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Article

Coupled Laplace-Decomposition Method for Solving Klein-Gordon Equation

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Abstract: In this paper, we consider a new approach to solve type of partial differential equation by using coupled Laplace transformation with decomposition method to find the exact solution for non–linear non–homogenous equation with initial conditions. The reliability for suggested approach illustrated by solving model equations such as second order linear and nonlinear Klein–Gordon equation. The application results show the efficiency and ability for suggested approach.

Keyword: Partial differential equation, Klein - Gordon equation, Laplace transformation and decomposition method.

Mathematics Subject Classification (2010): 35E99, 35Q99, 35R99

1. Introduction

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology [1, 2]. Many physical and chemistry phenomena can be modeled using the language of calculus. For example, compute the pollution of heavy metals in soil by using the mathematical model for that phenomena and when we solve that model we find the valued of pollution in soil by heavy metals for more details see [3-7].

There are many researches explain how to solve the partial differential equations using efficient method to get exact or approximate solution such ADM [8], VIM [9], HPM [10], and HAM [11]. In recent years many researchers used coupled method to solve types of PDEs such [12-16].

Here we solved important type of PDEs by combine Laplace transformation with ADM that is LT-ADM to find the exact solution for non-linear Klein - Gordon equation.

2. Klein - Gordon Equation

The Klein - Gordon equation it is one of the most important mathematical models in quantum field theory, appears in relativistic physics and described dispersive wave phenomena in general [2]. Here we used the suggested approach to solve this type of PDEs.

2.1. Solving Linear Klein - Gordon Equation

The standard form of linear Klein Gordon equation is given by

$$u_{tt}(x,t) - u_{xx}(x,t) + au(x,t) = h(x,t) \tag{1}$$

Where a is a constant and $h(x,t)$ is the source term with initial conditions

$$u(x,0) = f(x),\ u_t(x,0) = g(x) \tag{2}$$

To solve that equation 1 by using LT-ADM:

First we take the Laplace transformation for equation 1 to get:

$$L\{u_{tt}(x,t)\} - L\{u_{xx}(x,t)\} + aL\{u(x,t)\} = L\{h(x,t)\} \tag{3}$$

$$s^2L\{u(x,t)\} - s\,u(x,0) - u_t(x,0) - \frac{d^2}{dx^2}L\{u(x,t)\} + aL\{u(x,t)\} = L\{h(x,t)\}$$

Use the initial conditions in equation 3 to get:

$$s^2L\{u(x,t)\} - s\,f(x) - g(x) - \frac{d^2}{dx^2}L\{u(x,t)\} + aL\{u(x,t)\} = L\{h(x,t)\}$$

$$L\{u(x,t)\} = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u(x,t)\} - \frac{a}{s^2}L\{u(x,t)\} + \frac{1}{s^2}L\{h(x,t)\}$$

Now, by using the decomposition representation as follow:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{4}$$

$$L\{u_0(x,t)\} = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L\{h(x,t)\}$$

then, take the invers Laplace transformation to get $u_0(x,t)$

$$u_0(x,t) = f(x) + g(x)\,t + t\,L\{h(x,t)\}$$

$$L\{u_1(x,t)\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_0(x,t)\} - \frac{a}{s^2}L\{u_0(x,t)\}$$

$\vdots$

$$L\{u_{k+1}(x,t)\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_k(x,t)\} - \frac{a}{s^2}L\{u_k(x,t)\}$$

Apply the invers Laplace transformation to find $u_1(x,t), u_2(x,t)$, ...

Now, we take illustration example as follow:

Example 1

Consider a homogenous linear Klein - Gordon equation

$u_{tt} - u_{xx} + u = 0$, with initial conditions $u(x,0) = 0$, $u_t(x,0) = x$

Applying LT-ADM as follow

$$L\{u_{tt}\} - L\{u_{xx}\} + L\{u\} = 0$$

$$s^2L\{u\} - s\,u(x,0) - u_t(x,0) - \frac{d^2}{dx^2}L\{u\} + L\{u\} = 0 \qquad (5)$$

Using ICs equation 5 becomes

$$s^2L\{u\} - x - \frac{d^2}{dx^2}L\{u\} + L\{u\} = 0$$

$$L\{u\} = \frac{x}{s^2} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u\} - \frac{L\{u\}}{s^2}$$

Now, using the decomposition representation for the dependent variable u:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (6)$$

$$L\{u_0\} = \frac{x}{s^2}$$

take the invers Laplace transformation to get u_0

$$u_0 = x\,t$$

Hence,

$$L\{u_1\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_0\} - \frac{1}{s^2}L\{u_0\}$$

$$L\{u_1\} = -\frac{x}{s^4} \quad ; so, \quad u_1 = -\frac{xt^3}{3!}$$

$$L\{u_2\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_1\} - \frac{1}{s^2}L\{u_1\}$$

$$L\{u_2\} = \frac{x}{s^6} \quad ; \quad u_2 = \frac{xt^5}{5!}$$

$$L\{u_3\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_2\} - \frac{1}{s^2}L\{u_2\}$$

$$L\{u_3\} = -\frac{x}{s^8} \quad ; so, \quad u_3 = -\frac{xt^7}{7!}$$

$$L\{u_4\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_3\} - \frac{1}{s^2}L\{u_3\}$$

$$L\{u_4\} = \frac{x}{s^{10}} \quad , so \quad u_4 = \frac{xt^9}{9!}$$

And so on

$$L\{u_{k+1}\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_k\} - \frac{L\{u_k\}}{s^2}$$

Depending on equation 6, we obtain

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4 \ldots$$

i.e., $u(x,t) = xt - \frac{xt^3}{3!} + \frac{xt^5}{5!} - \frac{xt^7}{7!} + \frac{xt^9}{9!} - \cdots$

$$u(x,t) = x\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots\right)$$

The exact solution is $u(x,t) = x\, sinx$

Example 2

Consider inhomogenous linear Klein - Gordon equation

$u_{tt} - u_{xx} + u = 2\sin x$, with initial conditions $u(x,0) = \sin x$, $u_t(x,0) = 1$

applying Laplace transformation for the equation

$$L\{u_{tt}\} - L\{u_{xx}\} + L\{u\} = 2L\{\sin x\}$$

$$s^2L\{u\} - s\,u(x,0) - u_t(x,0) - \frac{d^2}{dx^2}L\{u\} + L\{u\} = \frac{2\sin x}{s}$$

Using initial conditions to get:

$$s^2L\{u\} - s\sin x - 1 - \frac{d^2}{dx^2}L\{u\} + L\{u\} = \frac{2\sin x}{s}$$

$$L\{u\} = \frac{\sin x}{s} + \frac{1}{s^2} + \frac{2\sin x}{s^3} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u\} - \frac{a}{s^2}L\{u\}$$

Now, we using the decomposition representation for dependent variable u

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (7)$$

$$L\{u_0\} = \frac{\sin x}{s} + \frac{1}{s^2} + \frac{2\sin x}{s^3}$$

take the invers Laplace transformation to get u_0

$$u_0 = \sin x + t + t^2\sin x$$

$$L\{u_1\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_0\} - \frac{1}{s^2}L\{u_0\}$$

$$L\{u_1\} = -\frac{2\sin x}{s^5} - \frac{\sin x}{s^3} - \left(\frac{\sin x}{s^2} + \frac{1}{s^4} + \frac{2\sin x}{s^5}\right)$$

$$L\{u_1\} = -\frac{4\sin x}{s^5} - \frac{2\sin x}{s^3} - \frac{1}{s^4}$$

$$u_1 = -\frac{t^4\sin x}{3!} - t^2\sin x - \frac{t^3}{3!}$$

$$L\{u_2\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_1\} - \frac{1}{s^2}L\{u_1\}$$

$$L\{u_2\} = \frac{4\sin x}{s^7} + \frac{2\sin x}{s^5} - \left(-\frac{4\sin x}{s^7} - \frac{2\sin x}{s^5} - \frac{1}{s^6}\right)$$

$$L\{u_2\} = \frac{8\sin x}{s^7} + \frac{4\sin x}{s^5} + \frac{1}{s^6}$$

$$u_2 = \frac{8t^6\sin x}{6!} + \frac{t^4\sin x}{3!} + \frac{t^5}{5!}$$

$$L\{u_3\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_2\} - \frac{1}{s^2}L\{u_2\}$$

$$L\{u_3\} = -\frac{8\sin x}{s^9} - \frac{4\sin x}{s^7} - \left(\frac{8\sin x}{s^9} + \frac{4\sin x}{s^7} + \frac{1}{s^8}\right)$$

$$L\{u_3\} = -\frac{16\sin x}{s^9} - \frac{8\sin x}{s^7} - \frac{1}{s^8}$$

$$u_3 = -\frac{16t^8\sin x}{8!} - \frac{8t^6\sin x}{6!} - \frac{t^7}{7!}$$

$$L\{u_4\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_3\} - \frac{1}{s^2}L\{u_3\}$$

$$L\{u_4\} = \frac{16\sin x}{s^{11}} + \frac{8\sin x}{s^9} \; - (-\frac{16\sin x}{s^{11}} - \frac{8\sin x}{s^9} - \frac{1}{s^{10}})$$

$$L\{u_4\} = \frac{32\,sin\,x}{s^{11}} + \frac{16\,sin\,x}{s^9} + \frac{1}{s^{10}}$$

$$u_4 = \frac{32\,t^{10}\sin x}{10!} + \frac{16t^8\sin x}{8!} + \frac{t^9}{9!}$$

And so on

$$L\{u_{k+1}\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_k\} - \frac{L\{u_k\}}{s^2}$$

Depending on equation 7, we have:

$$u(x,t) = u_\circ + u_1 + u_2 + u_3 + u_4 \ldots$$

$$u(x,t) = \sin x + t + t^2\sin x \; - \frac{t^4\sin x}{3!} - t^2\sin x - \frac{t^3}{3!} + \frac{8t^6\sin x}{6!} + \frac{t^4\sin x}{3!} + \frac{t^5}{5!} - \frac{16t^8\sin x}{8!} - \frac{8t^6\sin x}{6!} - \frac{t^7}{7!}\frac{32\,t^{10}\sin x}{10!} + \frac{16t^8\sin x}{8!} + \frac{t^9}{9!} - \cdots$$

$$u(x,t) = \sin x + t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots$$

So, the exact solution is $u(x,t) = \sin x + \sin t$

2.2. Solving Nonlinear Klein Gordon Equation

The standard form of nonlinear Klein Gordon equation is

$$u_{tt}(x,t) - u_{xx}(x,t) + au(x,t) + F(u(x,t)) = h(x,t) \qquad (8)$$

where a is constant, $h(x,t)$ is the source term and $F(u(x,t))$ is nonlinear function of $u(x,t)$. With initial conditions:

$$u(x,0) = f(x), \;\; u_t(x,0) = g(x) \qquad (9)$$

To solve equation 8, by using LT-ADM, first we take the Laplace transformation for equation (8) to get

$$L\{u_{tt}(x,t)\} - L\{u_{xx}(x,t)\} + aL\{u(x,t)\} + L\{F\big(u(x,t)\big)\} = L\{h(x,t)\}$$

$$s^2L\{u(x,t)\} - s\,u(x,0) - u_t(x,0) - \frac{d^2}{dx^2}L\{u(x,t)\} + aL\{u(x,t)\} + L\{F(u(x,t))\} = L\{h(x,t)\}$$

Take the initial conditions in equation 3 we get

$$s^2L\{u(x,t)\} - s\,f(x) - g(x) - \frac{d^2}{dx^2}L\{u(x,t)\} + aL\{u(x,t)\} + L\{F(u(x,t))\} = L\{h(x,t)\}$$

$$L\{u(x,t)\} = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u(x,t)\} - \frac{a}{s^2}L\{u(x,t)\} - \frac{1}{s^2}L\{F(u(x,t))\} + \frac{1}{s^2}L\{h(x,t)\}$$

Now, using the decomposition representation for dependent variable:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (10)$$

The nonlinear form is decomposed as follow:

$$F\big(u(x,t)\big) = \sum_{n=0}^{\infty} A_n \qquad (11)$$

Where A_n is Adomian polynomials can be computed by using the formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\sum_{i=0}^{\infty}(\lambda^i u_i)\right]_{\lambda=0} \quad , n = 0,1,2,\dots \qquad (12)$$

$$L\{u(x,t)\} = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u(x,t)\} - \frac{a}{s^2}L\{u(x,t)\} - \frac{1}{s^2}L\left\{\sum_{n=0}^{\infty} A_n\right\} + \frac{1}{s^2}L\{h(x,t)\}$$

$$L\{u_0(x,t)\} = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L\{h(x,t)\}$$

Now, applied the invers Laplace transformation to get $u_0(x,t)$

$$u_1(x,t) = f(x) + g(x)\,t + t\,L\{h(x,t)\}$$

$$L\{u_1(x,t)\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_0(x,t)\} - \frac{a}{s^2}L\{u_0(x,t)\} - \frac{1}{s^2}L\{A_0\}$$

$$L\{u_2(x,t)\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_1(x,t)\} - \frac{a}{s^2}L\{u_1(x,t)\} - \frac{1}{s^2}L\{A_1\}$$

And so on

$$L\{u_{k+1}(x,t)\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_k(x,t)\} - \frac{a}{s^2}L\{u_k(x,t)\} - \frac{1}{s^2}L\{A_1\}$$

Apply the invers Laplace transformation to find the $u_1(x,t), u_2(x,t), \dots$

Now, we illustrate the above steps by solve the following example

Example 3

Consider nonlinear nonhomogeneous Klein - Gordon equation

$u_{tt} - u_{xx} + u^2 = x^2t^2$, with initial conditions $u(x,0) = 0$, $u_t(x,0) = x$

Firstly, apply Laplace transformation

$$L\{u_{tt}\} - L\{u_{xx}\} + L\{F\big(u(x,t)\big)\} = L\{x^2t^2\}$$

$$s^2L\{u\} - s\,u(x,0) - u_t(x,0) - \frac{d^2}{dx^2}L\{u\} + L\{F(u(x,t))\} = \frac{2x^2}{s^3}$$

Using ICs to get

$$s^2L\{u\} - x - \frac{d^2}{dx^2}L\{u\} + L\{F(u(x,t))\} = \frac{2x^2}{s^3}$$

$$L\{u\} = \frac{x}{s^2} + \frac{2x^2}{s^5} + \frac{1}{s^2}\frac{d^2}{dx^2}L\{u\} - \frac{1}{s^2}L\{F(u(x,t))\}$$

Now, using the decomposition representation for u such:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \qquad (13)$$

$$L\{u_0\} = \frac{x}{s^2} + \frac{2x^2}{s^5}$$

Then, apply the invers Laplace transformation to get u_0

$$u_0 = xt + \frac{x^2t^4}{12}$$

$$L\{u_1\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_\circ\} - \frac{1}{s^2}L\{F(u_\circ(x,t))\}$$

Now, calculate the nonlinear form as

$$F(u(x,t)) = \sum_{n=0}^{\infty} A_n \qquad (14)$$

Where A_n are Adomian polynomials computed by:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\sum_{i=0}^{\infty}(\lambda^i u_i)\right]_{\lambda=0} , \qquad n = 0,1,2,\ldots \qquad (15)$$

So, $L\{u_1\} = \frac{1}{s^2}\frac{d^2}{dx^2}\left[\frac{x}{s^2} + \frac{2x^2}{s^5}\right] - \frac{1}{s^2}L\{A_0\}$

$$L\{u_1\} = \frac{4}{s^7} - \frac{1}{s^2}L\{u_0{}^2\}$$

$$L\{u_1\} = \frac{4}{s^7} - \frac{1}{s^2}L\left\{x^2t^2 + \frac{x^3t^5}{6} + \frac{x^4t^8}{144}\right\}$$

$$L\{u_1\} = \frac{4}{s^7} - \frac{2x^2}{s^5} - \frac{20x^3}{s^8} - \frac{280x^4}{s^{11}}$$

$$u_1 = \frac{4t^6}{6!} - \frac{2x^2t^4}{4!} - \frac{20x^3t^7}{7!} - \frac{280x^4t^{10}}{10!}$$

$$u_1 = \frac{t^6}{180} - \frac{x^2t^4}{12} - \frac{x^3t^7}{252} - \frac{x^4t^{10}}{12960}$$

$$L\{u_2\} = \frac{1}{s^2}\frac{d^2}{dx^2}L\{u_1\} - \frac{1}{s^2}L\{F(u_1(x,t))\}$$

$$L\{u_2\} = \frac{1}{s^2}\frac{d^2}{dx^2}\left[\frac{4}{s^7} - \frac{2x^2}{s^5} - \frac{20x^3}{s^8} - \frac{280x^4}{s^{11}}\right] - \frac{1}{s^2}L\{A_1\}$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{1}{s^2}L\{2u_0u_1\}$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{2}{s^2}L\left\{\left(xt + \frac{x^2t^4}{12}\right)\left(\frac{t^6}{180} - \frac{x^2t^4}{12} - \frac{x^3t^7}{252} - \frac{x^4t^{10}}{12960}\right)\right\}$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{2}{s^2}L\left\{\left(\frac{xt^7}{180} - \frac{x^3t^5}{12} - \frac{x^4t^8}{252} - \frac{x^5t^{11}}{12960} + \frac{x^2t^{10}}{2160} - \frac{x^4t^8}{144} - \frac{x^5t^{11}}{3024} - \frac{x^6t^{14}}{15120}\right)\right\}$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{2}{s^2}\left(\frac{28\,x}{s^8} - \frac{10x^3}{s^6} - \frac{8!\,x^4}{252s^9} - \frac{11!\,x^5}{12960s^{12}} + \frac{10!\,x^2}{2160s^{11}} - \frac{8!\,x^4}{144s^9} - \frac{11!\,x^5}{3024s^{12}} - \frac{14!\,x^6}{15120s^{15}}\right)$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{56\,x}{s^{10}} + \frac{20x^3}{s^8} + \frac{8!\,x^4}{126s^{11}} + \frac{11!\,x^5}{6480s^{14}} - \frac{10!\,x^2}{1080s^{13}} + \frac{8!\,x^4}{72s^{11}} + \frac{11!\,x^5}{1512s^{14}} + \frac{14!\,x^6}{7560s^{17}}$$

$$L\{u_2\} = -\frac{4}{s^7} - \frac{120x}{s^{10}} - \frac{3360x^2}{s^{13}} - \frac{56\,x}{s^{10}} + \frac{20x^3}{s^8} + \frac{320x^4}{s^{11}} + \frac{6160x^5}{s^{14}} - \frac{3360\,x^2}{s^{13}} + \frac{560x^4}{s^{11}} + \frac{26400x^5}{s^{14}} + \frac{11531520x^6}{s^{17}}$$

$$L\{u_2\} = -\frac{4}{s^7} + \frac{20x^3}{s^8} - \frac{176x}{s^{10}} + \frac{880x^4}{s^{11}} - \frac{6720x^2}{s^{13}} + \frac{32560x^5}{s^{14}} + \frac{11531520x^6}{s^{17}}$$

$$u_2 = -\frac{4t^6}{6!} + \frac{20x^3t^7}{7!} - \frac{176xt^9}{9!} + \frac{880x^4t^{10}}{10!} - \frac{6720x^2t^{12}}{12!} + \frac{32560x^5t^{13}}{13!} + \frac{11531520x^6t^{16}}{16!}$$

$$u_2 = -\frac{t^6}{180} + \frac{x^3t^7}{252} - \frac{11xt^9}{75600} + \frac{x^4t^{10}}{12960} - \frac{x^2t^{12}}{71280} + \frac{37x^5t^{13}}{7076160} + \frac{x^6t^{16}}{1814400}$$

And so on, then depending on equation 13 we have

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4 \ldots$$

$$u(x,t) = xt + \frac{x^2t^4}{12} + \frac{t^6}{180} - \frac{x^2t^4}{12} - \frac{x^3t^7}{252} - \frac{x^4t^{10}}{12960} - \frac{t^6}{180} + \frac{x^3t^7}{252} - \frac{11xt^9}{75600} + \frac{x^4t^{10}}{12960} - \frac{x^2t^{12}}{71280} + \frac{37x^5t^{13}}{7076160} + \frac{x^6t^{16}}{1814400} + \ldots$$

Here we have noise terms which are

$$\frac{x^2t^4}{12}, \frac{t^6}{180}, -\frac{x^3t^7}{252}, \frac{x^4t^{10}}{12960}, \frac{x^3t^7}{252}, \frac{11xt^9}{75600}, \frac{x^2t^{12}}{71280}, \frac{37x^5t^{13}}{7076160}, \frac{x^6t^{16}}{1814400}\ and \ldots so\ on$$

So, the remaining noncanceled term which it satisfies the equation, that is the exact solution is

$$u(x,t) = xt$$

3. Conclusions

In this paper, new approach suggested for solving important type of non-linear non-homogenous PDFs, based on combine Laplace transformation with Adomian decomposition method to get exact solution. The proposed approach shows that it is in good agreement with the other methods solution and it is better than ADM. The experimental results show that the LT-ADM is computationally efficient for solving non-linear evolution problems and can easily be implemented without using computer.

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