Problems

1- Proved that $R = \frac{A}{\rho B} = \mathbf{e}^{\frac{hc}{kT}} - 1$ (where R is the ratio between the spontaneous emission and the stimulated emission) and then calculate :

1. The value of R for a tungsten lamp operates at 2000 k and has an emission frequency of 5×10^{14} Hz
2. The value of R for yellow light (590 nm) room temperature is 300 Kelvin
3. At any temperature the stimulated emission rate is equal to the spontaneous emission rate for yellow light (590 nm)
4. What is the wavelength when R = 1 at room temperature

2- Calculate the ratio between the two levels of energy at room temperature 300 K according to Boltzmann statistic, if the wavelength of the wave falling on this system is 590 nm and then find the energy needed to make this ratio equal to half?

3- Compare between collision and Doppler broadening to the Neon gas spectrometer at room temperature of 300 k for the line laser (632 nm). Note that the molecular mass M 18.6 and collision time is 0.5×10^{-6} Sec ?

4- Calculate the amount of λ_m at 6000 k according to Wien's Displacement Law, and what is the color corresponding to this wavelength?

5- If the intensity of light passing through the laser medium has length 0.5 m .

1 - Calculate the coefficient of gain (without loss) ?

2 - Calculate the coefficient of gain , If the intensity increase by 5% for the same path?

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Planck equation $\quad \rho_v = \frac{8\pi h v^3/c^3}{e^{hv/kT}-1}$

$$\frac{A}{B} = \frac{8\pi h v^3}{c^3}$$

$$R = \frac{A}{\rho B} = \frac{A}{B} \times \frac{1}{\rho} = \frac{8\pi h v^3}{c^3} \times \frac{1}{\frac{8\pi h v^3/c^3}{e^{hv/kT}-1}} = e^{hv/kT} - 1$$

1. $\mathrm{R} = e^{hv/kT} - 1 = \left(e^{6.625\times10^{-34}\times5\times10^{14}/1.38\times10^{-23}\times2000}\right) - 1 = 1.63 \times 10^5$

This result shows the dominance of Spontaneous emission

2. $\mathrm{R} = e^{hc/kT\lambda} - 1 = \left(e^{6.625\times10^{-34}\times3\times10^{8}/1.38\times10^{-23}\times300\times5900\times10^{-10}}\right) - 1 = 2.25 \times 10^5$

3. $\mathrm{R} = 1 = e^{hc/kT\lambda} - 1 \rightarrow 2 = e^{\frac{hc}{kT\lambda}} \rightarrow \ln 2 = \frac{hc}{kT\lambda} \rightarrow T = \frac{hc}{k\lambda \ln 2} =$
$6.625 \times 10^{-34} \times 3 \times 10^8 / 1.38 \times 10^{-23} \times 5900 \times 10^{-10} \times 0.693 = 3.5 \times 10^4 K$

4. $\mathrm{R} = 1 = e^{hc/kT\lambda} - 1 \rightarrow 2 = e^{\frac{hc}{kT\lambda}} \rightarrow \ln 2 = \frac{hc}{kT\lambda} \rightarrow \lambda = \frac{hc}{k\,T\,\ln 2} =$
$6.625 \times 10^{-34} \times 3 \times 10^8 / 1.38 \times 10^{-23} \times 300 \times 0.693 = 6.9 \times 10^{-5}\, m$

2- Calculate the ratio between the two levels of energy at room temperature 300 K according to Boltzmann statistic, if the wavelength of the wave falling on this system is 590 nm and then find the energy needed to make this ratio equal to half?

$$\frac{N_2}{N_1} = e^{-\frac{h\nu}{kT}} = e^{-\frac{hc}{kT\lambda}} = \left(e^{-6.625\times10^{-34}\times3\times10^{8}/1.38\times10^{-23}\times300\times590\times10^{-9}}\right) = 81.401$$

$$When \quad \frac{N_2}{N_1} = 1/2$$

$$\frac{1}{2} = e^{-\frac{h\nu}{1.38\times10^{-23}\times300}} = e^{-\frac{h\nu}{4.14\times10^{-21}}}$$

$$\ln 1 - \ln 2 = 0 - 0.693 = ^{-}\frac{h\nu}{4.14\times10^{-21}}$$

$$h\nu = 2.86\times10^{-21}\ J$$

3- Compare between collision and Doppler broadening to the Neon gas spectrometer at room temperature of 300 k for the line laser (632 nm). Note that the molecular mass M 18.6 and collision time is **0.5×10−6** Sec ?

$$\Delta\nu_0)_c = \frac{1}{\pi\tau_c} = \frac{1}{3.14\times0.5\times10^{-6}} = 0.64\ MHz$$

$$\Delta\nu_0)_D = 7.16\times10^{-7}\nu_0\sqrt{\frac{T}{M}} = 7.16\times10^{-7}\frac{c}{\lambda}\sqrt{\frac{T}{M}} = 7.16\times10^{-7}\frac{3\times10^{8}}{632\times10^{-9}}\sqrt{\frac{300}{18.6}} = 1.365\ GHz$$

$$\frac{\Delta\nu_0)_D}{\Delta\nu_0)_c} = \frac{1.365\times10^{9}}{0.64\times10^{6}} = 2.133\times10^{3}$$

$$\therefore \quad \Delta\nu_0)_D \gg \Delta\nu_0)_c$$

The Doppler broadening is dominant

4- Calculate the amount of $\boldsymbol{\lambda_m}$ at 6000 k according to Wien's Displacement Law, and what is the color corresponding to this wavelength?

$$\lambda_m T = 2.9 \times 10^{-3}$$

$$\lambda_m(6000) = 2.9 \times 10^{-3} \quad \therefore \lambda_m = 4833 \times 10^{-10} m = 4833\ \dot{A}$$

The color corresponding to the wavelength is (greenish blue)

Violet	Indigo	Blue	Green	Yellow	Orange	Red
4200	4400	4700	5300	5800	6200	7000

5- If the intensity of light passing through the laser medium Doubled has length 0.5 m
1 - Calculate the coefficient of gain (without loss) ?
2 - Calculate the coefficient of gain , If the intensity increase by 5% for the same path?

$$\frac{I_x}{I_0} = e^{-\alpha x} \rightarrow \frac{2I_0}{I_0} = e^{-\alpha(0.5)} \rightarrow ln\, 2 = -0.5\alpha \rightarrow \alpha = -1.39\, m^{-1} \qquad -$$

$$G = -\alpha = 1.39\, m^{-1}$$

$$\frac{I_x}{I_0} = e^{-\alpha x} \rightarrow \frac{1.05\, I_0}{I_0} = e^{-\alpha(0.5)} \rightarrow ln\, 1.05 = -0.5\alpha \rightarrow \alpha = -0.098\, m^{-1} \qquad -\ 2$$

$$G = -\alpha = 0.098\, m^{-1}$$