

# Estimation Parameter for Non-Linear Regression by Using HGSABAT Algorithm

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**Abstract.** This study offers a new Mixed Meta Heuristics algorithm (HGSABAT) for estimating the parameter values of each of the six categories of Non-Linear regression models examined (*Misra1d, Meyer4, Meyer7, Militky4, Militky2, and MGH09*) by combining the Gravitational Search Algorithm and Bat Algorithm. Some models have different numbers of parameters. For example, the *Misra1d* and *Militky2* models of the Non-Linear Regression model have two parameters ( $\beta_1, \beta_2$ ). In contrast, the *MGH09* and *Militky4* models have four parameters (*MGH1, MGH2, MGH3, and MGH4*), in which location as the *Meyer4* and *Meyer7* models have three attributes (*Meyer1, MGH2, and MGH3*).

To examine the effectiveness of the suggested Hybrid Meta Heuristics algorithm (HGSABAT), a simulation study based on Mean Square Error criteria is employed. Furthermore, using three metaheuristic algorithms, the simulation results are compared using Practical Swarm Enhancement, Gravitational Search, and BAT.

The numerical results will be obtained using the *Matlab* program version 2020. Because the hybrid algorithm (HGSABAT) has a lower mean error than one of the other techniques, the mean error in the squares estimation techniques that were taken into consideration, the results demonstrated that it is satisfactory for parameter estimation and produces accurate estimates.

Keywords: Gravitational Search Algorithm, BAT Algorithm, Non-Linear Regression and Meta-heuristic.

## INTRODUCTION

Formally speaking, N-Linear Regression models are those in which the model expression contains at least one nonlinear model parameter. In many scientific and engineering fields, nonlinear models are crucial for simulating intricate connections among attributes. Nonlinear models are frequently utilised to explain economic, commercial, biological, and physical operations. Development, yield quantity, and dose-response formulae are a few of these mathematical equations [1].

Many issues remain unresolved in the nonlinear case, despite the mathematical framework of the linear system measuring parameters being nearly fully developed. Similar to Regression using linear equations, N-Linear Regression's fundamental concept is to relate a result  $y$  to a collection of predictor attributes  $x = (x_1, \dots, x_k)$ .

The result of the equation is an exponential response on one or more unknown parameters, which is the defining characteristic of N-Linear Regression. In contrast, linear Regression is frequently employed to construct a model. Whenever there's tangible evidence that the connection between the answer and the factors that predict it follows a specific operational structure, non-linear Regression, a completely observational model, is usually used. A Non-Linear Regression model is defined as  $y = f(x_i, \beta_i) + e_i$ , in which location  $e_i \sim N(0, \sigma_2)$ ,  $i = 1, 2, \dots, n$  [2]. Having a mean of nothing, the  $e_i$  are usually believed to be indistinguishable when the average of one.

In the estimation of the parameters issue, the structure of the N-Linear Reconstruction functional has been determined; however, the values of  $\beta_1, \beta_2, \dots, \beta_n$  Are unidentified. A prevalent technique for estimating undefined attributes in a non-linear regression function is the ordinary least squares method [3] and [4]. Utilising this method, the estimates of  $\beta_1, \beta_2, \dots, \beta_n$  are derived by minimizing the amount  $s(\beta) = \sum e_i^2 = \sum (y_i - f(x_i, \beta_i))^2$  from  $i=1$  to  $n$ , representing the prediction's total of the quadratic error rate. In a complicated estimation of the parameter issue, the objective function  $S(\beta)$  is minimised by iteration.

The estimation of N-linear regression models' parameters is the subject of many articles. Nevertheless, the nonlinearity model complicates the parameter estimation process and the statistical analysis of parameter estimates. The shortcomings of these (conventional) approaches to nonlinear parameter estimation are also discussed. Additionally, it requires a lot of additional data to function correctly and is difficult for practitioners to control. The target function's multidimensional nature and large number of parameters are the causes of the above problems [5].

Consequently, strong meta-heuristic algorithms, which have numerous benefits such as ease of implementation, reliability, robustness, and effectiveness, are necessary to overcome these challenges [6]. To solve N-Linear Regression models, four alternative meta-heuristic algorithms, Gravitational Search, Practical Swarm Enhancement, Bat, and Practical Swarm Enhancement with Gravitational Search Algorithm (GSABAT) were used in this work.

2020 saw Zhang and colleagues. Zhang et al. enhanced the performance of the competitive swarm optimiser (CSO) by including mutant representatives (MA), especially CSO-MA, thus obtaining c-optimal designs for predicting particular values in extremely complex N-Linear Regression theories. Eight benchmark functions with various mathematical properties were used in the simulation study to compare CSO-MA's performance with Cuckoo search [7].

Zaid and Bayda presented a gravitational search algorithm in 2021 to estimate the parameters of N-linear regression models. Additionally, the effectiveness of the proposed approach is compared with the most probable approach in a simulated exercise [8].

Zaid and Bayda estimated the N-Linear Regressive behaviour model parameters in 2022, utilising the BAT method. Employing maximum likelihood (MLE) and least squares (LS), the effectiveness of the proposed method was investigated using a computational exercise [9].

The study is organised into four sections, each of which illustrates a distinct stage of the investigation procedure to accomplish the objectives mentioned above. Consequently, the work has the following organisation: Practical Swarm Enhancement (PSO), the Bat Algorithm, the Gravitational Search Algorithm, and hybrid Meta-heuristic Approaches are the three meta-heuristic algorithms covered in Section 2.

The mathematical findings are presented in Section 3, which compares hybrid and meta-heuristic algorithms using a simulation study and the mathematical measure of the standard error of mean (MSE).

The findings and some suggestions for further research are included in Section 4.

## **First: Regression Without Linearity**

Regression analysis is one of the most popular statistical techniques for elucidating the connection among several attributes [10], an indispensable instrument for the exact assessment and full use of acquired data in experiments. It explains the connection across two variants of observations: the one that depends on response measurement, represented by  $Y$ , and the unrelated or predictor evaluations, represented by  $x = (x_1, x_2, \dots, x_k)$ .

The aim is to develop a method of Regression allowing the researcher to efficiently define, predict, and manipulate the response attribute using the multiple uncorrelated attributes [11] and [12].

A regression strategy general structure is  $y = f(x, \beta) + \varepsilon$ , in which location  $\beta$  is a vector of parameters (s),  $y$  is the response attribute,  $x$  is a vector representing the in-response attribute(s), and  $\varepsilon$  has zero mean and variance. Regression models come in two main varieties: linear and nonlinear.

The statistical inferential techniques known as linear regression analysis are highly prevalent and commonly utilised when the regression function  $f$  is linear in terms of the parameters  $\beta$  [5]. In 1894, Sir Francis Galton introduced the idea of linear Regression [13]. Nevertheless, non-linear regression models, in which  $f$  is nonlinear in  $\beta$ , are frequently required since linear models are not always suitable [14]. The regression models can be non-linear in terms of attributes, parameters, or both.

For scientific data analysis, non-linear Regression serves as a powerful instrument, especially when changing data to align with a model that uses linear Regression is necessary. Thus, it is possible to think of linear Regression as merely a particular instance of the more general non-linear Regression [12]. Numerous fields use non-linear models, including business, engineering, economists, management sciences, biology, and the physical and social sciences. In the applied mathematical sciences, nonlinear model building is a relatively recent and exciting study area. Developing a nonlinear model is undoubtedly challenging for mathematicians or other scientific researchers.

Numerous nonlinear models that relate to various issues in practical mathematics can be clearly defined in published works and effectively utilised in various real-world scenarios. However, many situations have not yet been nonlinearly modelled due to their complexity or intractable nature in mathematics or statistics [15]. Appendix Table 1 presents various non-linear regression models.

### Second: Algorithms that are meta-heuristic.

Parameter estimation of a model is a problem that can occasionally be solved and addressed by traditional mathematical methods [16] and [17]. In real time, the previously Meta Algorithm (MA) yielded an exceptionally effective answer [18]. The challenging procedures have gained a lot of popularity because of their benefits, which include being flexible and versatile, avoiding local optima, and being easy to implement [19]. As a result, a new class of enhancement algorithms has been developed that attempt to overcome the limitations of traditional enhancement procedures by combining fundamental heuristic techniques in advanced structures to effectively and efficiently explore a search space.

A brief overview of some renowned meta-heuristic algorithms was provided in this thesis, particularly those who existed applied in the following study:

### Third: Gravitational Search Algorithm Technique (GSA)

Rashedi et al. proposed the Gravitational Search Algorithm (GSA) in 2009 [20] as a remedy for enhancement issues. The searching algorithm established by the general population is grounded in the principles of gravity and mass connections. The ideas that arise within the GSA are designated employees who engage with each other through gravitational interactions. The success rate of each substance within the collection was assessed according to its mass. The most significant answer is the ideal one [21].

The gravitational force acting within each point in space is directly related to the intersection of their masses and oppositely linked to the amount of space separating them, as outlined in the law of attraction. These particles exhibit an attractive force towards each other.

A mass's present velocity is equal to its past velocity times the percentage change in that velocity plus the change in that velocity itself, by the principles of motion. When you calculate the force exerted on a structure and divide it by the total weight of the momentum, you get the speed or motion variation of the system in question.

Here is how the masses of the objects are lowly obeying gravity:

$$F_{ij} = G \frac{M_{aj} \times M_{pj}}{R^2} \quad \dots (1)$$

This equation states that for every two objects,  $M_{aj}$  and  $M_{pj}$  The distance between them is  $R$ , the magnitude of the gravitational force is  $F$ , and the gravitational constant is  $G$ .

$$a_i = \frac{F_{ij}}{M_{ii}} \quad \dots (2)$$

While searching, representatives will change their positions ( $W_i$ ) and accelerations ( $L_i$ ), in which location  $M_{ii}$  Is the total mass of inertia of representative  $i$ . The results are shown in expression 0, appropriately.

$$L_i^d(t+1) = rand_i \times L_i^d(t) + a_i(t) \quad \dots (3)$$

$$W_i^d(t+1) = W(t) + V_i^d(t+1) \quad \dots (4)$$

This specifies where they belong, where  $d$  represents the quantity of attributes, and  $rand_i$  Denotes an equally distributed arbitrary value inside the range of  $[0,1]$ .

To summarise the GSA, the fundamental step is as follows:

Step 1. Initialising the gravitational acceleration equations  $G_0 \propto$  It is the first step in the search process.  $G_0$  and  $\propto$  are set up at the start of the process, and the principles will decrease as the search progresses.  $T$  represents the overall sum of repetitions.

Step 2. A beginning population of  $N$  people was formed at random, and each attorney's orientation was selected as follows:

$$X_i(t) = \left( X_i^1(t). X_i^2(t) \dots \dots \dots X_i^n(t) \right), \quad i=0, \dots, N, \quad \dots(5)$$

Step 3: Repeat this procedure until the cessation of operations goals are achieved.

A. Recognised the best and worst representatives; at this point, all representatives were evaluated according to the population.

B. As previously stated, the gravitational factor has undergone modification.

C. Determine the energy using the following method:

$$F_{ij}^d = G(t) \frac{M_{aj}(t) \times M_{pj}(t)}{R_{ij}(t) + \epsilon} \left( W_j^d(t) - W_i^d(t) \right) \quad \dots(6)$$

$$R_{ij}(t) = \frac{W_i(t).W_j(t)}{2} \quad \dots(7)$$

The distance measured by the Euclidean separating representatives  $i$  and  $j$  is represented by  $R_{ij}(t)$ .

D. Compute the following total force at each algorithm generation working on representative  $I$ :

$$F_i^d(t) = \sum_{j \in k_{best}, j \neq i} rand_j F_{ij}^d(t) \quad \dots(8)$$

In which locations  $rand_j$  It is a random number in the range  $[0,1]$ ,  $k_{best}$  It is a collection of the initial  $K$  representatives with the most significant level of fitness and largest mass.

E. To determine the inertial mass, you can use the formula:

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad \dots(9)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)} \quad \dots(10)$$

In this case  $fit_i(t)$   $worst(t)$  and  $best(t)$  represent the representative's transfer function at time  $t$ , respectively. It is stated as having the following implication for something like a minimisation problem:

$$best(t) = \min fit_j(t) \quad \dots(11)$$

$$worst(t) = \max fit_j(t) \quad \dots(12)$$

F. An additional technique was used to compute the acceleration of the representative:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \quad \dots(13)$$

G. The formula used shows how the location and speed of representative  $I$  are established.

H. Each iteration count is increased up to the point where expulsion situations are satisfied.

Step 4. The fastest and best answer is found in step four.

#### **Fourth: Practical Swarm Enhancement (PSO)**

The synchronised behaviour observed among groups of fish and flocks of birds inspired the development of PSO computational methods [22]. The PSO consists of a collection of entities, each embodying an alternative to the addressed problem. Particles navigate a known search space of this issue, with their positional improvements influenced by the psychological cognitive tendency to emulate the successes of others. Every individual within a population, specifically a population of particles, possesses distinct life experiences and the ability to evaluate the event's significance. Their awareness of their neighbours' behaviour reflects the inherent social nature of individuals. The two classifications of expertise reflect the cognitive part (a person learning) and the societal

component (cultural environment). Consequently, every individual decides by considering both mental and social factors, leading the general population (the swarm) to participate in a natural way of doing something.

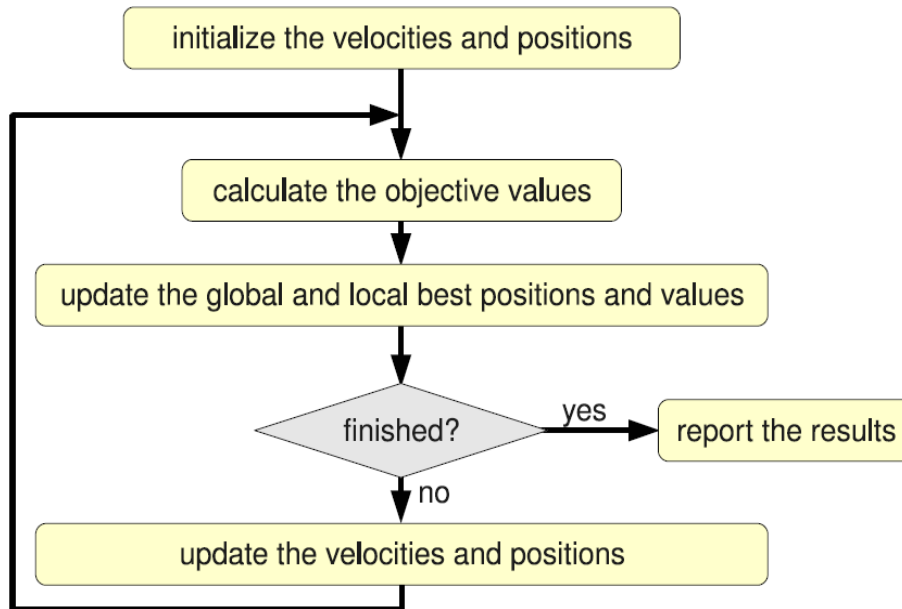


Figure 1. Flowchart of PSO

### Fifth: Bat Algorithm

The SI family encompasses the biologically inspired Bat Algorithm (BAT). The Bat, the following section was developed in [23] and has since gained widespread usage [24]. Bats utilise a sophisticated form of echolocation, relying on sonic signals known as sonar echolocation, to navigate their surroundings and detect potential threats. Yang concentrated on the three essential principles for effective bat operation:

While the volume and wavelength may fluctuate, bats consistently navigate at a constant arbitrary acceleration and regularity toward an appointed point. Bats exhibit a fascinating ability to modify their frequencies to mimic the sounds of their prey. Furthermore, all bats depend on noise signals to assess the distance of an object. In the final analysis, the author proposed modifying the audio settings from the loudest to the quietest rather than the reverse approach. To accurately represent the disparity in pulse respiration and clarity experienced by bats during hunting, BATA employs automatic zooming while it's looking for the time being [13].

### Sixth: The procedure of the BAT is in the following section:

Initiate the BAT population at  $(x, \beta)$  Establish the objective to perform at  $v_i$ , and determine the pulse frequency  $f_i$  at  $x_i$  by utilising the fitness parameter of the BAT Method,  $\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}$  Volume ( $A$ ) and heart rate ( $r_i$ ) are configured to their original settings,  $r_i \in \{0,1\}$ .

Second, modifying frequency generates a different response that modulates the positions and velocity measurements.

Third, if the chance number is greater than  $r_i$  Then choose the optimal strategy, shape it, and build an adjacent answer around it.

If this fails, allow the dice to fall and generate a novel idea.

The fifth phase is to determine whether the target function  $f(x)$  meets the requirements when (an arbitrary  $<A_i$  and  $f(x_i) < f(x_0)$ ). Adopt the updated recommendation, increase  $r_i$ , and decrease  $A_i$ .

Step 6, The optimal value ( $x_0$ ) may be ascertained.

Step 7: Whenever the repeated count attains its highest point. Analysis and subsequent outcomes of the process. Upon conclusion of the procedure, the ideal global approach is identified.

## Seventh: The Proposed Hybrid Method (HGSABAT)

The paper will start with a dictionary of MOP terminology. The structure of the suggested estimation parameter for nonlinear Regression using the gravitational searching approach is shown. The procedure for grade allocation will be addressed thereafter. Finally, we shall examine the mechanics of environmental modification and the reproduction methods.

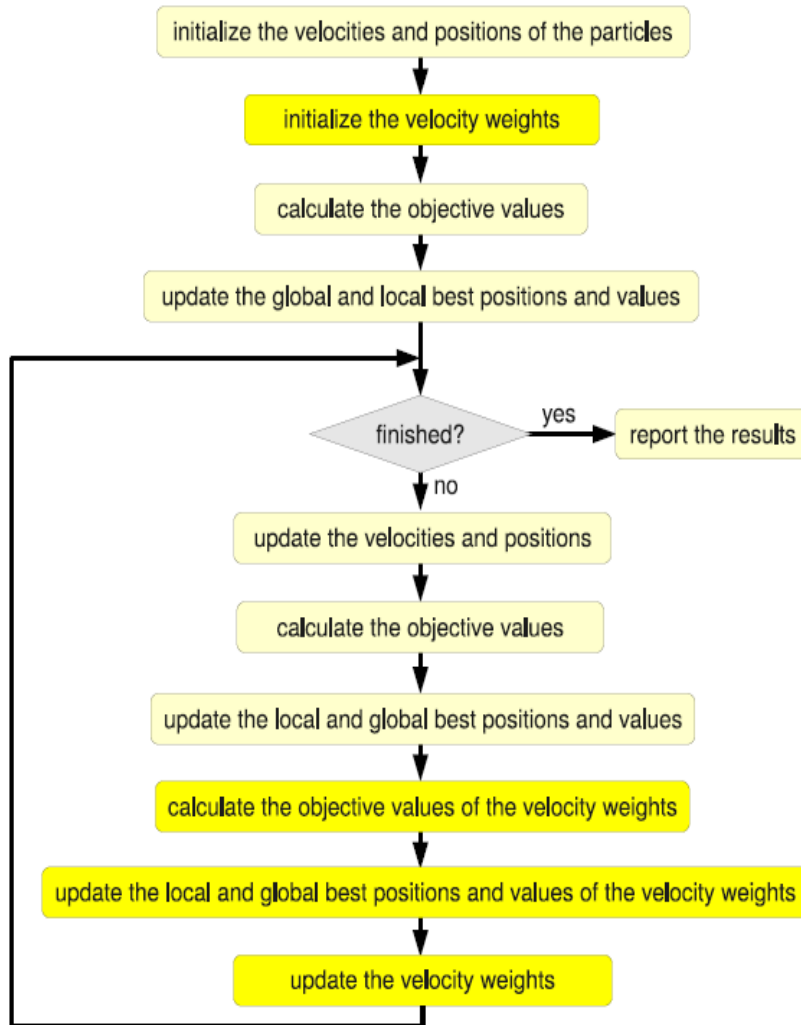


Figure 2: Flowchart of HGSABAT

Furthermore, a thorough examination of the correlations between the qualities and agreement rates has not been conducted. To develop a more effective optimum strategy for predicting Non-Linear Regression Illustrations, the investigator introduces a hybrid technique termed HGSABAT, which incorporates two more elements, depository and leader, identified in the MOPSO algorithm presented by [25]. The repository's principal role is to store and retrieve the most significant non-dominated and non-controllable Statistical solutions. The CPU (central processing unit) of the memory is also included.

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Pseudo-code of the HGSABAT

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Set  $k := 0$  and velocity = 0  $\mu=0.1, r_0 = 0.5, A = 0.6$ .

Set the starting point at an arbitrary  $P_i$  for  $n$ . population ;

Compute the physical fitness scores of the original group:  $f(P)$ ;

Identify the alternatives that are not dominated and establish the repository, including them.

WHILE ( The expulsion criteria have not been satisfied)

PSO Steps

$Q = T_{min} + (T_{min} - T_{max}) * rand$

$V_{ij}^{t+1} = V_{ij}^t + C_1 R_1 (B_{BEST_{ij}} - X_{ij}^t) + C_2 R_2 (G_{BEST_{ij}} - X_{ij}^t)$

$X_i^{t+1} = X_i^t + V_i^{t+1}$

If  $rand > r$

$P_{leader2} =$  Choose Archivist as their leader.

$X_{new} = X_{(t)} + rand * (X_{leader2} - X_{(t)})$

End

*if  $X_{new}$  dominated on  $X_{(t)}$  and ( $rand < A$ )*

$X_{(t)} = X_{new}$

End

If  $rand < (\frac{1-(k-1)}{Max\ iteration-1})^{1/\mu}$

$S =$  Mutation( $P_{(t)}$ )

*if  $X_{new}$  dominated on  $X_{(t)}$  and ( $rand < A$ )*

$P_{(t)} = S$

End

End

Discover the non-dominated answers.

Revise the collection pertaining to the acquired solutions that were not dominated.

Should the repository be at capacity

Execute the grid procedure to exclude one of the existing repository users.

Incorporate the new answer into the repository.

end if

If one of the newly added answers to the repository is situated far from the hypercube

Revise the diagrams to encompass the most recent solution(s).

end if

Inc

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rease r and reduce A

Set  $k := k + 1$ ;

End While

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## **Eighth: Modelling the previous study and Examination**

### **Eighth. A: Performance Measures**

The EPNGSA algorithm is derived from numerical relationships and evaluated against other algorithms. Pareto diagrams illustrating the outcomes of significant comparisons are presented. The statistical notions of gravitational distance (GD), inverted gravitational distance (IGD), and hyper volume (HV) [8] facilitate measurable assessments. Gravitational variation, often abbreviated as GD. The use of the term metric is appropriate when discussing the (GD) [26] and [27], as it examines the standard divisions of the structure of sets of Q about P: specifically, in assessing the potential inclusion of agreements of Q within the configuration of  $P^*$ .

$$GD = \frac{\sqrt[p]{\sum_{n=1}^R (d_i^p)}}{R} \quad \dots (14)$$

$$IGD = \frac{1}{R} (\sum_{n=1}^R \min ( \sqrt[p]{\sum_{n=1}^R (d_i^p)} )) \quad \dots (15)$$

The inverted accessible distance (IGD) represents the average distance between every possible solution in the dataset under Examination and the nearest fluid in the nearest remedy set, thereby indicating the expansion of alternatives. The IGD estimation, often referred to as the Pareto front (PF)  $P^*$  Assessment of measures the commonality of movement of picked up structures by examining the concept of dispersion and improvement.

The hyper quantity designation (Hyper size) (A) assesses the extent of the objective space that a PF approximation effectively occupies. The hyper quantity  $v^*$  It functions as a benchmark and signifies a general goal with a limit.  $v^*$  is characterised as potentially the most shockingly low measured values observed in A (for instance)  $v^*$  It is identified as one of A's most significant exceptionally small self- and outside-views (for instance) (A), and the hyper amount is expressed as:

$$HV(A) = \Lambda(\cup \{x | a < x < v^*, a \in A\}) \quad \dots (16)$$

Table 1 provides a list of the lingering repercussions of the IGD. Results obtained through the utilisation of hyper volume HV are presented in the additional table. To conclude, the p-estimation is shown in the very last column. The EPNGSA and other methods were subjected to two consecutive matched t-tests, and the first test's results revealed a substantial distinction between the two procedures. The deviation between the PF valid and the PF approximated should be calculated for each of the five assumptions being considered, as shown in Figure 1.

### **Eighth. B: Multi-Objective Test Functions**

Six comparative analysis functions [28] are employed and analysed to assess the suggested HGSABAT algorithm, showcasing its efficacy. The benchmarking tasks to evaluate the test ability of nonlinear systems are presented in detail in demonstration 1.

### **Eighth. C: Decision space**

The spectrum of choices illustrates the range of options available to us. The options we present will be the exclusive determinant in establishing the weights of the criterion. A comparable challenge within the decision-making framework can be characterised as an outcome. For example, during the item development process, we make choices regarding the design specifications, particularly about cost, and each of these choices influences the performance metrics we use for evaluation. A valid Pareto front represents an analysis of statistics used to identify the most significant options from various possibilities.

## **Eighth. D: Results and Discussion**

The topic of this paper centres on the precision of the technique presented. The proposed method (HGSABAT) is executed in MATLAB, and the duration of authorisation varies based on the topic at hand, ranging from a few mere seconds to within a second. Various elements are employed for testing, including the overall population ( $n=20, 40, 80, 160, \text{ and } 200$ ), the count of goals  $M$ , and the dimension  $D$ .

The experiment's results have validated the effectiveness of the proposed method in achieving a balance between accessibility and diversity. The extensive investigations demonstrated the effectiveness of an academic

**Table 1.** Comparative Results of HGSABAT, GSA, PSO, BAT and MSE for the Model one When  $\beta_1=500$ ,  $\beta_2=1$  and  $\beta_3=1$ 

N	Methods	Statics	$\beta_1$	$\beta_2$	$\beta_3$	MSE
20	HGSABAT	Estimated	518.3291709	0.910730555	1.05E+00	2.24E-12
		MSE	3.02E+03	2.05E-01	2.34E-02	
	GSA	Estimated	5.246E+02	1.227E+00	1.588E+00	1.417E+03
		MSE	1.056E+04	6.129E-01	3.423E-02	
	PSO	Estimated	6.145E+02	1.480E+00	1.506E+00	2.246E+01
		MSE	9.176E+03	1.766E-01	1.424E-03	
	BAT	Estimated	5.773E+02	1.404E+00	1.515E+00	1.640E+03
		MSE	22289.82665	1.069844882	0.034858133	
40	HGSABAT	Estimated	498.5497421	1.002126615	1.00E+00	9.897E+03
		MSE	2.00E+01	1.88E-05	1.68E-07	
	GSA	Estimated	5.455E+02	1.273E+00	1.543E+00	4.814E+02
		MSE	4.556E+03	3.682E-01	1.125E-02	
	PSO	Estimated	5.774E+02	1.480E+00	1.503E+00	1.031E+04
		MSE	5.107E+04	8.934E-02	4.435E-04	
	BAT	Estimated	6.025E+02	1.741E+00	1.489E+00	9.897E+03
		MSE	2.958E+04	9.598E-01	6.022E-03	
80	HGSABAT	Estimated	509.6823808	0.938666734	1.01E+00	2.14E+01
		MSE	2.91E+02	8.66E-02	1.29E-03	
	GSA	Estimated	5.070E+02	1.467E+00	1.509E+00	4.128E+0
		MSE	1.013E+04	1.928E-01	1.454E-03	
	PSO	Estimated	6.273E+02	1.429E+00	1.506E+00	3.673E+03
		MSE	1.213E+04	4.982E-02	4.006E-04	
	BAT	Estimated	4.484E+02	1.569E+00	1.497E+00	2.153E+05
		MSE	1.793E+05	9.103E-02	3.601E-04	
160	HGSABAT	Estimated	4.68E+02	1.004616382	1.00E+00	1.00E-12
		MSE	1.02E+04	3.24E-03	1.72E-05	
	GSA	Estimated	5.590E+02	1.154E+00	1.570E+00	9.830E+16
		MSE	1.532E+04	6.446E-01	2.749E-02	
	PSO	Estimated	5.192E+02	1.472E+00	1.501E+00	1.681E+16
		MSE	3.426E+04	6.284E-02	1.098E-04	
	BAT	Estimated	1.730E+02	1.406E+00	1.507E+00	7.912E+16
		MSE	5.590E+02	1.154E+00	1.570E+00	
200	HGSABAT	Estimated	480.8337505	1.048992482	1.00E+00	5.81E-08
		MSE	3.06E+04	1.36E-01	4.62E-04	
	GSA	Estimated	6.512E+02	1.681E+00	1.495E+00	1.350E+04

		MSE	<b>7.990E+04</b>	<b>3.787E-01</b>	<b>5.517E-04</b>	
	<b>PSO</b>	Estimated	<b>4.666E+02</b>	<b>1.457E+00</b>	<b>1.506E+00</b>	<b>6.960E+09</b>
		MSE	<b>2.069E+04</b>	<b>1.705E-01</b>	<b>6.726E-04</b>	
	<b>BAT</b>	Estimated	<b>2.890E+02</b>	<b>1.540E+00</b>	<b>1.500E+00</b>	<b>9.933E+08</b>
		MSE	<b>1.400E+05</b>	<b>1.070E-01</b>	<b>3.570E-04</b>	

The subject's policy is to balance being close and variety. Conversely, experts have created a variety of mathematical estimation methods for non-linear regression calculations using combined algorithms.

This section begins with an announcement regarding the MSE presentation, highlighting the application of the new HGSABAT computation alongside other PSO and BAT mathematical calculations.

The initial simulation (Meyer (7)) of Non-Linear Regression, which is alongside the set of variables ( $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ ) = (600,1.5,1.5) and (700,2,1), is the basis for the methods used (BAT and PSO). Tables 1 and 2 show the average square error of the Estimate obtained by different calculations in equivalent requirements. In particular, blue represents the best-estimated attributes.

According to these data tables, the HGSABAT algorithm outperforms other approaches in terms of validity for all samples, given that it has a lower Mean Square Error.

Several specifications for design have been utilised to examine the viability of the hybrid HGSABAT approach. An algorithm's actual worth becomes apparent when contrasted with other approaches that address the same issue. The novel HGSABAT algorithm is compared to the GSA, PSO, and BAT algorithms using the MSE criterion. The same basic functions solve all algorithms, employing deviation values.

The efficiency of the completely novel HGSABAT algorithm with the GSA, PSO, as well as BAT techniques is contrasted to the MSE measure with the parameters of Meyer (7), Meyer (4), Militky (4), Militky (2), Misra 1d, and MGH09 in Tables 1 and 2. The novel HGSABAT technique has a standard mean effectiveness in determining its median and computational mean in these locations. In contrast, the rank and the stronger are the other computational methods compared to it in the standards.

**Table 2.** Contrasting Findings of HGSABAT, GSA, PSO, BAT and MSE for Model one When  $\beta_1=700$ ,  $\beta_2=2$  and  $\beta_3=1$ 

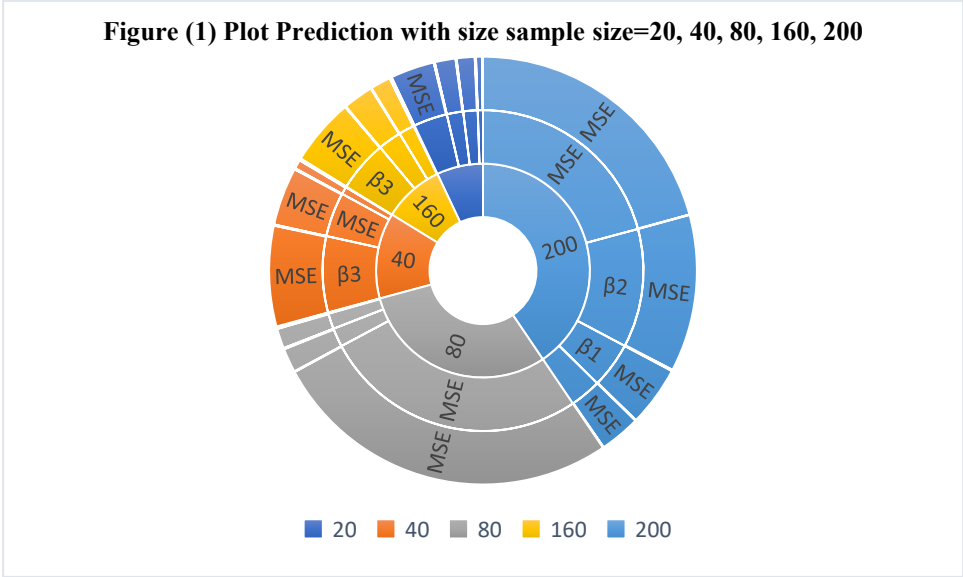
		700		2	1		
N	Methods	Statics	$\beta_1$	$\beta_2$	$\beta_3$	MSE	
20	HGSABAT	Estimated	703.9904304	1.753074048	1.01E+00	1.20E-13	
		MSE	1.03E+02	8.22E-01	1.16E-03		
	GSA	Estimated	600.3625531	1.10E+00	1.11E+00	1.80E+03	
		MSE	2.34E+04	1.02815145	0.026610648		
	PSO	Estimated	635.2227859	1.567576717	1.030009231	4.58E-05	
		MSE	49145.35846	0.671402495	0.00684904		
	BAT	Estimated	724.198291	1.607755894	1.040191683	5493.084317	
		MSE	62253.89049	1.06374377	0.006128501		
40	HGSABAT	Estimated	521.8442124	1.774335711	1.01E+00	1.22E-11	
		MSE	3.80E+03	1228E-01	2.24E-03		
	GSA	Estimated	629.8379786	1.05E+00	1.09E+00	3.19E+02	
		MSE	1.13E+03	1.041648181	0.01047274		
	PSO	Estimated	700.8884558	1.932221546	1.003110728	5.72E-10	
		MSE	7.89232288	0.045936859	9.68E-05		
	BAT	Estimated	650.9819254	4.876559547	1.007771297	1074.122974	
		MSE	35142.48365	94.57285581	0.009524098		
80	HGSABAT	Estimated	713.9606908	1.81088193	1.01E+00	1.28E-12	
		MSE	1.84E+02	2.47E-01	1.04E-02		
	GSA	Estimated	606.9944184	1.45E+00	1.04E+00	1.39E+03	
		MSE	18911.37891	0.568493155	0.004845854		
	PSO	Estimated	728.174662	1.999545506	1.000011216	4.78E-09	
		MSE	7938.114346	2.06E-06	1.25E-09		
	BAT	Estimated	753.6403571	1.974549382	1.002685229	295.7799145	
		MSE	90555.28186	0.497655844	0.001535721		
160	HGSABAT	Estimated	6.86E+02	1.928703	1.00E+00	1.21E+07	
		MSE	2.80E+04	2.53E-02	4.57E-05		
	GSA	Estimated	5.94E+02	1.29E+00	1.05E+00	1.45E+10	
		MSE	2.25E+04	0.696924646	0.003672247		

	PSO	Estimated	540.8003817	1.827364378	1.002236921	5157450.704
		MSE	28525.36058	0.018317116	2.36E-04	
	BAT	Estimated	604.2171801	1.604378484	1.01242594	5.03E+076
		MSE	101210.2331	0.432440034	0.001652577	
200	HGSABAT	Estimated	540.2090806	1.729504441	1.01E+00	1.03E-12
		MSE	1.16E+05	2.92E-01	8.02E-04	
	GSA	Estimated	546.3144824	1.72E+00	1.01E+00	3.99E+07
		MSE	3.11E+04	0.259405223	0.000704578	
	PSO	Estimated	720.7870626	1.684650089	1.019423862	118.6986807
		MSE	2232.371458	0.415247309	0.002021409	
	BAT	Estimated	422.9555071	1.596857927	1.01846512	1.73E+06
		MSE	182340.8134	0.25715727	0.000531839	

### Ninth: CONVERGENCE GRAPHS

Convergence charts were developed to illustrate the rate at which the traditional evaluation merges with the total number of repeats for samples similar to the examples. One hundred thousand runs were made for every collection. The efficacy of our proposed hybrid strategy in achieving the ideal value faster is seen in these subsequent images.

Figure (1) provides a clearer example of the comparative analysis of several Non-Linear Regression models using various sample sets for  $n=20, 40, 80, 160,$  and  $200$ . These statistics demonstrate that, compared to different designs, meta-heuristic algorithms provide the lowest MSE values. However, regarding precision, the comparison between GSA, BAT, and PSO showed that PSO was superior.



**Figure 1.** Comparative HGSABAT, BAT, PSO and GSA algorithms for Model one when the sample size is 20, 40, 60, 160, 200

## Conclusion and Future Work

Non-linear Regression in the context of the sequential improvement issue has been the focus of this paper's study. First, we provide a sectional summary of the study done in each chapter. By doing this, we describe our approach to address the difficulties with non-linear Regression, which was discussed at the outset of the thesis. Next, we provide guidelines for a detailed discussion of our hybrid algorithm (HGSABAT). Secondly, three Meta-heuristic methods were applied as an alternate method for estimating nonlinear regression models. Six distinct types of non-linear regression models with varying parameters were employed. Several samples (20, 40, 160, and 200) were also used. Finally, more tests and examinations of the suggestions are essential. The results of this study suggest that there are several ways to construct hybrid algorithms. These include classification options, multifunctional issues, and the HGSABAT algorithms. The findings of this method might be compared to other popular optimisation techniques in future research.

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