Mathematical Statistician and Engineering Applications
ISSN: 2094-0343
2326-9865

δ-Semi Normal and δ-Semi Compact Spaces

Afraa R. Sadek[1], Ayman A. Ali[2], Jalal H. Hussein[3] and Haider Jebur Ali[4]

[1,2]Department of mathematics, College of Science, Baghdad University

[3]Department of mathematics, College of Science Women, Baghdad University

[4] Department of mathematics, College of Science, Al-MustansiriyaUniversity

Article Info
Page Number: 2062 - 2069
Publication Issue:
Vol 71 No. 4 (2022)

Article History
Article Received*: 25 March 2022*
Revised*: 30 April 2022*
Accepted*: 15 June 2022*
Publication*: 19 August 2022*

Abstract

In this paper by using δ-semi.open sets we introduced the concept of weakly δ-semi.normal and δ-semi.normal spaces . Many properties and results were investigated and studied. Also we present the notion of δ-semi.compact spaces and we were able to compare with it δ-semi.regular spaces.

Keyword: Generalized separation axioms, $\mathcal{R}_1$,$\mathcal{R}_o$– spaces ,δ-semi.open ,regular and normal spaces.

1.**Introduction and preliminaries.**

In what follows $\mathcal{X}$, $\mathcal{Y}$ denoted topological spaces. Let $\mathscr{B}$ be a subset of a space $\mathcal{X}$, we denote the interior and closure of $\mathscr{B}$ by Int($\mathscr{B}$) and Cl($\mathscr{B}$) respectively.Levine [4] introduced the concept of semi.open sets and semi. continuouity in topological spaces. A subset $\mathscr{B}$ is said to be semi.open if and only if $\mathscr{B}\subset$Cl(Int($\mathscr{B}$)) .The complement of semi.open is called semi.closed . A subset $\mathscr{B}$ is called regular closed (resp.regular open) if Cl (Int ($\mathscr{B}$)) = $\mathscr{B}$ (resp.Int(Cl($\mathscr{B}$))= $\mathscr{B}$[5]. A subset $\mathscr{B}$ is said to be δ-open [5] if for each $x\in \mathscr{B}$ there exists regular open set $\mathscr{H}$ such that $x\in \mathscr{H}\subset\mathscr{B}$. A subset $\mathscr{B}$ is said δ-semi.open[2] if there exists a δ-open set $\mathcal{V}$ of $\mathcal{X}$ such that $\mathcal{V}\subset \mathscr{B}\subset$ Cl ($\mathcal{V}$).The complement of δ-semi.opencalled δ-semi.closed. We denote the family of all δ-semi.open(resp. δ-semi.closed) in a space $\mathcal{X}$ by δ-SO($\mathcal{X}$) (resp. δ-SC($\mathcal{X}$)).

The intersection of δ-semi.closed(resp. δ-semi.open) that contain a subset$\mathcal{B}$ is called the δ-semi.closure(resp.δ-semi.kernal) and denotes by s.Cl$_{\delta}$($\mathcal{B}$) (resp.sker$_{\delta}$($\mathcal{B}$)) [2]. Recall that [2] a space$\mathcal{X}$is δ-semi.$\mathcal{T}_1$(resp.δ-semi.$\mathcal{R}_o$,δ-semi.$\mathcal{R}_1$)if for any distinct pairs of point x and y in $\mathcal{X}$, there are two δ-semi.open$\mathcal{U}$ and $\mathcal{V}$ such that

$x \in \mathcal{U} - \mathcal{V}$ and $y \in \mathcal{V} - \mathcal{U}$ (resp. If every δ-semi.open set contains the δ-semi.closure of each of its singleton , if for x,y in $\mathcal{X}$ with s.Cl$_{\delta}$($\{x\}$) ≠ s.Cl$_{\delta}$($\{y\}$),there exist distinct δ-semi.open sets $\mathcal{U}$ and $\mathcal{V}$ such that s.Cl$_{\delta}$($\{x\}$)$\subset\mathcal{U}$ and s.Cl$_{\delta}$($\{y\}$)$\subset\mathcal{V}$). Finally A.A.Ali and A.R.Sadek [1] studied in depth the concept of δ-semi.regulatiry of spaces.

A space $\mathcal{X}$ is said to be δ-semi.regular if for each $x \in \mathcal{X}$ and each semi.closed set $\mathcal{H}$ such that $x \notin \mathcal{H}$ there exist disjoint $\mathcal{V}_1$,$\mathcal{V}_2 \in$ δ-SO($\mathcal{X}$) such that $x \in \mathcal{V}_1, \mathcal{H} \subset \mathcal{V}_2$. In this paper we introduced theδ-semi.normaland weakleyδ-semi.normal spaces. Many results were proved as well as we investigated the relationship between δ-semi normal and δ-semi.regular by us the δ-semi.compact spaces.

2.Weakly δ-semi.normal spaces .

We will start with the following definition

Definition 2.1. Aspace$\mathcal{X}$is called weaklyδ-semi.normal(w.δ-semi.normal for short),if for each distinct closed subset $\mathcal{E}_1$and $\mathcal{E}_2$ of$\mathcal{X}$ there exist disjoint δ- semi. open set $\mathcal{U}$, $\mathcal{V}$ such that $\mathcal{E}_1 \subset \mathcal{U}$, $\mathcal{E}_2 \subset \mathcal{V}$.

Example 2.2.let ($\mathcal{X}$, τ) be a topological space such that $\mathcal{X}=\{a, b, c, d\}$ and

τ= $\{\phi, \mathcal{X}, \{a\}, \{b\}, \{a, b\}\}$ implies $\tau^c = \{\phi, \mathcal{X}, \{b,c,d\}, \{a,c,d\}, \{c,d\}\}$.Clearlythefamily of all δ-open sets is the familyτ and δ.S.O($\mathcal{X}$)=$\{\phi, \mathcal{X}, \{a\}, \{b\}, \{a,b\}$

$\{a,c\}\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$.It is not difficult to check that ($\mathcal{X}$, τ) is not normal while it is w. δ-semi.normal space.

Theorem2.3.A space $\mathcal{X}$ is w.δ-semi.normal if for each closed set $\mathcal{F}$ of $\mathcal{X}$,the δ-semi.closedneighborhood of $\mathcal{F}$ form a basis of neighborhood of $\mathcal{F}$.

Proof: Let $\mathcal{F}$ be a closed set in $\mathcal{X}$ and let $\mathcal{N}$be a neighborhood of $\mathcal{F}$, so there is an open set $\mathcal{O}$in $\mathcal{X}$such that$\mathcal{F}\subset\mathcal{O}\subset\mathcal{N}$. Thus $\mathcal{F}$ and $\mathcal{X}$- $\mathcal{O}$ are distinct closed sets in

$\mathcal{X}$, andbythe w.δ-semi.normality, we have $\mathcal{U},\mathcal{V}\in$ δ.SO($\mathcal{X}$) such that $\mathcal{F}\subset\mathcal{U}$,($\mathcal{X}$-$\mathcal{O}$)$\subset\mathcal{V}$ and $\mathcal{U}\cap\mathcal{V}$=ϕ. Hence $\mathcal{F}\subset\mathcal{U}\subset$($\mathcal{X}$-$\mathcal{V}$) $\subset\mathcal{O}\subset\mathcal{N}$, thus $\mathcal{X}$-$\mathcal{V}$ is δ-semi.closedneighborhoodof $\mathcal{F}$ contains in $\mathcal{N}$.■

Recall that [1] a map f: $\mathcal{X}\to\mathcal{Y}$ is said to be δ-semi.open (resp. δ-semi.irresulute) if$f(\mathcal{V})\in$δ.SO($\mathcal{Y}$) where $\mathcal{V}\in$δ.SO($\mathcal{X}$) (resp. $f^{-1}(\mathcal{U})\in$δ.SO($\mathcal{X}$) where $\mathcal{U}\in$δ.SO($\mathcal{Y}$)).

Theorem2.4.let$f$: $\mathcal{X}\to\mathcal{Y}$ be a surjective, continuous and δ-semi.open map, then the image of w.δ-semi.normalspace is w.δ-semi.normal.

Proof. Let $\mathcal{E}_1$,$\mathcal{E}_2$ be a disjoint closed sets in the space $\mathcal{Y}$ .By the continuity of f we have $f^{-1}(\mathcal{E}_1)$,f^{-1} ($\mathcal{E}_2$) are closed in $\mathcal{X}$.Now $f^{-1}(\mathcal{E}_1)$ $\cap f^{-1}$ ($\mathcal{E}_2$) = ϕ [6], so by the w.δ-semi.normality of the space $\mathcal{X}$, there are two disjoint sets $\mathcal{U},\mathcal{V}\in$ δ.SO($\mathcal{X}$) such that $f^{-1}(\mathcal{E}_1)$ $\subset\mathcal{U}$ and f^{-1} ($\mathcal{E}_2$))$\subset\mathcal{V}$.Further$f$ ($f^{-1}(\mathcal{E}_1)$)= $\mathcal{E}_1\subset f$($\mathcal{U}$), f ($f^{-1}(\mathcal{E}_2)$)= $\mathcal{E}_2\subset f(\mathcal{V})$ and Since f is δ- semi. open the proof is complete ■

Theorem 2.5. Let f: $\mathcal{X}\to\mathcal{Y}$ be an injective, closed and δ-semi.irresolutemap.Then $\mathcal{X}$ is w.δ-semi.normalspaceif $\mathcal{Y}$ is δ-semi.normal.

Proof. Suppose $\mathcal{E}_1$, $\mathcal{E}_2$ are two are disjoint closed subsets of $\mathcal{X}$ and since f is injective and closed ,so $f(\mathcal{E}_1)$,$f(\mathcal{E}_2)$ are disjoint closed in $\mathcal{Y}$.But $\mathcal{Y}$ is w.δ-semi.normal, hence there are two disjoint $\mathcal{U},\mathcal{V}\in$δ.SO($\mathcal{Y}$) such that $f(\mathcal{E}_1)\subset\mathcal{U}$ $f(\mathcal{E}_2)\subset\mathcal{V}$.Now $\mathcal{E}_1\subset f^{-1}(f$ ($\mathcal{E}_1$)) $\subset f^{-1}$ ($\mathcal{U}$) and $\mathcal{E}_2\subset f^{-1}(f$ ($\mathcal{E}_2$)) $\subset f^{-1}$ ($\mathcal{V}$) [6] and since $\mathcal{U}\cap\mathcal{V}$=ϕ , therefore f^{-1} ($\mathcal{U}$) and f^{-1} ($\mathcal{V}$) are required subsets. ■

3. δ-semi.normaland δ-semi.compact spaces .

We present in this section a new type of spaces, we called it δ-semi.normal space. Many properties of this space were studied. First, we introduce the following definition.

Definition3.1.A space $\mathcal{X}$is said to be δ-semi.normalif for each disjoint semi.closed sets$\mathcal{F},\mathcal{K}$of $\mathcal{X}$.There exist $\mathcal{U},\mathcal{V}\in$ δ.SO($\mathcal{X}$) such that$\mathcal{F}\subset\mathcal{U},\mathcal{K}\subset\mathcal{V}$ and $\mathcal{U}\cap\mathcal{V}=\phi$.

Example 3.2. Consider the following topology $\tau=\{\phi, \mathcal{X},\{c_1\},\{c_2\},\{c_1,c_2\}\}$ on the set $\mathcal{X}=\{c_1,c_2,c_3\}$.Then δ.SO($\mathcal{X}$)$=\{\phi,\mathcal{X},\{c_1\},\{c_2\},\{c_1,c_2\},\{c_2,c_3\},\{c_1,c_3\}\}$ and the family of semi.closed are S.C($\mathcal{X}$)$=\{\phi,\mathcal{X},\{c_2,c_3\},\{c_1,c_3\}, \{c_3\},\{c_1\}, \{c_2\}\}$.Now it is not difficult to show that the space $\mathcal{X}$ is δ-semi.normal.

Some properties are holds in a δ-semi.normal spaces as shown in the next theorem and remark.

Theorem 3.3. Let $\mathcal{X}$ be a δ-semi.normal space, then for each semi.closed$\mathcal{F}\subset\mathcal{X}$and for every δ-semi.open set $\mathcal{V}$ containing$\mathcal{F}$there exists $\mathcal{U}\in$ δ.SO($\mathcal{X}$) such that$\mathcal{F}\subset\mathcal{U}\subset sCl_\delta(\mathcal{U})\subset\mathcal{V}$.

Proof. Let $\mathcal{F}\subset\mathcal{X}$ and let $\mathcal{V}$ be any δ-*s*emi.open set containing$\mathcal{F}$.Now$\mathcal{F}\cap(\mathcal{X}\text{-}\mathcal{V})=\phi$ and $(\mathcal{X}\text{-}\mathcal{V})$ is δ-*s*emi.closed ,hence it is semi.closed [5].By the δ-semi.normality of $\mathcal{X}$, there exist $\mathcal{U}$, G $\in$ δ.SO ($\mathcal{X}$) such that$\mathcal{F}\subset\mathcal{U}$, $(\mathcal{X}\text{-}\mathcal{V})\subset$G and $\mathcal{U}\cap$G$=\phi$.Thus $\mathcal{U}\subset\mathcal{X}$-G and since ($\mathcal{X}$-G) is δ-*s*emi.closed, hence $\mathcal{F}\subset\mathcal{U}\subset sCl_\delta(\mathcal{U})\subset(\mathcal{X}\text{-}G)\subset\mathcal{V}$. ■

Remark 3.4. If $\mathcal{X}$is δ-semi.normal space than for any two distinct semi.closed sets$\mathcal{F},\mathcal{K}$, then $sker_\delta(\mathcal{F})\cap sker_\delta(\mathcal{K})=\phi$.

Proof. From δ-semi.normality of $\mathcal{X}$ we have $\mathcal{U},\mathcal{V}\in$δ.SO($\mathcal{X}$) such that$\mathcal{F}\subset\mathcal{U},\mathcal{K}\subset\mathcal{V}$ and$\mathcal{U}\cap\mathcal{V}=\phi$ and since $sker_\delta(\mathcal{F})\subset\mathcal{U}$,$sker_\delta(\mathcal{K})\subset\mathcal{V}$[2],hence $sker_\delta(\mathcal{F})\cap sker_\delta(\mathcal{K})=\phi$.

Caldas[2] shows that any δ-semi$\mathcal{R}_1$ space is δ-semi$\mathcal{R}_o$.In the following preposition we will show the converse is true in δ-semi.normal spaces .

Proposition 3.5.A δ-semi$\mathcal{R}_o$and δ-semi.normal spaceis δ-semi $\mathcal{R}_1$.

Proof. Let $\mathcal{X}$ be a space and $x, y \in \mathcal{X}$ such that $sCl_\delta(\{x\}) \neq sCl_\delta(\{y\})$. Since $\mathcal{X}$

is δ-semi$\mathcal{R}_o$, hence $sCl_\delta(\{x\}) \cap sCl_\delta(\{y\}) = \phi$ [2, p.123].But $sCl_\delta(\{x\})$, $sCl_\delta(\{y\})$ are δ-semi.closed sets [2] consequently semi.closed sets [5], hence by the δ-semi.normality of $\mathcal{X}$ there exist $\mathcal{U}, \mathcal{V} \in \delta.SO(\mathcal{X})$ such that $sCl_\delta(\{x\}) \subset \mathcal{U}$, $sCl_\delta(\{y\}) = \mathcal{V}$ and $\mathcal{U} \cap \mathcal{V} = \phi$. Thus, $\mathcal{X}$ is δ-semi$\mathcal{R}_1$. ■

The following two theorems are good characterizations for δ-semi.normalspaces.

Theorem3.6. A space $\mathcal{X}$ is δ-semi.normalif and only if for eachdisjoint *s*emi.closed sets $\mathcal{E}$ and $\mathcal{F}$, there exists a δ-*s*emi.open set $\mathcal{U}$ containing $\mathcal{E}$ such that $\mathcal{F} \cap sCl_\delta(\mathcal{U}) = \phi$.

Proof. Suppose $\mathcal{X}$ is a δ-semi.normal space, and $\mathcal{F} \cap \mathcal{E} = \phi$, where $\mathcal{E}$ and $\mathcal{F}$ are *s*emi.closed sets in $\mathcal{X}$.Thus there exist $\mathcal{U}, \mathcal{V} \in \delta.SO(\mathcal{X})$ such that $\mathcal{F} \subset \mathcal{V}$, $\mathcal{E} \subset \mathcal{U}$ and $\mathcal{U} \cap \mathcal{V} = \phi$, hence $\mathcal{V}$ is δ-*s*emi.open containing $\mathcal{F}$ which not intersect $\mathcal{U}$, implies $\mathcal{F} \cap sCl_\delta(\mathcal{U}) = \phi$. For sufficiency since there exists $\mathcal{U} \in \delta.SO(\mathcal{X})$ containing $\mathcal{E}$ such that $\mathcal{F} \cap sCl_\delta(\mathcal{U}) = \phi$ where $\mathcal{F}$ and $\mathcal{E}$ as in the assumption,that is mean there exist a family $\{\mathcal{V}_\alpha, \alpha \in \Delta\}$ of δ-semi.open sets containing x_α for all $x_\alpha \in \mathcal{E}$ and $\mathcal{U} \cap \mathcal{V}_\alpha = \phi$, $\alpha \in \Delta$. But $\cup_{\alpha\in\Delta} \{\mathcal{V}_\alpha\}$ is δ-*s*emi.open[2] which complete the proof. ■

Theorem 3.7. In any topological space the following are equivalent.

(1) $\mathcal{X}$ is δ-semi.normal space.

(2) For any two distinct δ-semi.closed sets $\mathcal{F}$ and $\mathcal{K}$ there exists two δ-semi.open sets $\mathcal{V}_1$ and $\mathcal{V}_2$ such that $\mathcal{F} \subset \mathcal{V}_1$ and $\mathcal{K} \subset \mathcal{V}_2$ and $sCl_\delta(\mathcal{V}_1) \cap sCl_\delta(\mathcal{V}_2) = \phi$.

Proof.(2) ⟶ (1) obviously.

(1) ⟶ (2) Take any two distinct semi.closed sets $\mathcal{F}$ and $\mathcal{K}$ there exist disjoint δ-semi.open sets $\mathcal{V}_1$ and $\mathcal{V}_2$ such that $\mathcal{F} \subset \mathcal{V}_1$, and $\mathcal{K} \subset \mathcal{V}_2$. By [1] we have two δ-*s*emi.open sets $\mathcal{U}$ and $\mathcal{V}$ such that $\mathcal{F} \subset \mathcal{U} \subset sCl_\delta(\mathcal{U}) \subset \mathcal{V}_1$ and $\mathcal{K} \subset \mathcal{V} \subset sCl_\delta(\mathcal{V}) \subset \mathcal{V}_2$ since $\mathcal{V}_1 \cap \mathcal{V}_2 = \phi$ implies $sCl_\delta(\mathcal{U}) \cap sCl_\delta(\mathcal{V}) = \phi$ and we have done. ■

A δ-semi.normal space is δ-semi.regular under a given condition as shown in the following remark.

Remark. 3.8. A δ-semi.normaland δ-semi$\mathcal{T}_1$ space is δ-semi.regular.

Proof. Let $\mathcal{F}$be a semi.closed subset in $\mathcal{X}$ and let x be any point in $\mathcal{X}$ such that$x\notin\mathcal{F}$ .Now since $\mathcal{X}$ is δ-semi$\mathcal{T}_1$,then every singleton in $\mathcal{X}$ is δ-semi.closed and ,then is semi.closed [5] .Thus thereexist adisjoint $\mathcal{U},\mathcal{V}\in\delta.SO(\mathcal{X})$suchthat $x\in\mathcal{U}$ and$\mathcal{F}\in\mathcal{V}$, hence $\mathcal{X}$ is δ-semi.regular. ■

Corollary3.9. A δ-semi.normal and δ-semi$\mathcal{T}_1$ space is δ-semi$\mathcal{R}_1$.

Proof. Follows by remark (3.8)and proposition (3.5) sinceeach δ-semi.regular is δ-semi$\mathcal{R}_o$[1,p.956].■

Before we will give the next theorem, we present the following definitions.

Definition 3.10. A covering of a set $\mathcal{X}$ is a family $\mathfrak{B}$ of subsets of $\mathcal{X}$ such that

$\mathcal{X}=\cup\{\mathcal{U}:\mathcal{U}\in\mathfrak{B}\}$.If $\mathcal{X}$is a topological space and every member of $\mathfrak{B}$ is δ-semi.open, then $\mathfrak{B}$ is called δ-semi.open cover of $\mathcal{X}$.

Definition 3.11. A space $\mathcal{X}$ is said to be δ-semi.compact if every δ-semi. open cover of $\mathcal{X}$ has a finite subcover.

The following is an example of δ-semi.compact space

Example 3.12. Let $(\mathbb{R},\tau_{ind})$ the Indiscrete topological space on the set of the real number $\mathbb{R}$. It is not difficult to check that $(\mathbb{R},\tau_{ind})$ is δ-semi.compact space.

Theorem 3.13.A δ-semi.compact ,δ-semi.regular and δ-semi$\mathcal{T}_1$ space is δ-semi.normal space.

Proof. Suppose $\mathcal{X}$is δ-semi.regular and δ-semi$\mathcal{T}_1$ then $\mathcal{X}$ is δ-semi$\mathcal{T}_2$[1]. Now let $\mathcal{E}$, $\mathcal{F}$ be a disjoint semi.closed sets .Fix $e\in\mathcal{E}$ since $\mathcal{X}$ is δ-semi$\mathcal{T}_2$,then for each $f\in\mathcal{F}$ there exist δ-semi.open sets $\mathcal{U}_\alpha$,$\mathcal{V}_\alpha$such that $e\in\mathcal{V}_\alpha$, $f\in\mathcal{U}_\alpha$ and $\mathcal{V}_\alpha\cap\mathcal{U}_\alpha=\phi$. Thus $\{\mathcal{U}_\alpha;\ \alpha\in\Delta\}$ is δ-semi.open cover of $\mathcal{F}$ and since $\mathcal{F}$ is δ-semi.compact so there exists a finite sub cover $\{\mathcal{U}_{\alpha 1},\mathcal{U}_{\alpha 2},$

$\ldots,\mathcal{U}_{\alpha n}\}$,put $\mathcal{U}_e = \mathcal{U}_{\alpha 1}\cup\mathcal{U}_{\alpha 2}\cup\ldots\cup\mathcal{U}_{\alpha n}$ and $\mathcal{V}_e = \mathcal{V}_{\alpha 1}\cap\mathcal{V}_{\alpha 2}\cap\ldots\cap\mathcal{V}_{\alpha n}$is δ-semi.open [2] and since $\mathcal{X}$ is δ-semi.regular , then $\mathcal{V}_{\alpha i}$,i=1,2…n are also δ-semi.closed[1]. Thus $\mathcal{V}_e$ is δ-semi.closed [2]. Thus we have $\mathcal{F} \subset\mathcal{U}_e\in\mathcal{V}_e$ and $\mathcal{V}_e \cap \mathcal{U}_e = \phi$. Now let e vary throughout $\mathcal{E}$, so we obtain a δ-semi.open cover { $\mathcal{V}_e$; $e \in\mathcal{E}$}of $\mathcal{E}$.As $\mathcal{E}$ is δ-semi.compact there exists a finite sub cover $\mathcal{V}_{e1}$, $\mathcal{V}_{e2}$, …,$\mathcal{V}_{er}$ means $\mathcal{V}= \mathcal{V}_{e1}\cup\mathcal{V}_{e2}\cup\ldots\cup\mathcal{V}_{er}and\mathcal{U}= \mathcal{U}_{e1} \cap\mathcal{U}_{e2}\ldots. \cap\mathcal{U}_{en}$ .Now$\mathcal{V}$ is δ-semi.open [2] and the δ-semi.regularity$\mathcal{U}$ is also δ-semi.open. Then$\mathcal{F}\subset\mathcal{U}$, $\mathcal{E}\subset\mathcal{V}$ and$\mathcal{U}\cap\mathcal{V}= \phi$, hence $\mathcal{X}$ is δ-semi.normal. ■

Lemma3.14. [3]If f:$\mathcal{X}\rightarrow\mathcal{Y}$is continuous and open map, and if $\mathcal{A}$ is semi.open in $\mathcal{X}$,then $f(\mathcal{A})$ is semi.open in $\mathcal{Y}$.

Under the same condition of f in the above lemma we have the following theorem.

Theorem3.15. Let f: $\mathcal{X}\rightarrow\mathcal{Y}$be an injective, δ-semi.irresolute map, and $\mathcal{Y}$ is δ-semi.normal space, then $\mathcal{X}$ is δ-semi.normal space.

Proof. Suppose $\mathcal{E}_1$, $\mathcal{E}_2$are any disjoint semi.closed subset of $\mathcal{X}$. Now $f(\mathcal{E}_1)$,$f(\mathcal{E}_2)$ are semi.closed (3.14) and disjoint since f is injective .Thus by the δ-semi.normality of $\mathcal{Y}$, there exist disjoint $\mathcal{U},\mathcal{V}\in$δ.SO($\mathcal{Y}$) such that $f(\mathcal{E}_1)\subset\mathcal{U}$,$f(\mathcal{E}_2) \subset\mathcal{V}$. But $\mathcal{E}_1\subset f^{-1}(f(\mathcal{E}_1))\subset f^{-1}(\mathcal{U})$, $\mathcal{E}_2\subset f^{-1}(f(\mathcal{E}_2))\subset f^{-1} (\mathcal{V})$, clear that each of $f^{-1}(\mathcal{U})$,$f^{-1} (\mathcal{V})$ are δ-semi.open further they are disjoint [7]. Hence $\mathcal{X}$ is δ-semi.normal space■

Theorem 3.16. Let f:$\mathcal{X}\rightarrow\mathcal{Y}$ be an injective,continuous, and δ-semi.open, then $\mathcal{Y}$ is δ-semi.normal if $\mathcal{X}$ is δ-semi.normal space.

Proof. Suppose $\mathcal{X}$ is δ-semi.normalspace.Let$\mathcal{E}_1$, $\mathcal{E}_2$are two disjoint semi.closed subset in $\mathcal{Y}$.Now since f is bijective and irresolute continuous, thus $f^{-1}(\mathcal{E}_1)$,$f^{-1}(\mathcal{E}_2)$ are disjoint and semi.closed in $\mathcal{X}$,so by the δ-semi.normality of $\mathcal{X}$ there exist $\mathcal{U},\mathcal{V}\in$ δ.SO($\mathcal{X}$) such that $f^{-1}(\mathcal{E}_1)\subset\mathcal{U}$,$f^{-1}(\mathcal{E}_2)\subset\mathcal{V}$ and $\mathcal{U}\cap\mathcal{V}=\phi$, hence$f(f^{-1}(\mathcal{E}_1))=\mathcal{E}_1\subset f(\mathcal{U})$ and $f(f^{-1}(\mathcal{E}_2))=\mathcal{E}_2\subset f(\mathcal{V})$. It is clear $f(\mathcal{U})$, $f(\mathcal{V})$ are disjoint and since f is δ-semi.open map so we have done . ■

Reference

[1]A.A.Ali and A.R.Sadek,On regularδ-semi.openspace,Journal of interdisciplinary of mathematics 24(4),(2021),pp.953-960.

[2] M.Caldas and T.Noiri, More on δ-semi.open sets ,Note di Mathematica.22 (2)(2003),pp.113-126.

[3] S.G. Crossley and S.K. Hildebrand, Semi-topological properties, FundamentalMathematicae 74(3), (1972), pp.233-257.

[4] N.Levine, Semi.open sets and continuity in topological space, The American Mathematical Monthly ,70(1),(1963),pp.36-41.

[5] T. Noiri, Remarks on δ-semi.open sets and δ-preopen sets, Demonstratio Mathematica, 36,(2003),pp.1007-1020.

[6]S.Willard, General Topology, Dover Publications, (2012).

[7]A.R.Sadek,More on the minimal set of periods, Iraqi Journal of science 56(4A),(2015),pp.2453-2958.