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Some types of fan sets in supra topology

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Abstract

This paper work new and unprecedented definitions of sets, which we have named supra fan, supra. delta fan, supra. semi delta fan sets, which are generated by three sets of specific type of supra open sets, it was utilized supra open, supra delta open, supra. semi delta open sets with special conditions. It is highlighted many details of these new types of fan sets, their axis, blades and their annular sets using tables. Attention is given to the interior and the closure of these three types in supra topological spaces. The research was further enriched numerous and diverse examples. Subsequently, the focus shifted to supra. semi delta fan sets to prove lemma and theorem.

Subject Classification: *11B05, 11F23.*

Keywords: *Supra topology, Supra open, Supra delta open, Supra semi delta open.*

1. Introduction

In 1963, Levine [6] and several other scientists introduced generalization to the principles of topology such as semi. open, δ.open and pre-open … etc. and related notions such as interior, closure and other concepts adopted in topology, for more information see [2, 3]. Let (C, τ_C) be a topological space then the interior and closure operators on a subset B of the universal set C are usually represented by $\mathrm{int}(B)$ and $\mathrm{cl}(B)$ [4]. In 1968 Velicko [11] introduced the notions of regular open and then defined delta-open for short (δ.open) and δ.closed sets as follows: A subset B is termed δ.open if

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for every $c \in B$ there is a regular open set O such that $c \in O \subseteq B$, the complement of δ.open is referred to δ.closed, for more information you can see [1, 5, 10]. Recall that the sub collection τ_C^S of the power set of C is designated as supra topology on C if $\phi, C \in \tau_C^S$ and it is closed under arbitrary union see [7]. We symbolized the collection of supra δsemi.open (resp. supra δsemi.closed) sets by $\mathrm{S.\delta s.O}(\mathrm{C}, \tau_{\mathrm{C}}^{\mathrm{S}})$ (resp. $\mathrm{S.\delta s.C}(C, \tau_C^S)$) and the family of supra regular open (resp. supra δopen, supra semi-δopen) is symbolized by $\mathrm{S.regO}(\mathrm{C}, \tau_{\mathrm{C}}^{\mathrm{S}})$ (resp. $\mathrm{S.\delta O}(\mathrm{C}, \tau_{\mathrm{C}}^{\mathrm{S}})$, $\mathrm{S.\delta sO}(\mathrm{C}, \tau_{\mathrm{C}}^{\mathrm{S}})$) [8].

In this paper we created a new type of sets inspired by supra sets which we named supra fan and this type was generalized to supra delta fan and supra semi delta fan. The diversity of these sets gives rise to equally importance types for fan sets which we have named blade, supra delta blade and supra semi delta blade sets. Through this of blade-types, we have addressed new types in a new scientific research approach in the field of topology, which we have called supra annular, supra delta annular and supra delta semi annular sets. we were also able to create what was called axis with a new technology that serves our future work, but after generalizing it to supra-axis, supra delta-axis, and supra delta semi-axis. These components can be envisioned as a system of hidden fans, with all their elements forming patterns that vary according to the space from which they are created.

2. Supra Fan sets

In this section, we will introduce a new perception of sets in supra topology.

Definition 2.1: Let (C, τ_C^S) be a supra topological space, F is called supra fan set (for short S.fan set) if there are there supra open sets $O_i \in \tau_C^S, i = 1,2,3$ with , and $O_i \cap O_j = O_{ij} \neq \phi, \forall i, j = 1,2,3$, and $O_{12} \cap O_{13} = \phi$, $O_{12} \cap O_{23} = \phi$, $O_{13} \cap O_{23} = \phi$ such that $F = O_{12} \cup O_{13} \cup O_{23}$. If there is a term $c \in C$ such that $c \notin F$ with $c \in \mathrm{S.cl}(O_{12}) \cup \mathrm{S.cl}(O_{23}) \cup \mathrm{S.cl}(O_{13})$, then c is called the supra axis or S.axis for short of the S.fan set F and O_{ij} is called S.blade set. The complement of S.fan set is called S.air set.

Definition 2.2: Let F be a S.fan set generated by the sets $O_1, O_2, O_3 \in (C, \tau_C^S)$ with S.axis c of F. Then any set $B = \{x, y, z\}$ in C such that $x \in O_{12}$, $y \in O_{23}$, $z \in O_{13}$ is called supra annular (briefly S.annular) set of the S.fan set F, as shown in Figure 1.

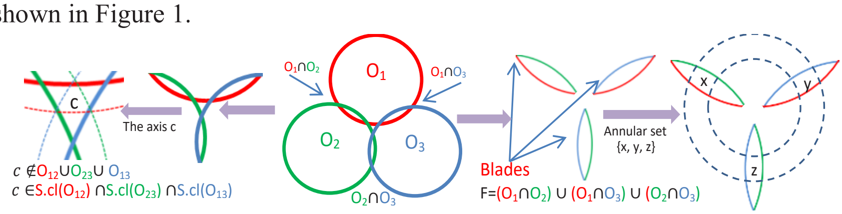

Figure 1
Fan set and its axis with blades and annular sets

Example 2.3: Let $C = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ with supra topology $\tau_C^S = \{\phi, C,$ $\{v_1, v_2\}$,$\{v_1, v_4\}$, $\{v_4, v_2\}$, $\{v_1, v_4, v_2\}$, $\{v_5, v_4, v_6\}$, $\{v_1, v_3, v_4, v_6\}$, $\{v_3, v_4, v_2, v_6\}$, $\{v_2, v_1, v_3, v_4, v_6\}$, $\{v_3, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_5, v_4, v_2\}$, $\{v_1, v_3, v_4, v_2\}$, $\{v_3, v_2, v_5\}$, $\{v_3, v_4, v_5\}$, $\{v_1, v_3, v_2, v_5\}$, $\{v_1, v_3, v_4, v_5\}$, $\{v_3, v_4, v_2, v_5\}$, $\{v_5, v_2, v_1, v_3, v_4\}$, $\{v_1, v_3, v_5\}$, $\{v_3, v_4, v_6, v_5\}$, $\{v_5, v_1, v_3, v_4, v_6\}$, $\{v_2, v_3, v_4, v_5, v_6\}\}$. The Family of S.fan sets is $\{C$, $\{v_1, v_4, v_2\}$, $\{v_6, v_1, v_4, v_2, v_3\}$, $\{v_1, v_3, v_2\}$, $\{v_1, v_3, v_4, v_2\}$, $\{v_6, v_1, v_4, v_2, v_3\}$, $\{v_1, v_3, v_5, v_2\}$, $\{v_3, v_4, v_2\}$, $\{v_1, v_4, v_2, v_3, v_5\}$, $\{v_1, v_3, v_4\}$, $\{v_1, v_3, v_5, v_4\}$, $\{v_3, v_5, v_2, v_4\}\}$. We note that the S.fan set C generated by $\{v_1, v_2\}$, $\{v_6, v_1, v_4, v_5, v_3\}$, $\{v_6, v_4, v_2, v_5, v_3\}$ which is component three S.blads are $\{v_1\}$, $\{v_6, v_5, v_3, v_4\}$, $\{v_2\}$ with S.axis equal to ϕ . The S.fan set $\{v_1, v_4, v_2\}$ generated by $\{v_1, v_2\}$, $\{v_1, v_4\}$, $\{v_4, v_2\}$ which is component three S.blads are $\{v_1\}$, $\{v_4\}$, $\{v_2\}$ with free S.axis.

Remark 2.4: Any (S. fan, S.δfan, S.sδfan) set contain from three elements then that annular of its self. See example (2.3) $\{v_1, v_4, v_2\}$ generated by several ways, the only S.annular is $\{v_1, v_4, v_2\}$. Also referring to the example(2.3), we note that $\{v_6, v_1, v_2\}$ is a annular set to $\{v_6, v_1, v_4, v_2, v_3\}$, while it does not form a fan set.

Regardless of the details of the calculations, the following example shows that S.axis is not an empty set.

Example 2.5: Let $C = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8\}$ with supra topology $\tau_C^S = \{\phi, C, \{o_1, o_2\}$, $\{o_6, o_1, o_3\}$, $\{o_5, o_2, o_3\}$, $\{o_6, o_1, o_2, o_3\}$, $\{o_5, o_1, o_2, o_3\}$, $\{o_2, o_5, o_6, o_1, o_3\}\}$. See that the S.fan set $\{o_1, o_2, o_3\}$ which is generated by $\{o_1, o_2\}$, $\{o_6, o_1, o_3\}$, $\{o_5, o_2, o_3\}$has S.axis of three elements are o_2, o_7, o_8.

3. S. delta fan and S. semi delta fan sets

In this section we will present the previous concepts in supra structure under influence of δ-open sets.

Definition 3.1: Let (C, τ_C^S) be a supra topological space, F is called S.δfan set (resp. S.sδfan) if there are three $S.\delta$open (resp. $S.s\delta$open) sets $O_i \in \tau_C^S, i = 1,2,3$ with $O_i \cap O_j = O_{ij} \neq \phi, \forall i, j = 1,2,3$, and $O_{12} \cap O_{13} = \phi$, $O_{12} \cap O_{23} = \phi$, $O_{13} \cap O_{23} = \phi$ such that $F = O_{12} \cup O_{13} \cup O_{23}$. If there is a term $c \in C$ such that $c \notin F$ with $c \in S.cl_\delta(O_{12}) \cup S.cl_\delta(O_{23}) \cup S.cl_\delta(O_{13})$ (resp. $c \in$ $\mathrm{S}._\mathrm{s}\,\mathrm{cl}_\delta(O_{12}) \cup c \in \mathrm{S}._\mathrm{s}\,\mathrm{cl}_\delta(O_{23}) \cup c \in \mathrm{S}._\mathrm{s}\,\mathrm{cl}_\delta(O_{13})$), then c is called S. δaxis (resp. S.$s\delta$axis) of the S. δfan (resp. S.$s\delta$fan) set F and O_{ij} is called S. δblade (resp. S. $s\delta$blade) set. The complement of S. δfan (resp. S. $s\delta$fan) set is called S. δair (resp. S. $s\delta$air) set.

Definition 3.2: Let F be a S. δfan (resp. S.$s\delta$fan) set generated by the sets O_1, O_2, O_3 in a supra space (C, τ_C^S) with S. δaxis (resp. S.$s\delta$axis) c of F. Then any set $B = \{x, y, z\}$ in C such that $x \in O_{12}$, $y \in O_{23}$, $z \in O_{13}$ is called S. δannular (resp. S.sδannular) set of the S. δfan (resp. S.sδfan) set F.

Remark 3.3: It is known that any S. δ-open is S.open, therefore any three S. δopen sets generate S. δfan set actually generate S.fan set.

Example 3.4: Let $C = \{v_1, v_2, v_3, v_4\}$ and let $\tau_C^S = \{\phi, C, \{v_1\}, \{v_2\}, \{v_2, v_1\}, \{v_3, v_1\}, \{v_3, v_2\}, \{v_3, v_2, v_1\}\}$, then the family of S.δO$(C, \tau_C^S) = \{\phi, C, \{v_3, v_2, v_1\}, \{v_3, v_2\}, \{v_3, v_1\}, \{v_2, v_1\}, \{v_2\}, \{v_1\}\}$. There is only one S. δfan set is $\{v_3, v_2, v_1\}$ generated by $\{v_2, v_1\}$, $\{v_3, v_1\}$ and $\{v_3, v_2\}$, so the S. δblades sets are $\{v_1\}, \{v_2\}, \{v_3\}$ and the S. δannular is $\{v_3, v_2, v_1\}$, the S.δaxis of the S. δfan set is $\{v_4\}$. It is clear that $\{v_3, v_2, v_1\}$ is also S.fan set.

To generalize the previous concepts for the purpose of benefiting from them, we will use supra semi delta open sets.

Example 3.5: Let $C = \{o_1, o_2, o_3, o_4\}$ with supra topology $\tau_C^S = \{\phi, C, \{o_1, o_2\}, \{o_3, o_1\}, \{o_3, o_2\}, \{o_3, o_1, o_2\}\}$, the collection of S.$\delta O(C, \tau_C^S)$ = S.$s\delta O(C, \tau_C^S) = \{\phi, C\}$. Thus (C, τ_C^S) has only one S.fan set $\{o_3, o_1, o_2\}$ generated by $\{o_1, o_2\}, \{o_3, o_1\}, \{o_3, o_2\}$ with three δ.blads are $\{o_1\}, \{o_2\}, \{o_3\}$ and have S.axis which is the element o_4, the only S.annular set of S.fan is $\{o_3, o_1, o_2\}$ and hasn't any S.δfan or S.$s\delta$fan set.

Example 3.6: Let $C = \{o_1, o_2, o_3, o_4\}$ with supra topology $\tau_C^S = \{\phi, C, \{o_2\}, \{o_3\}, \{o_1, o_2\}, \{o_3, o_2\}, \{o_3, o_1\}, \{o_3, o_1, o_2\}\}$, the family of S. $\delta O(C, \tau_C^S)$ is $\{\phi, C, \{o_3, o_1, o_2\}, \{o_3, o_2\}, \{o_3, o_1\}, \{o_3\}, \{o_1, o_2\}, \{o_2\}\}$ and S.$s\delta O(C, \tau_C^S)$ is $\{\phi, C, \{o_3, o_1, o_2\}, \{o_3, o_2\}, \{o_4, o_3, o_2\}, \{o_3, o_1\}, \{o_4, o_3, o_1\}, \{o_3\}, \{o_4, o_3\}, \{o_1, o_2\}, \{o_4, o_1, o_2\}, \{o_2\}, \{o_4, o_2\}\}$. Note that $\{o_3, o_1, o_2\}$ is the only S.fan set generated from $\{\{o_1, o_2\}, \{o_3, o_2\}, \{o_3, o_1\}\}$ and $\{o_3, o_1, o_2\}$ is also S.δfan set, while there are many sets of type S. $s\delta$fan set such as C, $\{o_4, o_3, o_2\}$, $\{o_4, o_3, o_1\}$, $\{o_4, o_1, o_2\}$ and others.

4. Properties of fan sets via supra. δ and supra semi δ sets

Definition 4.1: In a supra space (C, τ_C^S), the supra interior fan (resp. supra interior δfan, supra semi interior δfan) set of $K \subseteq C$ is symbolized by S.int_fan (K) (resp. $\mathrm{S.int}_{\delta\mathrm{fan}}(K)$, $\mathrm{S._s\,int}_{\delta\mathrm{fan}}(K)$) is defined by $\mathrm{S.int}_{\mathrm{fan}}(K) = \cup\{F \subseteq C : F \text{ is S.fan set and } F \subseteq K\}$(resp.$\mathrm{S.int}_{\delta\mathrm{fan}}(K) = \cup\{F \subseteq C : F \text{ is S.}\delta\text{fan set and } F \subseteq K\}$, $\mathrm{S._s\,int}_{\delta\mathrm{fan}}(K) = \cup\{F \subseteq C : F \text{ is S.s}\delta\text{fan set and } F \subseteq K\}$).

Definition 4.2: In a supra space (C, τ_C^S), the supra closure fan (resp. supra closure δfan, supra semi closure δfan) set of $K \subseteq C$ is symbolized $S.cl_{fan}(K)$ (resp. $S.cl_{\delta fan}(K)$, $S._s\,cl_{\delta fan}(K)$) and defined by $\mathrm{S.cl}_{\mathrm{fan}}(K) = \cap\{A \subseteq C : A \text{ is S.air and S.}_{\mathrm{dorm}} \text{ air set with } K \subseteq A\}$ (resp. $\mathrm{S.cl}_{\delta\mathrm{fan}}(K) = \cap\{A \subseteq C : A \text{ is S.}\delta\text{air and S.}_{\mathrm{dorm}}\, \delta\text{air set with } K \subseteq A\}$, $\mathrm{S.cl}_{\mathrm{s}\delta\mathrm{fan}}(K) = \cap\{A \subseteq C : A \text{ is S.s}\delta\text{air and S.}_{\mathrm{dorm}}\, \mathrm{s}\delta\text{air set with } K \subseteq A\}$).

Example 4.3: In example (2.3), the family of S.$s\delta O(C, \tau_C^S)$ is $\{\phi, C, \{v_3, v_1, v_5, v_6, v_4\}, \{v_6, v_2, v_5, v_4\}, \{v_6, v_2, v_5\}, \{v_2, v_1, v_3\}, \{v_3, v_1\}, \{v_2\}\}$.

$\mathrm{S.int_{fan}}(\{v_1\})$=$\mathrm{S.int_{\delta fan}}(\{v_1\}) = \mathrm{S.int_{\delta fan}}(\{v_1\}) = \phi$, $\mathrm{S.cl_{fan}}(\{v_1\}) = \mathrm{S.cl_{\delta fan}}(\{v_1\}) = \{v_6, v_1\}$ and $\mathrm{S.cl_{s\delta fan}}(\{v_1\}) = \{v_1\}$.

Remark 4.4: It is not necessary that $\mathrm{S.cl_{fan}}(\phi)$, $\mathrm{S.cl_{\delta fan}}(\phi)$ is equal to ϕ, also $\mathrm{S.int_{fan}}(C)$, $\mathrm{S.int_{\delta fan}}(C)$ may not equal to C as the following examples shows.

Example 4.5: In example (3.6) , the family of S.reg$O(C,\tau_C^S)$ is $\{\phi, C, \{o_2\}, \{o_2, o_1\}, \{o_3\}, \{o_3, o_1\}\}$. $\mathrm{S.cl_{fan}}(\phi) = \mathrm{S.cl_{\delta fan}}(\phi) = \{o_4\}$, $\mathrm{S.int_{fan}}(C) = \mathrm{S.int_{\delta fan}}(C) = \{o_1, o_3, o_2\}$.

Lemma 4.6: *Let B be a subset of a supra space (C,τ_C^S), then $\left(S.cl_{s\delta fan}(B)\right)^c = S.int_{s\delta fan}(B^c)$.*

Proof: Let $c \in (\mathrm{S.cl_{s\delta fan}}(B))^c$, so $c \notin \mathrm{S.cl_{s\delta fan}}(B)$ that's mean there is a S.sδair set A such that $B \subseteq A$ with $c \notin A$ (4.2). Thus A^c is S.sδfan set which containing c such that $A^c \subseteq B^c$ implies $c \in \mathrm{S.int_{s\delta fan}}(B^c)$ (4.1).

Proposition 4.7: Let B be a subset of a supra space (C,τ_C^S), then $c \in \mathrm{S.cl_{s\delta fan}}(B)$ if and only if for every S.sδfan set F containing c, $F \cap B \neq \phi$.

Proof: Let $\mathrm{c} \in \mathrm{S.cl_{s\delta fan}}(\mathrm{B})$ and assume there is a S.sδfan set F containing c with $\mathrm{F} \cap \mathrm{B} = \phi$, hence $\mathrm{F^c}$ is S.sδair set that containing B. Now $\mathrm{S.cl_{s\delta fan}}(\mathrm{B})$ is the intersection of S.sδair sets which containing B (4.2), hence $\mathrm{c} \in \mathrm{F^c}$ which is a contradiction. Conversely: Suppose $\mathrm{c} \notin \mathrm{S.cl_{s\delta fan}}(\mathrm{B})$, so $\mathrm{c} \in (\mathrm{S.cl_{s\delta fan}}(\mathrm{B}))^{\mathrm{c}}$ implies $\mathrm{c} \in \mathrm{S.int_{s\delta fan}}(\mathrm{B^c})$ (4.6). Therefore $\mathrm{c} \in \bigcup\{\mathrm{F} \subseteq \mathrm{C}: \mathrm{F}$ is S.sδfan set and $\mathrm{F} \subseteq \mathrm{B^c}\}$ that's mean there is a S.sδfan set F containing c with $\mathrm{F} \subseteq \mathrm{B^c}$, thus $\mathrm{F} \cap \mathrm{B} = \phi$ which is a contradiction.

Theorem 4.8: *Let X, Y be two subsets of a supra space (C,τ_C^S), then the following properties hold:*

1. $(\mathrm{S.int_{s\delta fan}}(\mathrm{X}))^{\mathrm{c}} = \mathrm{S.cl_{s\delta fan}}(\mathrm{X^c})$;
2. if $\mathrm{X} \in \mathrm{S.s\delta air}(\mathrm{C},\tau_\mathrm{C}^\mathrm{S})$, then $\mathrm{S.cl_{s\delta fan}}(\mathrm{X}) = \mathrm{X}$ and if $\mathrm{X} \in \mathrm{S.s\delta fan}(\mathrm{C},\tau_\mathrm{C}^\mathrm{S})$, then $\mathrm{S.int_{s\delta fan}}(\mathrm{X}) = \mathrm{X}$;
3. if $\mathrm{X} \subseteq \mathrm{Y}$, then $\mathrm{S.int_{s\delta fan}}(\mathrm{X}) \subseteq \mathrm{S.int_{s\delta fan}}(\mathrm{Y})$ and $\mathrm{S.cl_{s\delta fan}}(\mathrm{X}) \subseteq \mathrm{S.cl_{s\delta fan}}(\mathrm{Y})$;
4. $\mathrm{X} \subseteq \mathrm{S.cl_{s\delta fan}}(\mathrm{X})$ and $\mathrm{S.int_{s\delta fan}}(\mathrm{X}) \subseteq \mathrm{X}$;
5. $\mathrm{S.cl_{s\delta fan}}\left(\mathrm{S.cl_{s\delta fan}}(\mathrm{X})\right) = \mathrm{S.cl_{s\delta fan}}(\mathrm{X})$ and $\mathrm{S.int_{s\delta fan}}\left(\mathrm{S.int_{s\delta fan}}(\mathrm{X})\right) = \mathrm{S.int_{s\delta fan}}(\mathrm{X})$;
6. $\mathrm{S.cl_{s\delta fan}}(\mathrm{X}) \cup \mathrm{S.cl_{s\delta fan}}(\mathrm{Y}) \subseteq \mathrm{S.cl_{s\delta fan}}(\mathrm{X} \cup \mathrm{Y})$ and $\mathrm{S.cl_{s\delta fan}}(\mathrm{X} \cap \mathrm{Y}) \subseteq \mathrm{S.cl_{s\delta fan}}(\mathrm{X}) \cap \mathrm{S.cl_{s\delta fan}}(\mathrm{Y})$;
7. $\mathrm{S.int_{s\delta fan}}(\mathrm{X} \cap \mathrm{Y}) \subseteq \mathrm{S.int_{s\delta fan}}(\mathrm{X}) \cap \mathrm{S.int_{s\delta fan}}(\mathrm{Y})$ and $\mathrm{S.int_{s\delta fan}}(\mathrm{X}) \cup \mathrm{S.int_{s\delta fan}}(\mathrm{Y}) \subseteq \mathrm{S.int_{s\delta fan}}(\mathrm{X} \cup \mathrm{Y})$.

Proof:

1. Follows from lemma (4.6)
2. Follows from definitions (4.1 and 4.2)
3. Let $c \in S.\mathrm{int}_{s\delta fan}(X)$, then there is a S.sδfan set F contain c such that $c \in F \subseteq X$, since $X \subseteq Y$ then $c \in F \subseteq Y$, hence $c \in S.\mathrm{int}_{s\delta fan}(Y)$. Now let $c \in S.\mathrm{cl}_{s\delta fan}(X)$, then for all S.sδfan set F contain c, $F \cap X \neq \phi$ (4.10) which leads to $F \cap Y \neq \phi$ for all S.sδfan set F contain c implies $c \in S.\mathrm{cl}_{s\delta fan}(Y)$.
4. Directly followed by (4.1, 4.2)
5. Obvious due to the nature of the definitions.
6. Consider $c \in S.\mathrm{cl}_{s\delta fan}(X) \cup S.\mathrm{cl}_{s\delta fan}(Y)$, then $c \in S.\mathrm{cl}_{s\delta fan}(X)$ or $c \in S.\mathrm{cl}_{s\delta fan}(Y)$. If $c \in S.\mathrm{cl}_{s\delta fan}(X)$, hence by (3) $c \in S.\mathrm{cl}_{s\delta fan}(X \cup Y)$. If $c \in S.\mathrm{cl}_{s\delta fan}(Y)$ then in the same way it will be $c \in S.\mathrm{cl}_{s\delta fan}(X \cup Y)$. Now let $c \in S.\mathrm{cl}_{s\delta fan}(X \cap Y)$, hence $F \cap (X \cap Y) \neq \phi$ for each S.sδfan set F contain c (4.7) and hence $F \cap X \neq \phi$ and $F \cap Y \neq \phi$ [9]. Thus by (4) we have $c \in S.\mathrm{cl}_{s\delta fan}(X)$ and $c \in S.\mathrm{cl}_{s\delta fan}(Y)$ implies $c \in S.\mathrm{cl}_{s\delta fan}(X) \cap S.\mathrm{cl}_{s\delta fan}(Y)$.
7. Follows from (1) and (6).

5. Conclusion

In this research, an unconventional type of sets was presented, that essentially depends on the union of sets, which justifies our reliance on the supra topology. Due to the large number of calculations required to obtain such sets, the computer was used. The introduced set is a new set that has not been previously discussed. We called it the set of fans. From it, related sets were derived, including axes, blades and annular sets. This study serves as a foundation for our study of new topological applications about amino acid chains and their use in therapeutic and diagnostic issues, such as cancer and diabetes.

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