228.905	116.342	68.429	44.367	30.731	22.294	16.721	12.851	10.058	1.374
	8	Bias	0.000	0.087	0.173	0.259	0.344	0.428	0.510	0.590	0.669	0.745	0.818	1.362
		R.Eff _A	826.688	429.944	173.690	85.096	48.271	30.108	20.021	13.939	10.050	7.452	5.657	0.887
		R.Eff _B	599.574	357.423	159.452	81.043	46.698	29.355	19.605	13.685	9.883	7.335	5.572	0.873
	12	Bias	0.000	0.086	0.171	0.256	0.340	0.422	0.503	0.581	0.657	0.731	0.802	1.321
		R.Eff _A	776.624	399.674	160.249	78.220	44.258	27.554	18.302	12.740	9.193	6.829	5.200	0.870
		R.Eff _B	507.090	311.960	142.809	73.268	42.353	26.652	17.810	12.442	8.998	6.695	5.102	0.855
	20	Bias	0.000	0.085	0.170	0.254	0.337	0.418	0.498	0.575	0.650	0.722	0.791	1.297
		R.Eff _A	744.428	380.807	152.076	74.114	41.904	26.082	17.329	12.073	8.724	6.494	4.957	0.864
		R.Eff _B	452.060	284.259	132.652	68.592	39.787	25.086	16.788	11.747	8.512	6.348	4.852	0.847

Table (8) Showed Bias Ratio [B(-)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_i when $k_i = 0.1$ and $u = 12$

α	n_1	λ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.800
		R.Eff _A	1300.000	718.232	306.604	156.815	93.123	61.176	43.103	31.949	24.603	19.517	15.854	4.000
		R.Eff _B	1300.000	718.232	306.604	156.815	93.123	61.176	43.103	31.949	24.603	19.517	15.854	4.000
	8	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.629	0.719	0.809	0.898	1.759
		R.Eff _A	1300.207	717.637	306.015	156.339	92.701	60.771	42.694	31.519	24.141	19.014	15.300	2.725
		R.Eff _B	1298.430	717.056	305.887	156.294	92.679	60.759	42.685	31.513	24.136	19.009	15.296	2.723
	12	Bias	0.000	0.090	0.180	0.269	0.359	0.449	0.538	0.627	0.716	0.804	0.892	1.691
		R.Eff _A	1295.494	711.835	302.065	153.579	90.513	58.867	40.935	29.833	22.489	17.376	13.670	1.814
		R.Eff _B	1282.535	707.687	301.193	153.298	90.386	58.796	40.889	29.800	22.464	17.356	13.653	1.81-
	20	Bias	0.000	0.090	0.179	0.269	0.358	0.447	0.535	0.624	0.711	0.798	0.884	1.631
		R.Eff _A	1281.636	699.568	294.790	148.896	87.059	56.066	38.524	27.685	20.539	15.593	12.040	1.444
		R.Eff _B	1248.046	688.960	292.624	148.222	86.766	55.908	38.425	27.618	20.490	15.555	12.010	1.437
0.01	4	Bias	0.000	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900	1.790
		R.Eff _A	1300.106	718.217	306.562	156.775	93.083	61.135	43.058	31.898	24.544	19.449	15.774	3.583
		R.Eff _B	1299.963	718.169	306.551	156.771	93.081	61.134	43.057	31.897	24.544	19.448	15.773	3.582
	8	Bias	0.000	0.089	0.177	0.266	0.354	0.441	0.528	0.613	0.698	0.781	0.862	1.516
		R.Eff _A	1214.131	649.016	267.972	133.028	76.301	48.064	32.223	22.557	16.293	12.052	9.085	1.093
		R.Eff _B	1101.048	613.985	261.105	131.005	75.475	47.647	31.981	22.402	16.187	11.975	9.027	1.084
	12	Bias	0.000	0.088	0.176	0.263	0.350	0.436	0.521	0.605	0.695	0.786	0.867	1.448
		R.Eff _A	1144.543	601.030	244.360	119.969	68.064	42.392	28.099	19.460	13.925	10.223	7.666	0.989
		R.Eff _B	958.282	543.565	233.266	116.780	66.801	41.774	27.750	19.243	13.780	10.121	7.591	0.978
	20	Bias	0.000	0.087	0.175	0.261	0.347	0.432	0.516	0.598	0.678	0.756	0.832	1.408
		R.Eff _A	1092.360	566.271	227.901	111.187	62.736	38.870	25.643	17.698	12.621	9.251	6.936	0.947
		R.Eff _B	854.541	492.779	213.771	107.173	61.171	38.117	25.224	17.432	12.452	9.133	6.850	0.934
0.01	4	Bias	0.000	0.090	0.179	0.269	0.358	0.447	0.536	0.624	0.712	0.799	0.885	1.641
		R.Eff _A	1284.668	702.098	296.240	149.808	87.716	56.586	38.961	28.064	20.874	15.891	12.304	1.490
		R.Eff _B	1255.220	692.780	294.329	149.210	87.455	56.445	38.872	28.003	20.830	15.856	12.277	1.484
	8	Bias	0.000	0.086	0.173	0.258	0.343	0.426	0.508	0.588	0.666	0.742	0.815	1.357
		R.Eff _A	1017.769	517.859	205.631	99.628	55.925	34.503	22.687	15.618	11.136	8.171	6.142	0.910
		R.Eff _B	711.470	422.416	187.252	94.458	53.940	33.564	22.174	15.308	10.935	8.032	6.042	0.894
	12	Bias	0.000	0.085	0.171	0.255	0.338	0.420	0.501	0.579	0.655	0.728	0.798	1.316
		R.Eff _A	950.715	475.384	186.635	90.008	50.400	31.054	20.413	14.063	10.048	7.395	5.582	0.889
		R.Eff _B	589.437	361.381	164.479	83.795	48.036	29.946	19.814	13.705	9.816	7.236	5.468	0.872
	20	Bias	0.000	0.085	0.169	0.253	0.335	0.416	0.496	0.573	0.647	0.719	0.788	1.292
		R.Eff _A	909.761	449.926	175.471	84.442	47.254	29.121	19.159	13.220	9.465	6.985	5.290	0.882
		R.Eff _B	518.824	325.308	151.021	77.583	44.653	27.909	18.507	12.831	9.215	6.814	5.168	0.863

Table (9) Showed Bias Ratio [B(.)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_i when $k_i = 0.5$ and $u = 0.5$

α	n_1	λ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	1.000	
		R.Eff _A	6.000	5.941	5.769	5.505	5.172	4.800	4.412	4.027	3.659	3.315	3.000	1.200	
		R.Eff _B	6.000	5.941	5.769	5.505	5.172	4.800	4.412	4.027	3.659	3.315	3.000	1.200	
	8	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.299	0.299	0.349	0.399	0.449	0.498	0.985
		R.Eff _A	6.010	5.951	5.781	5.519	5.189	4.819	4.434	4.051	3.685	3.344	3.031	2.732	1.232
		R.Eff _B	6.010	5.951	5.781	5.519	5.189	4.819	4.434	4.051	3.686	3.344	3.031	2.732	1.232
	12	Bias	0.000	0.050	0.099	0.149	0.199	0.247	0.298	0.298	0.347	0.396	0.445	0.494	0.966
		R.Eff _A	6.056	5.998	5.831	5.572	5.247	4.881	4.499	4.119	3.755	3.413	3.099	2.745	1.245
		R.Eff _B	6.056	5.998	5.831	5.573	5.247	4.882	4.500	4.120	3.756	3.414	3.100	2.747	1.247
	20	Bias	0.000	0.049	0.099	0.148	0.197	0.246	0.295	0.295	0.344	0.392	0.441	0.489	0.955
		R.Eff _A	6.124	6.067	5.902	5.645	5.323	4.959	4.577	4.196	3.829	3.483	3.164	2.841	1.241
		R.Eff _B	6.125	6.068	5.902	5.647	5.324	4.960	4.578	4.198	3.830	3.485	3.166	2.843	1.243
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.995	
		R.Eff _A	6.001	5.942	5.771	5.506	5.175	4.803	4.415	4.031	3.663	3.320	3.006	2.713	
		R.Eff _B	6.001	5.942	5.771	5.506	5.175	4.803	4.415	4.031	3.663	3.320	3.006	2.713	
	8	Bias	0.000	0.048	0.097	0.145	0.193	0.241	0.289	0.289	0.336	0.383	0.430	0.477	0.940
		R.Eff _A	6.346	6.289	6.123	5.866	5.539	5.168	4.776	4.381	3.997	3.632	3.293	2.998	1.207
		R.Eff _B	6.350	6.292	6.127	5.870	5.544	5.173	4.781	4.386	4.002	3.638	3.298	2.998	1.211
	12	Bias	0.000	0.048	0.095	0.142	0.190	0.237	0.284	0.284	0.330	0.377	0.423	0.469	0.935
		R.Eff _A	6.545	6.487	6.317	6.052	5.714	5.330	4.921	4.508	4.105	3.721	3.362	3.062	1.176
		R.Eff _B	6.552	6.494	6.324	6.059	5.722	5.338	4.930	4.517	4.113	3.729	3.370	3.070	1.180
	20	Bias	0.000	0.047	0.094	0.141	0.187	0.234	0.280	0.280	0.326	0.372	0.418	0.464	0.934
		R.Eff _A	6.696	6.636	6.461	6.187	5.839	5.441	5.018	4.589	4.170	3.771	3.399	3.099	1.154
		R.Eff _B	6.706	6.646	6.471	6.198	5.850	5.452	5.029	4.601	4.181	3.782	3.409	3.099	1.159
0.01	4	Bias	0.000	0.049	0.099	0.148	0.197	0.247	0.296	0.296	0.344	0.393	0.441	0.490	0.956
		R.Eff _A	6.111	6.054	5.888	5.632	5.309	4.945	4.563	4.183	3.816	3.471	3.153	2.843	1.243
		R.Eff _B	6.112	6.055	5.889	5.633	5.310	4.946	4.565	4.184	3.818	3.473	3.155	2.845	1.245
	8	Bias	0.000	0.049	0.092	0.138	0.184	0.230	0.276	0.276	0.321	0.367	0.413	0.458	0.934
		R.Eff _A	6.924	6.861	6.675	6.386	6.017	5.596	5.147	4.693	4.250	3.828	3.435	3.099	1.125
		R.Eff _B	6.940	6.877	6.692	6.403	6.034	5.612	5.164	4.709	4.265	3.843	3.449	3.099	1.130
	12	Bias	0.000	0.045	0.091	0.136	0.181	0.226	0.272	0.272	0.317	0.362	0.408	0.453	0.935
		R.Eff _A	7.150	7.082	6.885	6.576	6.183	5.735	5.259	4.778	4.310	3.867	3.455	3.099	1.099
		R.Eff _B	7.173	7.105	6.908	6.599	6.206	5.757	5.281	4.799	4.330	3.885	3.472	3.099	1.104
	20	Bias	0.000	0.045	0.090	0.135	0.180	0.224	0.269	0.269	0.314	0.360	0.405	0.451	0.963
		R.Eff _A	7.300	7.229	7.022	6.699	6.288	5.820	5.325	4.826	4.341	3.884	3.461	3.099	1.083
		R.Eff _B	7.329	7.258	7.050	6.727	6.316	5.848	5.351	4.850	4.364	3.905	3.481	3.099	1.089

Table (10) Showed Bias Ratio [B(·)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_j when $k_j = 0.5$ and $u = 1$

α	n_1	λ_j	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	1.000	
		R.Eff _A	8.000	7.921	7.692	7.339	6.897	6.400	5.882	5.369	4.878	4.420	4.000	1.600	
		R.Eff _B	8.000	7.921	7.692	7.339	6.897	6.400	5.882	5.369	4.878	4.420	4.000	1.600	
	8	Bias	0.000	0.50	0.100	0.150	0.200	0.249	0.299	0.349	0.399	0.399	0.448	0.497	0.977
		R.Eff _A	8.018	7.940	7.713	7.634	6.925	6.434	5.921	5.412	4.925	4.471	4.054	4.054	1.642
		R.Eff _B	8.018	7.940	7.713	7.634	6.925	6.434	5.921	5.412	4.925	4.471	4.054	4.054	1.642
	12	Bias	0.000	0.050	0.099	0.149	0.198	0.247	0.296	0.345	0.394	0.394	0.442	0.491	0.950
		R.Eff _A	8.097	8.021	7.800	7.458	7.027	6.543	6.036	5.531	5.045	4.590	4.170	4.170	1.644
		R.Eff _B	8.097	8.021	7.800	7.458	7.027	6.543	6.036	5.531	5.045	4.590	4.170	4.170	1.644
	20	Bias	0.000	0.049	0.098	0.147	0.196	0.244	0.293	0.341	0.389	0.389	0.436	0.483	0.932
		R.Eff _A	8.215	8.140	7.922	7.585	7.159	6.677	6.171	5.663	5.173	4.709	4.279	4.279	1.619
		R.Eff _B	8.215	8.140	7.922	7.585	7.159	6.677	6.171	5.663	5.173	4.709	4.279	4.279	1.619
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.993	
		R.Eff _A	8.002	7.923	7.695	7.343	6.900	6.405	5.888	5.376	4.886	4.429	4.011	1.619	
		R.Eff _B	8.002	7.923	7.695	7.343	6.900	6.405	5.888	5.376	4.886	4.429	4.011	1.619	
	8	Bias	0.000	0.048	0.095	0.142	0.189	0.236	0.283	0.329	0.375	0.420	0.465	0.465	0.910
		R.Eff _A	8.600	8.525	8.308	7.964	7.538	7.045	6.519	5.986	5.464	4.964	4.495	4.495	1.530
		R.Eff _B	8.600	8.525	8.308	7.964	7.538	7.045	6.519	5.986	5.464	4.964	4.495	4.495	1.530
	12	Bias	0.000	0.046	0.092	0.139	0.184	0.230	0.275	0.320	0.365	0.365	0.409	0.454	0.903
		R.Eff _A	8.950	8.873	8.649	8.298	7.849	7.332	6.778	6.212	5.653	5.117	4.610	4.610	1.461
		R.Eff _B	8.950	8.873	8.649	8.298	7.849	7.332	6.778	6.212	5.653	5.117	4.610	4.610	1.461
	20	Bias	0.000	0.045	0.091	0.136	0.181	0.226	0.270	0.315	0.359	0.359	0.403	0.446	0.901
		R.Eff _A	9.219	9.139	8.907	8.542	8.074	7.534	6.953	6.358	5.770	5.204	4.670	4.670	1.416
		R.Eff _B	9.219	9.139	8.907	8.542	8.074	7.534	6.953	6.358	5.770	5.204	4.670	4.670	1.416
0.01	4	Bias	0.000	0.049	0.098	0.147	0.196	0.245	0.293	0.342	0.390	0.437	0.484	0.935	
		R.Eff _A	8.193	8.117	7.899	7.561	7.135	6.653	6.147	5.640	5.151	4.689	4.261	1.624	
		R.Eff _B	8.193	8.117	7.899	7.561	7.135	6.653	6.147	5.640	5.151	4.689	4.261	1.624	
	8	Bias	0.000	0.044	0.088	0.132	0.176	0.220	0.263	0.307	0.350	0.350	0.394	0.437	0.900
		R.Eff _A	9.635	9.549	9.300	8.908	8.403	7.820	7.191	6.548	5.912	5.302	4.728	4.728	1.357
		R.Eff _B	9.635	9.549	9.300	8.908	8.403	7.820	7.191	6.548	5.912	5.302	4.728	4.728	1.357
	12	Bias	0.000	0.043	0.086	0.129	0.172	0.215	0.258	0.301	0.344	0.387	0.430	0.430	0.902
		R.Eff _A	10.058	9.965	9.693	9.266	8.717	8.083	7.402	6.706	6.021	5.366	4.756	4.756	1.307
		R.Eff _B	10.058	9.965	9.693	9.266	8.717	8.083	7.402	6.706	6.021	5.366	4.756	4.756	1.307
	20	Bias	0.000	0.042	0.085	0.127	0.169	0.212	0.254	0.297	0.340	0.383	0.426	0.426	0.904
		R.Eff _A	10.347	10.248	9.959	9.504	8.921	8.249	7.529	6.796	6.078	5.395	4.761	4.761	1.277
		R.Eff _B	10.347	10.248	9.959	9.504	8.921	8.249	7.529	6.796	6.078	5.395	4.761	4.761	1.277

Table (11) Showed Bias Ratio [B(-)] and relative efficiency [R.Eff.] of \hat{A} and \hat{B} w.r.t. α , n_1 and λ_i when $k_i = 0.5$ and $u = 9$

α	n_1	λ_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	1.000
		R.Eff _A	24.000	23.762	23.077	22.018	20.690	19.200	17.647	16.107	14.634	13.260	12.000	4.800
		R.Eff _B	24.000	23.762	23.077	22.018	20.690	19.200	17.647	16.107	14.634	13.260	12.000	4.800
	8	Bias	0.000	0.050	0.100	0.150	0.199	0.249	0.299	0.348	0.398	0.447	0.496	0.961
		R.Eff _A	24.051	23.813	23.137	22.088	20.769	19.289	17.744	16.209	14.735	13.355	12.084	4.378
		R.Eff _B	24.050	23.815	23.136	22.086	20.768	19.287	17.742	16.206	14.732	13.352	12.080	4.370
	12	Bias	0.000	0.049	0.099	0.148	0.197	0.245	0.294	0.342	0.390	0.437	0.484	0.916
		R.Eff _A	24.245	24.014	23.344	22.305	20.994	19.514	17.956	16.395	14.881	13.449	12.13	3.748
		R.Eff _B	24.238	24.006	23.336	22.296	20.984	19.502	17.943	16.380	14.865	13.431	12.095	3.727
	20	Bias	0.000	0.048	0.097	0.145	0.193	0.241	0.288	0.335	0.381	0.427	0.472	0.887
		R.Eff _A	24.489	24.258	23.589	22.548	21.226	19.722	18.128	16.517	14.943	13.441	12.032	3.337
		R.Eff _B	24.470	24.239	23.568	22.525	21.200	19.694	18.097	16.484	14.907	13.404	11.993	3.306
4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.449	0.499	0.988	
	R.Eff _A	24.006	23.769	23.085	22.028	20.701	19.214	17.664	16.127	14.656	13.283	12.024	4.715	
	R.Eff _B	24.006	23.769	23.084	22.028	20.701	19.214	17.664	16.127	14.655	13.282	12.024	4.713	
8	Bias	0.000	0.046	0.092	0.137	0.182	0.227	0.271	0.315	0.358	0.400	0.442	0.850	
	R.Eff _A	25.110	24.872	24.178	23.091	21.697	20.092	18.371	16.614	14.885	13.229	11.676	2.728	
	R.Eff _B	25.035	24.795	24.099	23.006	21.606	19.996	18.270	16.509	14.778	13.121	11.570	2.679	
12	Bias	0.000	0.044	0.087	0.131	0.174	0.217	0.259	0.301	0.342	0.382	0.423	0.838	
	R.Eff _A	25.554	25.305	24.581	23.441	21.975	20.281	18.460	16.599	14.768	13.021	11.392	2.430	
	R.Eff _B	25.407	25.157	24.428	23.281	21.807	20.107	18.281	16.418	14.589	12.845	11.223	2.375	
20	Bias	0.000	0.042	0.084	0.126	0.168	0.210	0.250	0.291	0.331	0.371	0.411	0.835	
	R.Eff _A	25.860	25.603	24.854	23.674	22.154	20.397	18.507	16.577	14.682	12.878	11.202	2.265	
	R.Eff _B	25.645	25.386	24.632	23.444	21.916	20.153	18.260	16.331	14.441	12.647	10.984	2.206	
4	Bias	0.000	0.049	0.097	0.146	0.194	0.242	0.289	0.336	0.383	0.429	0.474	0.891	
	R.Eff _A	24.446	24.215	23.546	22.506	21.187	19.689	18.102	16.501	14.937	13.448	12.016	3.398	
	R.Eff _B	24.429	24.198	23.528	22.486	21.165	19.664	18.075	16.471	14.906	13.414	12.016	3.368	
8	Bias	0.000	0.040	0.080	0.120	0.160	0.200	0.239	0.278	0.317	0.356	0.396	0.834	
	R.Eff _A	26.322	26.051	25.263	24.021	22.419	20.568	18.578	16.548	14.562	12.680	10.942	2.069	
	R.Eff _B	25.976	25.704	24.910	23.660	22.052	20.198	18.211	16.191	14.221	12.360	10.648	2.008	
12	Bias	0.000	0.038	0.077	0.115	0.153	0.191	0.229	0.268	0.306	0.345	0.384	0.837	
	R.Eff _A	26.816	26.531	25.701	24.392	22.705	20.755	18.662	16.532	14.454	12.496	10.699	1.917	
	R.Eff _B	26.302	26.015	25.181	23.868	22.179	20.235	18.156	16.049	14.003	12.083	10.330	1.856	
20	Bias	0.000	0.037	0.074	0.112	0.149	0.186	0.224	0.261	0.299	0.338	0.377	0.840	
	R.Eff _A	27.185	26.889	26.028	24.669	22.919	20.897	18.728	16.526	14.384	12.373	10.539	1.832	
	R.Eff _B	26.532	26.235	25.372	24.013	22.268	20.260	18.116	15.951	13.855	11.897	10.119	1.771	

Table (12) Showed Bias Ratio [B(·)] and relative efficiency [R.Eff.] of \tilde{A} and \tilde{B} w.r.t. α , n_1 and λ_1 when $k_1 = 0.5$ and $n = 12$

α	n_1	λ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	1.000
		R.Eff _A	52.000	51.485	50.000	47.706	44.828	41.600	38.235	34.899	31.707	28.729	26.000	10.400
		R.Eff _B	52.000	51.485	50.000	47.706	44.828	41.600	38.235	34.899	31.707	28.729	26.000	10.400
	8	Bias	0.000	0.050	0.100	0.150	0.199	0.249	0.299	0.348	0.397	0.446	0.495	0.957
		R.Eff _A	52.039	51.524	50.040	47.744	44.856	41.609	38.209	34.819	31.552	28.474	25.619	7.642
		R.Eff _B	52.036	51.521	50.037	47.740	44.851	41.603	38.202	34.811	31.542	28.464	25.607	7.622
	12	Bias	0.000	0.049	0.098	0.147	0.196	0.245	0.293	0.341	0.389	0.436	0.483	0.907
		R.Eff _A	52.002	51.478	49.961	47.604	44.619	41.233	37.652	34.041	30.519	27.164	24.023	5.426
		R.Eff _B	51.981	51.456	49.937	47.577	44.588	41.198	37.613	33.998	30.473	27.115	23.972	5.383
	20	Bias	0.000	0.048	0.097	0.145	0.192	0.240	0.287	0.333	0.379	0.424	0.469	0.875
		R.Eff _A	51.694	51.150	49.572	47.112	43.982	40.415	36.626	32.797	29.062	25.520	22.23	4.433
		R.Eff _B	51.638	51.092	49.510	47.004	43.907	40.33	36.538	32.702	28.964	25.419	22.129	4.472
0.01	4	Bias	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.449	0.499	0.987
		R.Eff _A	52.007	51.493	50.009	47.717	44.839	41.612	38.246	34.906	31.707	28.718	25.971	9.564
		R.Eff _B	52.007	51.493	50.009	47.716	44.839	41.611	38.245	34.905	31.706	28.716	25.970	9.559
	8	Bias	0.000	0.045	0.091	0.136	0.181	0.225	0.272	0.311	0.354	0.395	0.436	0.833
		R.Eff _A	49.883	49.287	47.559	44.867	41.81	37.577	33.504	29.447	25.572	21.987	18.755	3.308
		R.Eff _B	49.673	49.074	47.338	44.634	41.45041.205	37.322	33.242	29.183	25.310	21.732	18.511	3.222
	12	Bias	0.000	0.043	0.086	0.129	0.171	0.213	0.254	0.295	0.335	0.375	0.414	0.821
		R.Eff _A	47.967	47.351	45.571	42.809	39.327	35.414	31.341	27.333	23.551	20.098	17.023	2.854
		R.Eff _B	47.579	46.960	45.170	42.396	38.902	34.981	30.908	26.907	23.140	19.708	16.659	2.758
	20	Bias	0.000	0.041	0.083	0.124	0.165	0.205	0.245	0.285	0.324	0.363	0.401	0.817
		R.Eff _A	46.572	45.954	44.171	41.413	37.948	34.073	30.060	26.131	22.442	19.089	16.115	2.617
		R.Eff _B	46.026	45.405	43.613	40.843	37.370	33.492	29.487	25.575	21.913	18.595	15.661	2.518
0.01	4	Bias	0.000	0.048	0.097	0.145	0.193	0.241	0.288	0.335	0.381	0.426	0.471	0.879
		R.Eff _A	51.767	51.227	49.660	47.219	44.115	40.579	36.824	33.027	29.323	25.804	22.529	4.563
		R.Eff _B	51.719	51.177	49.606	47.159	44.049	40.506	36.745	32.943	29.234	25.712	22.437	4.505
	8	Bias	0.000	0.039	0.078	0.117	0.156	0.194	0.233	0.271	0.308	0.346	0.384	0.816
		R.Eff _A	44.706	44.098	42.344	39.638	36.249	32.471	28.571	24.763	21.198	17.963	15.101	2.348
		R.Eff _B	43.876	43.265	41.506	38.793	35.405	31.637	27.762	23.993	20.478	17.305	14.509	2.248
	12	Bias	0.000	0.037	0.074	0.111	0.148	0.185	0.222	0.259	0.296	0.333	0.371	0.819
		R.Eff _A	43.232	42.640	40.931	38.294	34.992	31.309	27.505	23.787	20.303	17.141	14.344	2.146
		R.Eff _B	42.060	41.466	39.757	37.123	33.836	30.184	26.430	22.780	19.380	16.313	13.616	2.047
	20	Bias	0.000	0.036	0.072	0.107	0.143	0.179	0.215	0.252	0.288	0.326	0.364	0.822
		R.Eff _A	42.473	41.890	40.210	37.616	34.362	30.727	26.966	23.284	19.829	16.692	13.920	2.035
		R.Eff _B	41.029	40.448	38.772	36.190	32.965	29.380	25.692	22.105	18.762	15.748	13.103	1.937

Table (13) Showed probability for avoiding the Second Sample of \tilde{A} and \tilde{B} when $\alpha = 0.01$

λ_1 \ n_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
4	0.992	0.992	0.992	0.992	0.992	0.991	0.991	0.991	0.991	0.991	0.990	0.990	0.988	0.980	0.968
8	0.992	0.991	0.991	0.991	0.991	0.991	0.990	0.987	0.986	0.985	0.984	0.977	0.934	0.842	0.693
12	0.992	0.990	0.988	0.987	0.986	0.985	0.984	0.982	0.979	0.976	0.972	0.946	0.867	0.729	0.545
16	0.992	0.987	0.986	0.985	0.984	0.982	0.980	0.977	0.974	0.969	0.964	0.927	0.829	0.674	0.481
20	0.992	0.987	0.984	0.982	0.978	0.974	0.968	0.960	0.951	0.941	0.928	0.832	0.679	0.486	0.297

Table (14) Showed Expected Sample size of \tilde{A} and \tilde{B} when $\alpha = 0.01$ and $n_1 = 12$

λ_1 \ n	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
0.5	12.048	12.059	12.074	12.077	12.082	12.088	12.097	12.108	12.123	12.141	12.165	12.322	12.799	13.622	14.732
1	12.096	12.120	12.147	12.154	12.168	12.176	12.193	12.217	12.246	12.283	12.329	12.644	13.599	15.249	17.464
2	12.192	12.240	12.295	12.307	12.326	12.352	12.388	12.433	12.492	12.566	12.659	13.287	15.198	18.489	22.929
3	12.289	12.359	12.442	12.461	12.489	12.529	12.581	12.649	12.738	12.849	12.987	13.931	16.797	21.734	28.393
4	12.385	12.478	12.589	12.615	12.652	12.705	12.775	12.866	12.983	13.132	13.317	14.575	18.396	24.979	33.857
5	12.481	12.599	12.737	12.767	12.815	12.881	12.969	13.083	13.229	13.419	13.646	15.218	19.995	28.224	39.322
10	12.962	13.198	13.474	13.537	13.631	13.762	13.938	14.166	14.459	14.829	15.293	18.437	27.991	44.448	66.644

Table (15) Showed Proportion of Expected Sample size of \tilde{A} and \tilde{B} when $\alpha = 0.01$ and $n_1 = 12$

λ_1 u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
0.5	0.669	0.669	0.671	0.671	0.671	0.671	0.672	0.672	0.673	0.674	0.676	0.684	0.711	0.757	0.818
1	0.502	0.505	0.506	0.506	0.507	0.507	0.508	0.509	0.510	0.511	0.514	0.527	0.566	0.635	0.728
2	0.335	0.339	0.341	0.342	0.342	0.343	0.344	0.345	0.346	0.349	0.352	0.369	0.409	0.514	0.637
3	0.251	0.257	0.259	0.259	0.260	0.261	0.262	0.263	0.265	0.267	0.270	0.290	0.349	0.453	0.591
4	0.206	0.208	0.209	0.210	0.211	0.211	0.212	0.214	0.216	0.216	0.222	0.243	0.306	0.416	0.564
5	0.167	0.175	0.177	0.177	0.177	0.178	0.180	0.182	0.184	0.186	0.189	0.211	0.278	0.392	0.546
10	0.098	0.099	0.102	0.102	0.103	0.104	0.105	0.107	0.109	0.112	0.115	0.139	0.212	0.337	0.505

Table (16) Showed Percentage of Second Sample save of \tilde{A} and \tilde{B} when $\alpha = 0.01$ and $n_1 = 12$

λ_1 U	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	2.5	3.0
0.5	33.066	33.000	32.923	32.906	32.880	32.843	32.794	32.731	32.650	32.547	32.418	31.545	28.891	24.319	18.154
1	49.599	49.501	49.386	49.359	49.320	49.265	49.192	49.097	48.975	48.821	48.628	47.318	43.337	36.480	27.231
2	66.132	66.000	65.847	65.812	65.760	65.686	65.589	65.462	64.300	64.094	63.836	63.090	57.782	48.639	36.308
3	74.398	74.251	74.079	74.039	73.980	73.898	73.788	73.646	73.463	73.231	72.942	70.977	65.005	54.720	40.847
4	79.358	79.201	79.017	78.975	78.912	78.824	78.708	78.556	78.360	78.113	77.804	75.708	69.229	58.368	43.570
5	82.665	82.501	82.309	82.265	82.200	82.108	81.987	81.828	81.625	81.368	81.046	78.863	72.228	60.799	45.385
10	90.180	90.001	89.792	89.744	89.673	89.573	89.440	89.268	89.046	88.765	88.414	86.032	78.794	66.327	49.511

حول مقدرات الاختبار الأولي المقلصة ذي المرحلتين المعدلة لتقدير معالم أنموذج الانحدار الخطي البسيط

عباس نجم سلمان، مها عبد الجبار محمد

قسم الرياضيات، كلية التربية- ابن الهيثم، جامعة بغداد

استلم البحث في 7 شباط 2010

قبل البحث في 25 نيسان 2010

الخلاصة

في هذا البحث اقترح مقدر الاختبار الأولي المقلص المعدل ذو المرحلتين لتقدير معالم أنموذج الانحدار الخطي البسيط وذلك باختيار عامل تقلص موزون $\psi(\cdot)$ في المرحلة الأولى، فضلا عن اقتراح تقنية معدلة في المرحلة الثانية، واشتقت معادلات التحيز، متوسط مربعات الخطأ، والكفاية النسبية، وحجم العينة المتوقع واحتمالية تجنب المرحلة الثانية (التوقف في المرحلة الأولى) للمقدرات المقترحة، واعطيت بعض النتائج العددية الخاصة بنسبة التحيز $[B(\cdot)]$ والكفاية النسبية $[R.Eff(\cdot)]$ للمقدرات المذكورة ولمختلف الثوابت فيها، ثم اعطيت بعض الخواص لهذه المقدرات وقورنت مع المقدرات الكلاسيكية وكذلك قورنت مع المقدرات المقترحة من بعض الباحثين لبيان كفايتها. الكلمات المفتاحية:- التقدير، معالم، النقل، معلومات مسبقة، المقدر ذو المرحلتين، الانحدار الخطي البسيط

جامعة بغداد - كلية التربية - ابن الهيثم



مجلة ابن الهيثم للعلوم الصرفة والتطبيقية



ISSN 1609 - 4042

العدد (1)

2011

المجلد (24)