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FIBREWISE IJ-PERFECT BITOPOLOGICAL SPACES

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Abstract : The main purpose of this paper is to introduce a some concepts in fibrewise bitopological spaces which are called fibrewise ij , fibrewise ij -closed, fibrewise ij $-$compact, fibrewise ij-perfect, fibrewise weakly ij-closed, fibrewise almost ij-perfect, fibrewise ij^*-bitopological space respectively. In addition the concepts as ij-contact point, ij-adherent point, filter, filter base, ij-converges to a subset, ij-directed toward a set, ij-continuous, ij-closed functions, ij-rigid set, ij-continuous functions, weakly ij-closed, ij-H-set, almost ij-perfect, ij^*-continuous, pairwise Urysohn space, locally ij-QHC bitopological space are introduced and the main concept in this paper is fibrewise ij-perfect bitopological spaces. Several theorems and characterizations concerning with these concepts are studied.

Keywords : bitopological spaces, closed bitopological space, filter base, Fibrewise IJ-Perfect Bitopological Spaces

1. Introduction and Preliminaries.

In order to begin the category in the classification of fibrewise (briefly, F.W.) sets over a given set, named the base set, which say B. A F.W. set over B consists of a set M with a function $p: M \rightarrow B$, that is named the projection. The fibre over b for every point b in B is the subset $M_b = p^{-1}(b)$ of M. Perhaps, fibre will be empty since we do not require p is surjectve, also, for every subset B^* of B, we consider $M_{B^*} = p^{-1}(B^*)$ as a F.W. set over B^* with the projection determined by p. The alternative notation of M_{B^*} is sometime referred to as $M \mid B^*$. We consider the Cartesian product $B \times T$, for every set T, as a F.W. set over B by the first projection.

The bitopological spaces were first created by Kelly [٧] in 1963 and after that a large number of researches have been completed to generalize the topological ideas to bitopological setting. A set M with two topologies τ_1 and τ_2 is called bitopological space [7] and is denoted by (M, τ_1, τ_2). By τ_i-open (resp., τ_i-closed), we shall mean the open (resp., closed) set with respect to τ_i in M, where $i = 1,2$. A is open (resp., closed) if it is both τ_1-open (resp., τ_1-closed) and τ_2-open (resp., τ_2-closed) in M. As well as, we built on some of the results in [1, 8, 13, 14, 15, 16,17, 18]. For other notations or notions which are not mentioned here we go behind closely I. M. James [5], R. Engelking [٤] and N. Bourbaki [3].

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Definition:1.1. [5] If M and N with projections p_M and p_N, respectively, are F.W. sets over B, a function $\varphi: M \to N$ is named F.W. function if $p_N \, o \, \varphi = p_M$, or $\varphi(M_b) \subset N_b$ for every $b \in B$.

Definition: 1.2. [5] Let (B, Λ) be a topological space. The F.W. topology on a F.W. set M over B mean any topology on M makes the projection p is continuous.

Definition: 1.3. [5] The F.W. function $\varphi: M \to N$, where M and N are F.W. topological spaces over B is named:

Continuous if for every $x \in M_B$; $b \in B$, the inverse image of every open set of $\varphi(x)$ is an open set of x.

Open if for every $x \in M_B$; $b \in B$, the direct image of every open set of x is an open set of $\varphi(x)$.

Definition:1.4. [5] The F.W. topological space (M, τ) over (B, Λ) is named F.W. closed, (resp. F.W. open) if the projection p is closed (resp. open).

Definition: 1.5. [7] The triple (M, τ_1, τ_2) where M is a non-empty set and τ_1 and τ_2 are topologies on M is named bitopological spaces.

Definition:1.6. [7] A function $\varphi: (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is said to be τ_i-continuous (resp. τ_i-open and τ_i-closed), if the functions $\varphi: (M, \tau_i) \to (N, \sigma_i)$ are continuous (resp. open and closed), φ is named continuous (resp. open and closed) if it is τ_i-continuous (resp. τ_i-open and τ_i-closed) for every $i = 1,2$.

Definition:1.7. [13] Let $(B, \Lambda_1, \Lambda_2)$ be a bitopological space. The F.W. bitopology on a F.W. set M over B mean any bitopology on M makes the projection p is continuous.

Definition 1.8. [6] A point x in (M, τ_1, τ_2) is called an ij-contact point of a subset $A \subseteq M$ if and only if for every τ_i-open nbd U of x, $(\tau_{j\_}cl(U)) \cap A \neq \emptyset$. The set of all ij-contact points of A is called the ij-closure of A and is denoted by $ij - cl(A)$. $A \subset M$ is called ij-closed if and only if $A = ij - cl(A)$, where $i, j = 1, 2, (i \neq j)$.

Definition 1.9 [3] A filter $\mathcal{F}$ on a set M is a nonempty collection of nonempty subsets of M with the properties:

(a) If $F_1, F_2 \in \mathcal{F}$, then $F_1 \cap F_2 \in \mathcal{F}$.
(b) If $F \in \mathcal{F}$ and F $\subseteq F^* \subseteq$ M, then $F^* \in \mathcal{F}$.

Definition 1.10. [3] A filter base $\mathcal{F}$ on a set M is a nonempty collection of nonempty subsets of M such that if $F_1, F_2 \in \mathcal{F}$ then $F_3 \subset F_1 \cap F_2$ for some $F_3 \in \mathcal{F}$.

Definition 1.11. [3] If $\mathcal{F}$ and $\mathcal{G}$ are filter bases on M, we say that $\mathcal{G}$ is finer than $\mathcal{F}$ (written as $\mathcal{F} < \mathcal{G}$) if for each $F \in \mathcal{F}$, there is $G \in \mathcal{G}$ such that $G \subseteq F$ and that $\mathcal{F}$ meets $\mathcal{G}$ if $F \cap G \neq \varphi$ for every $F \in \mathcal{F}$ and $G \in \mathcal{G}$.

Definition 1.12. [10] A filter base $\mathcal{F}$ on M i s said to be ij-converges to a subset A of M (written as $\mathcal{F} \xrightarrow{ij-con}$ A) if and only if for every τ_i-open cover $\mathcal{U}$ of A, there is a finite subfamily $\mathcal{U}_0$ of $\mathcal{U}$ and a number F of $\mathcal{F}$ such that $F \subset \cup\{\tau_j - cl(U): U \in \mathcal{U}_0\}$. If $x \in M$, we say $\mathcal{F} \xrightarrow{ij-con} x$ if and only if $\mathcal{F} \xrightarrow{ij-con} \{x\}$ or equivalently, τ_j-closure of every τ_i-open nbd of x contains some members of $\mathcal{F}$.

Definition 1.13. [2] A function $f: (M, \tau_1, \tau_2) \rightarrow (N, \sigma_1, \sigma_2)$ is called ij-continuous if and only if for any σ_i-open nbd V of $f\ (x)$, there exists a τ_i-open nbd U of x such that $f\ (\tau_j\text{-}cl(U)) \subset \sigma_j\text{-}cl(V)$, where $i, j\ = 1, 2$.

Definition 1.14. [2] A point x in a bitopological space (M, τ_1, τ_2) is called an ij-adherent point of a filter base $\mathcal{F}$ on M if and only if it is an ij-contact point of every number of $\mathcal{F}$. The set of all ij-adherent points of $\mathcal{F}$ is called the ij-adherence of $\mathcal{F}$ and is denoted by ij-ad $\mathcal{F}$, where $i, j\ = 1, 2$.

2. Fibrewise IJ-Perfect Bitopological Spaces.

In this section, we introduce the notion of ij-perfect bitopological, ij-rigidity spaces and investigate some of their basic properties.

Definition 2.1. Let $(B, \Lambda_1, \Lambda_2)$ be a bitopological space. The F.W. ij-bitopology on a F.W. set M over B mean any bitopology on M for which the projection p is ij-continuous, where $i, j\ = 1, 2$.

Definition 2.2. A function $f: (M, \tau_1, \tau_2) \rightarrow (N, \sigma_1, \sigma_2)$ is called ij-closed if the image of each ij-closed set in M is ij-closed set in N, where $i, j\ = 1, 2$.

Theorem 2.3. A function $f: (M, \tau_1, \tau_2) \rightarrow (N, \sigma_1, \sigma_2)$ is ij-closed if and only if $ij - cl(f(A)) \subset f(ij - cl(A))$ for each $A \subset M$, where $i, j\ = 1, 2$.

Proof. (⇒) Suppose that f is ij-closed. Let $A \subset M$, since f is ij-closed then $f(\mathrm{ij} - \mathrm{cl(A)})$ is ij-closed set in N, since $\mathrm{ij} - \mathrm{cl(A)}$ is closed in M. so, $\mathrm{ij} - \mathrm{cl(f(A))} \subset f(\mathrm{ij} - \mathrm{cl(A)})$.

(⇐) Suppose that A is ij-closed set in M, so $A = \mathrm{ij} - \mathrm{cl(A)}$, but we have $\mathrm{ij} - \mathrm{cl(f(A))} \subset f(\mathrm{ij} - \mathrm{cl(A)})$, thus $ij - cl(f(A)) \subset f(A)$, so $f(A)$ is ij-closed in M, therefore f is ij-closed.

Definition 2.4. A filter base $\mathcal{F}$ on bitopological space (M, τ_1, τ_2) is said to be ij −converges to a point $x \in M$ (written as $\mathcal{F} \xrightarrow{ij-con}$ x) if and only if every τ_i-open nbd U of x contains some elements of $\mathcal{F}$, where $i, j\ = 1, 2$.

Definition 2.5. A filter base $\mathcal{F}$ on bitopological space (M, τ_1, τ_2) is said to be ij-directed toward a set $A \subseteq M$, written as $\mathcal{F} \xrightarrow{ij-d} A$, if and only if every filter base $\mathcal{G}$ finer than $\mathcal{F}$ has an ij −adherent point in A, i.e. $(ij - ad\ \mathcal{G}) \cap A \neq \varphi$. We write $\mathcal{F} \xrightarrow{ij-d} x$ to mean $\mathcal{F} \xrightarrow{ij-d}$ {x}, where $x \in M$, where $i, j\ = 1, 2$.

Theorem 2.6. A point x in bitopological space (M, τ_1, τ_2) is an ij-adherent point of a filter base $\mathcal{F}$ on M if and only if there exists a filter base $\mathcal{F}^*$ finer than $\mathcal{F}$ such that $\mathcal{F}^* \xrightarrow{ij-con} x$, where $i, j = 1, 2$.

Proof. (⇒) Let x be an ij −adherent point of a filter base $\mathcal{F}$ on M, so it is an ij −contact point of every number of $\mathcal{F}$, then for every τ_i-open nbd U of x, we have $\tau_j - cl(U) \cap F \neq \varphi$ for every number F in $\mathcal{F}$. And thus $\tau_j - cl(U)$ contains a some member of any filter base $\mathcal{F}^*$ finer than $\mathcal{F}$, so that $\mathcal{F}^* \xrightarrow{ij-con} x$.

(⇐) Suppose that x is not an ij −adherent point of a filter base $\mathcal{F}$ on M, so there exist $F \in \mathcal{F}$ such that x is not an ij −contact of F. Then there exists an τ_i-open nbd U of x such that $\tau_j - cl(U) \cap F = \varphi$. Denote by $\mathcal{F}^*$ the family of sets $F^* = F \cap (M - \tau_j\text{-}cl(U))$ for $F \in \mathcal{F}$, then the sets F^* are nonempty. Also $\mathcal{F}^*$ is a filter base and indeed it is finer than $\mathcal{F}$, because given $F_1^* = F_1 \cap (M\text{-}\tau_j\text{-}cl(U))$ and $F_2^* = F_2 \cap (M\text{-}\tau_j\text{-}cl(U))$, there is an $F_3 \subseteq F_1 \cap F_2$ and this gives $F_3^* = F_3 \cap (M\text{-}\tau_j\text{-}cl(U)) \subseteq F_1 \cap F_2 \cap (M\text{-}\tau_j\text{-}cl(U)) = F_1 \cap (\text{M-}\tau_j\text{-}cl(U)) \cap F_2 \cap (M\text{-}\tau_j\text{-}cl(U))$, by construction $\mathcal{F}^*$ not ij-convergent to x. This is a contradiction, and thus x is an ij-adherent point of a filter base $\mathcal{F}$ on M.

Theorem 2.7. Let $\mathcal{F}$ be a filter base on bitopological space (M, τ_1, τ_2), and a point $x \in M$, then $\mathcal{F} \xrightarrow{ij-con} x$ if and only if $\mathcal{F} \xrightarrow{ij-d} x$, where $i, j = 1, 2$.

Proof. (⇐) If $\mathcal{F}$ does not ij-converge to x, then there exists a τ_i-open nbd U of x such that $F \not\subset \tau_j\text{-}cl(U)$, for all $F \in \mathcal{F}$. Then $\mathcal{G} = \{(M\text{-}\tau_j\text{-}cl(U) \cap F : F \in \mathcal{F}\}$ is a filter base on M finer than $\mathcal{F}$, and clearly $x \notin ij$-adherence of $\mathcal{G}$. Thus $\mathcal{F}$ cannot be ij-directed towards x which is contradiction. So $\mathcal{F}$ is ij-converge to x.

(⇒) Clear.

Definition 2.8. A function $f : (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is said to be ij-perfect if and only if for each filter base $\mathcal{F}$ on $f\ (M)$, ij-directed towards some subset A of $f(M)$, the filter base $f^{-1}(\mathcal{F})$ is ij-directed towards f^{-1}(A) in M. f is called pairwise ij-perfect if and only if f is 12 and 21-perfect, where $i, j = 1, 2$.

Definition 2.9. The F.W. bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. ij-perfect if and only if the projection p is ij-perfect, where $i, j = 1, 2$.

In the following theorem we show that only points of N could be sufficient for the Subset A in definition [2.8] and hence ij-direction can be replaced in view of theorem [2.6] by ij-convergence.

Theorem 2.10. Let (M, τ_1, τ_2) be a F.W. bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$. Then the following are equivalent:

(a) $(\text{M}, \tau_1, \tau_2)$ is F.W. ij-perfect bitopological space.

(b) For each filter base $\mathcal{F}$ on $p(M)$, which is ij-convergent to a point b in B, $M_{\mathcal{F}} \xrightarrow{ij-d} M_b$.

(c) For any filter base $\mathcal{F}$ on M, ij-ad $p(\mathcal{F}) \subset p$ (ij-ad $\mathcal{F}$).

Proof. (a) ⇒ (b) Follows from theorem (2.7).

(b) ⇒ (c) Let $b \in$ ij-ad $p\ (\mathcal{F})$. Then by theorem (2.6), there is a filter base $\mathcal{G}$ on $p(M)$ finer than $p\ (\mathcal{F})$ such that $\xrightarrow{ij-con} b$. Let $\mathcal{U} = \{M_{\mathcal{G}} \cap \text{F} : G \in \mathcal{G}$ and $F \in \mathcal{F}\}$. Then $\mathcal{U}$ is a filter base on M finer than $M_{\mathcal{G}}$. Since $\mathcal{G} \xrightarrow{ij-d} b$, by theorem (2.7) and p is ij-perfect, $M_{\mathcal{G}} \xrightarrow{ij-d} M_b$. $\mathcal{U}$ being finer than $M_{\mathcal{G}}$, we have $M_b \cap$ (ij-ad $\mathcal{U}) \neq \varphi$. It is then clear that $M_b \cap$ (ij-ad $\mathcal{F}) \neq \varphi$. Thus $b \in p\ (ij - ad\ F)$.

(c) ⇒ (a) Let $\mathcal{F}$ be a filter base on $p(M)$ such that it is ij-directed towards some subset A of $p(M)$. Let $\mathcal{G}$ be a filter base on M finer than $M_{\mathcal{F}}$. Then $p(\mathcal{G})$ is a filter base on $p(M)$ finer than $\mathcal{F}$ and hence $A \cap (ij - ad\ p(\mathcal{G})) \neq \varphi$. Thus by (c) A ∩ p (ij − ad $\mathcal{G}$) ≠ φ so that $M_A \cap (ij - ad\ \mathcal{G}) \neq \varphi$. This shows that $M_{\mathcal{F}}$ is ij-directed towards M_A. Hence p is ij-perfect.

Definition 2.11. The function $f: (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is called ij-compact function if it is ij −continuous, ij −closed and for each filter base $\mathcal{F}$ in N then $f^{-1}(\mathcal{F})$ is filter base in M, where $i, j = 1, 2$.

Definition 2.12 The F.W. ij −bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. ij −compact if and only if the projection p is ij −compact, where $i, j = 1, 2$.

For example the bitopological product $B \times_B T$ is F.W. ij-compact over B, for all ij −compact space T, where $i, j = 1, 2$.

Definition 2.13. The F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. ij-closed if and only if the projection p is ij-closed, where $i, j = 1, 2$.

Theorem 2.14. If the F.W. bitopological space (M, τ_1, τ_2) over a bitopological space $(B, \Lambda_1, \Lambda_2)$ is ij-perfect, then it is ij-closed, where $i, j = 1, 2$.

Proof. Assume that M is a F.W. ij perfect bitopological space over B, then the projection $p_M : M \to B$ is ij-perfect, to prove that it is ij-closed, by (2. 10 (a)⇒(c)) for any filter base $\mathcal{F}$ on M ij-ad p($\mathcal{F}$) ⊂ p(ij-ad ($\mathcal{F}$)), by theorem (2. 12) f is ij-closed if ij − cl f(A) ⊂ f(ij − cl(A) for all $A \subset M$, therefore p is ij-closed where $\mathcal{F} = \{A\}$.

Definition 2.15. A subset A of bitopological space (M, τ_1, τ_2) is said to be ij-rigid in M if and only if for each filter base $\mathcal{F}$ on M with $(ij - ad\ \mathcal{F}) \cap A = \varphi$, there is τ_i-open set U and $F \in \mathcal{F}$ such that $A \subset U$ and $\tau_j\text{-}cl(U) \cap F = \varphi$, or equivalently, if and only if for each filter base $\mathcal{F}$ on M whenever $A \cap (ij - ad\ \mathcal{F}) = \varphi$, then for some $F \in \mathcal{F}, A \cap (ij - cl(F)) = \varphi$, where $i, j = 1, 2$.

Theorem 2.16. If (M, τ_1, τ_2) is F.W. ij-closed bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$ such that each $M_b$where $b \in B$ is ij-rigid in M, then (M, τ_1, τ_2) is F.W. ij-perfect, where $i, j = 1, 2$.

Proof. Assume that M is a F.W. ij-closed bitopological space over B, then the projection $p_M : M \to B$ exist, to prove that it is ij-perfect. Let $\mathcal{F}$ be a filter base on $p\ (M)$ such that $\mathcal{F} \xrightarrow{ij-con} b$ in B, for some $b \in B$. If $\mathcal{G}$ is a filter base on M finer than the filter base $M_{\mathcal{F}}$, then $p(\mathcal{G})$ is a filter base on B, finer than $\mathcal{F}$. Since $\mathcal{F} \xrightarrow{ij-d} b$ by theorem (2.6), $b \in$ ij-ad p($\mathcal{G}$), i.e., $b \in \cap \{ij - ad\ p(G) : G \in \mathcal{G}\}$ and hence $\{b \in \cap \{p(ij - ad\ (G) : G \in \mathcal{G}\}$ by theorem (2.12), since p is ij-closed. Then M_b ∩ ij-ad (G) ≠ φ, for all $G \in \mathcal{G}$. Hence for all $U \in \tau_i$ with $M_b \subset$ U, τ_j-cl(U) ∩ $G \neq \varphi$, for all $G \in \mathcal{G}$. Since M_b is ij-rigid, it then follows that M_b ∩ (ij-ad $\mathcal{G}$)≠ φ. Thus $M_{\mathcal{F}} \xrightarrow{ij-d} M_b$. Hence by theorem 2.10 (b)⇒(a), p is ij-perfect.

Theorem 2.17. If F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is ij-perfect, then it is ij-closed and for each $b \in B, M_b$ is ij-rigid in M, where $i, j = 1, 2$.

Proof. Assume that M is a F.W. ij-bitopological space over B, then the projection $p_M : M \to B$ exist and it is ij-continuous. Since p is an ij-perfect so it is ij-closed. To prove the other part, let $b \in B$, and suppose $\mathcal{F}$ is a filter base on M such that (ij-ad $\mathcal{F}$) ∩ M_b= φ. Then $b \notin p$ (ij-ad $\mathcal{F}$).

Since p is ij-perfect, by theorem (2.10 (a)⇒(c)) b $\notin$ ij $-$ ad p($\mathcal{F}$). Thus there exists an $F \in \mathcal{F}$ such that $b \notin$ ij-ad $p(F)$. There exists an Λ_i-open nbd V of b such that $\Lambda_j - cl(V) \cap p(F) = \varphi$. Since p is ij-continuous, for each $x \in M_b$ we shall get a τ_i-open nbd U_x of x such that $p(\tau_j - cl(U_x) \subset \Lambda_j - cl(V) \subset B - p(F)$. Then $p(\tau_j - cl(U_x) \cap p(F) = \varphi$, so that τ_j-cl-(U_x) $\cap F = \varphi$. Then $x \notin$ ij-cl(F), for all $x \in M_b$, so that $M_b \cap$ (ij-cl(F)) = φ, Hence M_b is ij-rigid in M.

Corollary 2.18. A F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is ij-perfect if and only if it is ij-closed and each M_b, where $b \in B$ is ij-rigid in M, where $i, j = 1, 2$.

Next we show that the above theorem remains valid if F.W. ij-closedness bitopological space replaced by a strictly weak condition which we shall called F.W. weak ij-closedness bitopological space. Thus we define as follows.

Definition 2.19. A function $f : (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is said to be weakly ij-closed if for every $y \in f(M)$ and every τ_i-open set U containing $f^{-1}(y)$ in M, there exists a σ_i open nbd V of y such that f^{-1} (σ_j-cl(V)) $\subset \tau_j$-$cl(U)$, where $i, j = 1, 2$.

Definition 2.20. The F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. weakly ij-closed if and only if the projection p is weakly ij-closed, where $i, j = 1, 2$.

Lemma 2.21. [6] In space (M, τ_1, τ_2) if $U \in \tau_j$, then $ij - cl(U) = \tau_j - cl(U)$, where $i, j = 1, 2$.

Theorem 2.22. The F.W. ij-closed bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is weakly ij-closed, where $i, j = 1, 2$.

Proof. Assume that M is a F.W. weak ij-closed bitopological space over B, then the projection $p_M : M \to B$ exist and its weakly ij-closed. Let $b \in p(M)$ and let U be a τ_i-open set containing M_b in M. Now, by lemma (2.21) $\tau_j - cl\left(M - \tau_j - cl(U)\right) = ij - cl\left(M - \tau_j - cl(U)\right)$ and hence by theorem [2.1 ٢] and since p is ij-closed, we have ij-cl $p(M - \tau_j - cl(U)) \subset p[ij - cl(M - \tau_j - cl(U)]$. Now since $b \notin p[ij - cl(M - \tau_j - cl(U)]$, $b \notin ij - cl\, p(M - \tau_j$-$cl(U))$ and thus there exists an σ_i-open nbd V of b in B such that σ_j-$cl(V) \cap p(M - \tau_j - cl(U)) = \varphi$ which implies that $M_{(\sigma_j - \text{cl(V)})} \cap (M - \tau_j - cl(U)) = \varphi$ i.e., $M_{(\sigma_j - \text{cl(V)})} \subset \tau_j$-cl(U), and thus p is weakly ij-closed.

A F.W. weakly ij-closed is not necessarily to be F.W. ij-closed and the following example show This.

Example 2.23. Let $\tau_1, \tau_2, \Lambda_1$ and Λ_2 be any topologies and p : $(M, \tau_1, \tau_2) \to (B, \Lambda_1, \Lambda_2)$ be a constant function, then p is weakly ij-closed for $i, j = 1, 2$ and $(i \neq j)$. Now, let $M = B = IR$. If Λ_1 or Λ_2 is the discrete topology on B, then $p : (M, \tau_1, \tau_2) \to (B, \Lambda_1, \Lambda_2)$ given by $p(x) = 0$, for all $x \in M$, is neither 12-closed nor 21-closed, irrespectively of the topologies τ_1, τ_2 and Λ_2 (or Λ_1).

Theorem 2.24. Let (M, τ_1, τ_2) be F.W. ij-bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$. Then (M, τ_1, τ_2) is F.W. ij-perfect if:

(a) (M, τ_1, τ_2) is F.W. weakly ij-closed bitopological space, and

(b) M_b is ij-rigid, for each $b \in B$.

Proof. Assume that M is a F.W. ij-bitopological space over B satisfying the conditions (a) and (b), then the projection $p_M : M \to B$ exist. To prove that p is ij-perfect we have to show in view of theorem [2.1 7] that p is ij-closed. Let $b \in ij - cl\, p(A)$, for some non-null subset A of M, but $b \notin p\,(ij - cl(A))$. Then $\mathcal{H} = \{A\}$ is a filter base on M and (ij-ad $\mathcal{H}$) $\cap$ $M_b = \varphi$. By ij-rigidity of M_b, there is a τ_i-open set U containing M_b such that $\tau_j - cl(U) \cap A = \varphi$. By weak ij-closedness of p, there exists an Λ_i –open nbd V of b such that $M_{(\Lambda_j-\mathrm{cl(V)})} \subset \tau_j$-cl(U), which implies that $M_{(\Lambda_j-\mathrm{cl(V)})} \cap A = \varphi$, i.e., $(\Lambda_j - cl(V)) \cap p(A) = \varphi$, which is impossible since $b \in ij - cl\, p(A)$. Hence $b \in p(ij - cl(A))$. So f is ij-closed.

Definition 2.25.[11] A subset A in bitopological space (M, τ_1, τ_2) is called ij-H-set in M if and only if for each τ_i-open cover $\mathcal{A}$ of A, there is a finite sub collection $\mathcal{B}$ of $\mathcal{A}$ such that $A \subset \cup \{\tau_j - cl(U) : U \in \mathcal{B}\}, i, j = 1,2$. A is called a pairwise-H-set if and only if it is a 12- and 21-H-set. If A is an ij-H-set (pairwise-H-set) and $A = M$, then the space is called an ij-QHC (resp. pairwise QHC) space, where $i, j = 1, 2$.

Lemma 2.26.[10] A subset A of a bitopological space (M, τ_1, τ_2) is an ij-H-set if and only if for each filter base $\mathcal{F}$ on A, $(ij - ad\,\mathcal{F}) \cap A \neq \varphi$, where $i, j = 1, 2$.
Proof. (⇒) Clear.
(⇐) Let $\mathcal{A}$ be a τ_i-open cover of A such that the union of τ_j-closure of any finite Sub collection of $\mathcal{A}$ is not cover A. Then $\mathcal{F}$ $\{A \setminus \cup_{\mathcal{B}}\, \tau_j$-cl(B) : $\mathcal{B}$ is finite sub collection of $\mathcal{A}\}$is a filter base on A and (ij-ad $\mathcal{F}$) $\cap A = \varphi$. This contradiction so that A is ij-set.

Theorem 2.27. If (M, τ_1, τ_2) is F.W. ij-perfect bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$ and $B^* \subset B$ is an ij-H-set in B, then M_{B^*} is an ij-H-set in M, where $i, j = 1, 2$.
Proof. Assume that M is a F.W. ij-perfect bitopological space over B, then the projection $p_M : M \to B$ exist. Let $\mathcal{F}$ be a filter base on M_{B^*}, then $p(\mathcal{F})$ is a filter base on B^*. Since B^* is an ij-H-set in B, $B^* \cap ij - ad\, p(\mathcal{F}) \neq \varphi$ by lemma (2.26). By theorem (2.10 (a)⇒(c)), $B^* \cap p(ij - ad\,(F)) \neq \varphi$, so that $M_{B^*} \cap$ ij-ad $(\mathcal{F}) \neq \varphi$. Hence by lemma (2.26), M_{B^*} is an ij-H-set in M. The converse of the above theorem is not true, is shown in the next example.

Example 2.28. Let $M = B = IR, \tau_1$ and τ_2 be the cofinite and discrete topologies on M and Λ_1, Λ_2 respectively denote the indiscrete and usual topologies on B. Suppose $p : (M, \tau_1, \tau_2) \to (B, \Lambda_1, \Lambda_2)$ is the identity function. Each subset of either of (M, τ_1, τ_2) and $(B, \Lambda_1, \Lambda_2)$ is a 12-set. Now, any non-void finite set $A \subset M$ is 12-closed in M, but $p(A)$ (i.e., A) is not 12-closed in B (in fact, the only 12-closed subsets of B are B and φ).

Definition 2.29. A function $f : (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is said to be almost ij-perfect if for each ij-H-set K in N, $f^{-1}(K)$ is an ij-H-set in M, where $i, j = 1, 2$.

Definition 2.30. The F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. almost ij-perfect if and only if the projection p is almost ij-perfect, where $i, j = 1, 2$. *By analogy to theorem (2.16), a sufficient condition for a function to be almost ij-perfect, is proved as follows.*

Theorem 2.31. Let (M, τ_1, τ_2) be F.W. ij- bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$ such that:
(a) $M_b$is ij-rigid, for each $b \in B$, and
(b) (M, τ_1, τ_2) is F.W. weakly ij-closed bitopological space.
Then (M, τ_1, τ_2) is F.W. almost ij-perfect bitopological space.

Proof. Assume that M is a F.W. ij-bitopological space over B, then the projection $p_M : M \to B$ exist and it is ij-continuous. Let B^* be an ij-H-set in B and let $\mathcal{F}$ be a filter base on M_{B^*}. Now $p(\mathcal{F})$ is a filter base on B^* and so by theorem (2.26), $(ij - ad\, p(\mathcal{F})) \cap B^* \neq \varphi$. Let $b \in [ij - ad\, p(\mathcal{F})] \cap B^*$. Suppose that $\mathcal{F}$ has no ij-ad point in M_{B^*} so that (ij-ad $(\mathcal{F})$) $\cap\, M_b = \varphi$. Since M_b is ij-rigid, there exists an $F \in \mathcal{F}$ and a τ_i-open set U containing M_b such that $F \cap \tau_j - cl(U) = \varphi$. By weak ij-closedness of p, there is a Λ_i-open nbd V of b such that $M_{(\Lambda_j - \mathrm{cl(V)})} \subset \tau_j - cl(U)$ which implies that $M_{(\Lambda_j - \mathrm{cl(V)})} \cap F = \varphi$, i.e., Λ_j-$cl(V) \cap p(F) = \varphi$, which is a contradiction. Thus by theorem (2.26), M_{B^*} is an ij-H-set in M and hence p is almost ij-perfect.

We now give some applications of ij-perfect functions. The following characterization theorem for an ij-continuous function is recalled to this end.

Theorem 2.32. A bitopological space (M, τ_1, τ_2) is F.W. ij-bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$. if and only if $p(ij - cl(A)) \subset ij - cl(p(A))$, for each $A \subset M$, where $i, j = 1, 2$.

Proof. (⇒) Assume that M is a F.W. ij-bitopological space over B, then the projection $p_M : M \to B$ exist and it is ij-continuous. Suppose that $x \in ij - cl(A)$ and V is Λ_i-open nbd of $f(x)$. Since p is ij-continuous, there exists an τ_i-open nbd U of x such that $p(\tau_j - cl(U)) \subset \Lambda_j - cl(V)$. Since τ_j - $cl\,(U) \cap A \neq \varphi$, then Λ_j - $cl(V) \cap p(A) \neq \varphi$. So, $p(x) \in ij - cl(p(A))$. This shows $that\, p(ij - cl(A)) \subset ij - cl(p(A))$.

(⇐) Clear.

Theorem 2.33. Let (M, τ_1, τ_2) be a F.W. ij-perfect bitopological space over bitopological space $(B, \Lambda_1, \Lambda_2)$. Then M_A preserves ij-rigidity, where $i, j = 1, 2$.

Proof. Assume that M is a F.W. ij-bitopological space over B, then the projection $p_M : M \to B$ exist and it is ij-continuous. Let A be an ij-rigid set in B and let $\mathcal{F}$ be a filter base on M such that $M_A \cap$ (ij- ad$(\mathcal{F})$) = φ. Since p is ij-perfect and $A \cap p(ij - ad(\mathcal{F})) = \varphi$ by theorem (2.10 (a) ⇒ (c)) we get $A \cap (ij - ad\, p(\mathcal{F})) = \varphi$. Now A being an ij-rigid set in B, there exists an $F \in \mathcal{F}$ such that $A \cap ij - clp(F) = \varphi$. Since p is ij-continuous, by theorem (2.32) it follows that $A \cap p(ij - cl(F)) = \varphi$. Thus $M_A \cap (ij - cl(F)) = \varphi$. This proves that M_A is ij-rigid.

In order to investigate for the conditions under which a F.W. almost ij-perfect bitopological space may be F.W. ij-perfect bitopological space, we introduce the following definition.

Definition 2.34. A function $f: (M, \tau_1, \tau_2) \to (N, \sigma_1, \sigma_2)$ is said to be ij^*-continuous if and only if for any σ_j-open nbd V of $f(x)$, there exists a τ_i-open nbd U of x such that $f(\tau_j$-$cl(U)) \subset \sigma_i$-$cl(V)$, where $i, j = 1, 2$.

Definition 2.35. The F.W. ij-bitopological space (M, τ_1, τ_2) over bitopological space $(B, \Lambda_1, \Lambda_2)$ is called F.W. ij^*-bitopological space if and only if the projection p is ij^*-continuous, where $i, j = 1, 2$.

The relevance of the above definition to the characterization of F.W. ij-perfect bitopological space is quite apparent from the following result.

Definition 2.36. A bitopological space (M, τ_1, τ_2) is said to be pairwise Urysohn space if for $x, y \in M$ with $x \neq y$, there are τ_i-open nbd U of x and τ_j-open nbd V of y such that $\tau_j - cl(U) \cap \tau_i - cl(V) = \varphi$, where $i, j = 1, 2$.

Theorem 2.37. If (M, τ_1, τ_2) is F.W. ij^*-bitopological space on a pairwise Urysohn space $(B, \Lambda_1, \Lambda_2)$, then it is F.W. ij-perfect bitopological space if and only if for every filter base $\mathcal{F}$ on M, if $p(\mathcal{F}) \xrightarrow{ij-con} b$ wher $b \in B$, then $ij - ad\ \mathcal{F} \neq \varphi$, where $i, j = 1, 2$.

Proof. (⇒) Let $(\mathrm{M}, \tau_1, \tau_2)$ be a F.W. ij^*-bitopological space on a pairwise Urysohn space $(\mathrm{B}, \Lambda_1, \Lambda_2)$, then there is a ij^*-continuous projection function $p: (\mathrm{M}, \tau_1, \tau_2) \to (\mathrm{B}, \Lambda_1, \Lambda_2)$ and $p(\mathcal{F}) \xrightarrow{ij-con.} b$ where $b \in B$, for a filter base $\mathcal{F}$ on M. Then $M_{p(\mathcal{F})} \xrightarrow{ij-dir.} M_b$. Since $\mathcal{F}$ is finer than $M_{p(\mathcal{F})}$, $M_b \cap ij - ad\ \mathcal{F} \neq \varphi$, so that $ij - ad\ \mathcal{F} \neq \varphi$.

(⇐): Suppose that for every filter base $\mathcal{F}$ on M, $p(\mathcal{F}) \xrightarrow{ij-con.} b$ where $b \in B$ implies $ij - ad\ \mathcal{F} \neq \varphi$. Let $\mathcal{G}$ be a filter base on B such that $\mathcal{G} \xrightarrow{ij-con.} b$, and suppose that $\mathcal{G}^*$ is a filter base on M such that $\mathcal{G}^*$ is finer than $M_{\mathcal{G}}$. Then $p(\mathcal{G}^*)$ is finer than $\mathcal{G}$. So $p(\mathcal{G}^*) \xrightarrow{ij-con.} b$. Hence $ij - ad\ \mathcal{G}^* \neq \varphi$. Let $z \in B$ such that $z \neq b$. Then since B is pairwise Urysohn, there exist a Λ_i-open nbd U of b and Λ_j-open nbd V of z such that $\left(\Lambda_j - cl\ (U)\right) \cap \left(\Lambda_i - cl(V)\right) = \varphi$. Since $p(\mathcal{G}^*) \xrightarrow{ij-con.} b$, there exist a $G \in \mathcal{G}^*$ such that $p(G) \subset \Lambda_j - cl(U)$. Now, since p is ij^*-continuous, corresponding to each $x \in M_z$ there is a τ_i-open nbd W of x such that $p\left(\tau_j - cl(W)\right) \subset \Lambda_i - cl(V)$. Thus $\Lambda_j - cl(W) \cap G = \varphi$. It follows that $M_z \cap ij - \mathcal{G}^* = \varphi$, for each $z \in B - \{b\}$. Consequently $M_b \cap ij - ad\ \mathcal{G}^* \neq \varphi$, and p is ij- perfect and hence $(\mathrm{M}, \tau_1, \tau_2)$ is F.W. ij^*-bitopology.

Definition 2.38. [9] A bitopological space (M, τ_1, τ_2) is said to be locally ij-QHC bitopological space if and only if for every $x \in M$, there is a τ_i-open nbd of x, which is an ij-H-set, where $i, j = 1, 2$.

Lemma 2.39. [10] In a pairwise Urysohn bitopological space (M, τ_1, τ_2) an ij-H-set is ij-closed, where $i, j = 1, 2$.

Corollary 2.40. Let (M, τ_1, τ_2) be a F.W. ij^*-bitopological space and ij-QHC on a pairwise Urysohn bitopological space $(B, \Lambda_1, \Lambda_2)$, then (M, τ_1, τ_2) is F.W. ij-perfect bitopological space, where $i, j = 1, 2$.

Theorem 2.41. Let (M, τ_1, τ_2) be a F.W. ij^*-bitopological space and locally ij-QHC on a Urysohn space $(B, \Lambda_1, \Lambda_2)$, then (M, τ_1, τ_2) is F.W. ij^*-bitopological space if and only if it is F.W. almost ij-perfect, where $i, j = 1, 2$.

Proof. (⇒) If $(\mathrm{M}, \tau_1, \tau_2)$ is F.W. ij^*-bitopological space, then by corollary (2.40.), it is F.W. almost ij-perfect.

(⇐) Let $(\mathrm{M}, \tau_1, \tau_2)$ is F.W. almost ij-perfect, then there exist almost ij-perfect projection function $p: (\mathrm{M}, \tau_1, \tau_2) \to (\mathrm{B}, \Lambda_1, \Lambda_2)$, and let $\mathcal{F}$ be any filter base on M and let $p(\mathcal{F}) \xrightarrow{ij-con.} b$ where $b \in B$. There are an ij-H-set B^* in B and Λ_i-open nbd V of b such that $b \in V \subseteq B^*$. Let $\mathcal{H} = \{\Lambda_j - cl(U) \cap p(F) \cap B^*; F \in \mathcal{F}$ and U is a Λ_i-open nbd of $b\}$. By lemma (2.39), B^* is ij-closed and hence no member of $\mathcal{H}$ is void. In fact, if not, let for some Λ_i-open nbd U of b and some $F \in \mathcal{F}$, $\Lambda_j - cl(U) \cap p(F) \cap B^* = \varphi$. Then $W = U \cap V$ since $y \in U \cap V \in \Lambda_i$ and $\Lambda_j - cl(W) = ij - cl(W) \subset ij - cl(B^*) = B^*$ by lemma (2.21). Now $\varphi = \Lambda_j - cl(W) \cap p(F) \cap B^* = \Lambda_j - cl(W) \cap p(F)$, which is not possible, since $p(\mathcal{F}) \xrightarrow{ij-con.} b$. Thus $\mathcal{H}$ is filter base on B, and is clearly finer than $p(\mathcal{F})$, so that $\mathcal{H} \xrightarrow{ij-con.} b$. Also $\mathcal{G} = \{M_H \cap F: H \in \mathcal{H}$ and $F \in \mathcal{F}\}$ is clearly a filter on M_{B^*}. Since p is almost ij-perfect, M_{B^*} is an ij-H-set and hence $ij - ad\ \mathcal{G} \cap M_{B^*} \neq \varphi$. Thus $ij - ad\ \mathcal{F} \neq \varphi$. Thus p is ij-perfect and by theorem (2.37) $(\mathrm{M}, \tau_1, \tau_2)$ is F.W. ij^*-bitopological space.

We now give some application of F.W. ij-perfect bitopological space. The following characterization theorem for a F.W. ij-bitopological space is recalled to this end.

Theorem 2.42. A F.W. set M over B is F.W. ij-bitopological space if and only if $p(ij\text{-}cl(A)) \subset ij\text{-}clp(A)$ for each $A \subset M$, where $i, j = 1, 2$.
Proof: (⟺) Since M is a F.W. set over B, then there is projection p where $p: M \to B$. Now we have to prove that p is ij-continuous. But it directly by theorem (2.32).

Lemma 2.43. It was proved in (Sen and Nandi 1993) [12] that a bitopological space (M, τ_1, τ_2) is pairwise Hausdorff if and only if $\{m\} = ij - cl\{m\}$, for each $m \in M$. It then follows immediately in view of theorem (2.14).

Theorem 2.44. If (M, τ_1, τ_2) is a F.W. ij-perfect surjection bitopological space with M is a pairwise Hausdorff space on a bitopological space $(B, \Lambda_1, \Lambda_2)$, Then B is also pairwise Hausdorff.
Proof: Let $b_1, b_2 \in B$ such that $b_1 \neq b_2$. Since p is onto, then $M_{b1}, M_{b2} \in M$ and since p is one to one, then $M_{b1} \neq M_{b2}$. Since p is ij-perfect, so by theorem (2.14) it is ij-closed. By lemma (2.43) we have $\{ M_{b1}\} = ij - cl\{M_{b1}\}$ and $\{ M_{b2}\} = ij - cl\{M_{b2}\}$. Since p is pairwise Hausdorff. Now $p(ij - cl\{ M_{b1}\}) = ij - cl\{b_1\}$ and $p(ij - cl\{ M_{b2}\}) = ij - cl\{b_2\}$ since p is ij-closed. This mean $b_1 = ij - cl\{b_1\}$ and $b_2 = ij - cl\{b_2\}$. Hence B is pairwise Hausdorff.

Our next theorem give a characterization of an important class of F.W. bitopological space viz. the ij-QHC spaces in terms of F.W. ij-perfect bitopological space.

Theorem 2.45. For a bitopological space (M, τ_1, τ_2), the following statement are equivalent:

a) M is ij-QHC
b) The F.W. $(\mathrm{M}, \tau_1, \tau_2)$ is ij-perfect bitopological space with constant projection over B^* where B^* is a singleton with two equal bitopologies viz. the unique bitopology on B^*.
c) The F.W. $(\mathrm{B} \times \mathrm{M}, Q_1, Q_2)$ is ij-perfect bitopological space over $(B, \Lambda_1, \Lambda_2)$, where $Q_i = \Lambda_i \times \tau_j$. $i, j = 1, 2$ and $i \neq j$.

Proof: (a)⇒(b) Let $p: (\mathrm{M}, \tau_1, \tau_2) \to (B^*, \Lambda_1, \Lambda_2)$ is a constant projection over B^* where B^* is a singleton with two equal bitopologies viz the unique bitopology on B^*. P is clearly ij-closed. Also, M_{B^*}, i.e. M is obviously ij-rigid since B^* is ij-QHC. Then by theorem (2.16) p is ij-perfect.
(b)⇒(a) Follows from theorem (2.33).

(a)⇒(C) Suppose that $(\mathrm{B} \times \mathrm{M}, Q_1, Q_2)$ is F.W. bitopological space over $(B, \Lambda_1, \Lambda_2)$ where $Q_i = \Lambda_i \times \tau_j$, $i, j = 1, 2$ and $i \neq j$, then there is a projection $p = \pi_i: (\mathrm{B} \times \mathrm{M}, Q_1, Q_2) \to (B, \Lambda_1, \Lambda_2)$. We show that π_i is ij- closed and for each $b \in B, M_B$ is ij-rigid in $B \times M$. Then the result will follow from theorem (2.16). Let $A \subset B \times M$ and $a \notin \pi_i(ij - cl(A))$. For each $m \in M, (a, m) \notin ij - cl(A)$, so that there exist a Λ_j-open nbd G_m of a and a τ_i-open nbd H_m of m such that $[Q_i - cl(G_m \times H_m)] \cap A = \varphi$. Since M is ij-QHC, $\{a\} \times M$ is a ij –H-set in $B \times M$. Thus there exist finitely many elements $m_1, m_2, m_3, \ldots, m_n$ with $\{a\} \times M \subset \bigcup_{k=1}^{n} Q_i - cl(G_{m_k} \times H_{m_k})$. Now, $a \in \bigcap_{k=1}^{n} G_{m_k} = G$ which is a Λ_i-open nbd of a such that $(\Lambda_i - cl(G) \cap \pi_i(A) = \varphi$. Hence $a \notin ij - cl\pi_i(A)$ and thus $ij - cl\pi_i(A) \subset \pi(ij - cl(A))$. So π is ij-closed, by theorem (2.12). Next, let $b \in B$. To show that $(B \times M)_b = \pi_i^{-1}(\mathrm{b})$ to be ij-rigid in $B \times M$. Let $\mathcal{F}$ be a filter base on $B \times M$ such that $\pi_i^{-1}(\mathrm{b}) \cap ij - ad\, \mathcal{F} = \varphi$. For each $m \in M, (b, m) \notin ij - ad\, \mathcal{F}$. Thus there exist Λ_j-open nbd U_m of b in B, a τ_i –open nbd V_m of m in M and an $F_m \in \mathcal{F}$ such that $Q_i - cl(U_m \times V_m) \cap F_m = \varphi$. As show above, there exist finitely many elements $m_1, m_2, m_3, \ldots, m_n$ of M such that $\{b\} \times M \subset \bigcup_{k=1}^{n} Q_i - cl(U_{m_k} \times V_{m_K})$. Putting $U = \bigcap_{k=1}^{n} U_{m_k}$ and choosing $F \in \mathcal{F}$ with $F \subset$

$\bigcap_{k=1}^{n} F_{m_k}$, we get $\{b\} \times M \subset U \times M \subset Q_j$ such that $Q_i - cl(U \times M) \cap F = \varphi$. Thus $\big(ij - cl(F)\big) \cap [\pi_i{}^{-1}$(b)]$= \varphi$. Hence $\pi_i{}^{-1}$(b) is ij-rigid in $B \times M$.
(c)⇒(a) Taking $B^* = B$, we have that $p = \pi_i : B^* \times B \times \to B^*$ is ij-perfect . Therefore by theorem. (2.27) $B^* \times M$ is an ij-H-set and Hence M is ij-QHC.

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